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THE MIXED PARAMETRIC AND SELF-OSCILLATIONS: THE EFFECT OF NONLINEAR PARAMETRIC EXCITATION AND DELAYS ON THEM CAUSING SELF-OSCILLATIONS

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Abstract. In the paper, the mixed parametric and self-oscillations in the presence of a nonlinear parametric action and a delay in friction causing self-oscillations have been studied. For this purpose, a model of a frictional self-oscillating system interacting with an energy source of limited power was used. The solution of nonlinear differential equations of motion was constructed using the direct linearization method. The relations of nonstationary and stationary oscillation modes were obtained. By application of the Routh – Hurwitz criteria, the conditions for the stability of stationary oscillations were derived. The calculations were carried out in order to gain information about the dynamics of fluctuations. An analysis of the data showed that the amplitude-frequency dependence changed its behavior under the influence of the delay. The latter also led to the appearance of stability regions on the lower branches of the amplitude curves, and a dependence arose on the steepness of the energy source characteristic.

Keywords: nonlinearity, parametric excitation, delay, mixed oscillations, parametric oscillations, self-oscillations, method, linearization, energy source

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СМЕШАННЫЕ ПАРАМЕТРИЧЕСКИЕ И АВТОКОЛЕБАНИЯ: ДЕЙСТВИЕ НА НИХ НЕЛИНЕЙНОГО ПАРАМЕТРИЧЕСКОГО ВОЗБУЖДЕНИЯ И ЗАПАЗДЫВАНИЯ, ВЫЗЫВАЮЩЕГО АВТОКОЛЕБАНИЯ

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Аннотация. В работе изучены смешанные параметрические и автоколебания при наличии нелинейного параметрического воздействия и запаздывания в трении, вызывающего автоколебания. Для этого использована модель фрикционной автоколебательной системы, взаимодействующей с источником энергии ограниченной мощности. Решение нелинейных дифференциальных уравнений движения достигнуто методом прямой линеаризации. Получены соотношения между нестационарными и стационарными режимами колебаний. С применением критериев Рауса – Гурвица

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установлены условия устойчивости стационарных колебаний. Проведены расчеты с целью получения информации о динамике колебаний. Анализ полученных данных показал, что под действием запаздывания амплитудно-частотная зависимость изменяет свое поведение. Запаздывание приводит также к появлению участков устойчивости на нижних ветвях амплитудных кривых, и возникает зависимость от крутизны характеристики источника энергии.

Ключевые слова: нелинейность, параметрическое возбуждение, запаздывание, смешанные колебания, параметрические колебания, автоколебания, линеаризация

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Introduction

A large number of studies have considered various oscillatory processes. According to the well-known classification, oscillation theory distinguishes four types of oscillations, which form four classes of mixed oscillations (interactions between types) [1–3]. One of these classes comprises mixed parametric oscillations and self-oscillations.

The goal of this study is to analyze mixed parametric oscillations and self-oscillations, accounting for three factors: nonlinear parametric excitation, time delay in friction that causes self-oscillations, and the limited power of the energy source.

Energy consumption has become increasingly important in the modern world, and time delays can occur in many systems [4–10].

Parametric oscillations, described mathematically by differential equations with periodic coefficients, can arise in various systems. A comprehensive study of parametric oscillations under quadratic parametric excitation is provided in [11].

In this paper, the solution of the nonlinear differential equations for mixed parametric oscillations and self-oscillations is constructed using linearization and the change of variables with averaging [17].

Equations of motion

Let us consider mixed parametric oscillations and self-oscillations adopting a commonly used model of a self-oscillating system [1, 2, 15]. In this model, a spring (with fixed ends) and a damper are connected to a body lying on a belt that moves continuously at a certain velocity V. The parameters c_0 and γ are the coefficients of the linear and nonlinear parts of the spring force, respectively; k_0 is the damping coefficient (all three parameters are constants). The velocity V is given by $V = r_0\dot{\varphi}$. The body has mass m and velocity $\dot{x}$ in the direction of motion x. The belt is driven by a motor with rotor angular velocity $\dot{\varphi}$ and torque characteristic $M(\dot{\varphi})$, which includes the resisting torque. A friction force $T(U)$ arises between the body and the belt, causing self-oscillations and depending on the relative velocity $U = r_0\dot{\varphi} - \dot{x}$.

The system of differential equations describing mixed parametric oscillations and self-oscillations under nonlinear parametric excitation and delay takes the following form:

$$\begin{gathered} m\ddot{x} + k_0\dot{x} + c_0 x = T(U_\Delta) - \gamma x^3 - bx^3\cos\nu t, \\ I\ddot{\varphi} = M(\dot{\varphi}) - r_0 T(U_\Delta), \end{gathered} \tag{1}$$

where $bx^3\cos(\nu t)$ is the term representing nonlinear parametric excitation (b = const, ν = const); r_0 is the radius at which the friction force is applied (r_0 = const); U_Δ is the relative velocity ($U_\Delta = r_0\dot{\varphi} - \dot{x}_\Delta$, $\dot{x}_\Delta = \dot{x}(t-\Delta)$, Δ = const is the delay); I is the moment of inertia of the motor rotor, I = const.

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The characteristic of the friction force $T(U_\Delta)$ is taken in the form

$$T(U_\Delta) = T_0(\operatorname{sgn} U_\Delta - \alpha_1 U_\Delta + \alpha_3 U_\Delta{}^3), \tag{2}$$

where T_0 is the normal reaction force, T_0 = const; the coefficients α_1 and α_3 are constants; $\operatorname{sgn}(U_\Delta) = +1$ for $U_\Delta > 0$, $\operatorname{sgn}(U_\Delta) = -1$ for $U_\Delta < 0$.

If $U_\Delta = 0$, the characteristic $T(0)$ lies in the range

$$-T_0 \le T(0) \le T_0.$$

Interestingly, characteristic (2) also occurred in space experiments [16].

Using the direct linearization method [17–20], we replace expression (2) with a function of the form

$$T(U_\Delta) = T_0[\operatorname{sgn}(U_\Delta) + B_T + k_T \dot{x}],$$

where the linearization coefficients B_T and k_T are determined by the expressions

$$\begin{gathered} B_T = -\alpha_1 u + \alpha_3 u^3 + 3N_2\alpha_3 u p^2 a^2,\ k_{\dot{O}} = -\alpha_3 \bar{N}_3(p^2a^2 - h),\ h = 3(u_0^2 - u^2)/\bar{N}_3, \\ u_0^2 = \alpha_1/3\alpha_3,\ N_2 = (2r+1)/(2r+3),\ \bar{N}_3 = (2r+3)/(2r+5), \\ a = max|x| \text{ is the amplitude.} \end{gathered} \tag{3}$$

Using the direct linearization method, we also replace the nonlinear part of the spring force γx^3 with a linear one $c_f x$ with the linearization coefficient $c_f = \bar{N}_3 \gamma a^2$. After linearization, Eqs. (1) take the form

$$\begin{gathered} m\ddot{x} + k_0\dot{x} + c\,x = T_0(B_{\dot{O}} + \operatorname{sgn} U_\Delta + k_T \dot{x}_\Delta) - bx^3 \cos\ t \\ I\ddot{\varphi} = M(\dot{\varphi}) - r_0 T_0(\operatorname{sgn} U_\Delta + B_T + k_T \dot{x}_\Delta), \end{gathered} \tag{4}$$

where $c = c_0 + c_f$.

Solution of the equations and stability conditions for stationary oscillations

The solutions of Eqs. (4) obtained by change of variables with averaging [17], taking into account the expression

$$\dot{x}_\Delta = -ap\sin(\psi - p\Delta),$$

take the following form:

$$x = a\cos\psi,\ \dot{x}_\Delta = -ap\sin\psi,\ \psi = pt + \xi,\ p = \nu/2,\ \dot{\varphi} = \Omega. \tag{5}$$

The solutions of Eqs. (4) differ for the cases $U_\Delta > 0$ and $U_\Delta < 0$. The equations for determining the nonstationary values of the amplitude a, oscillation phase ξ, and velocity u, given by $u = r_0\Omega$, take the following form.

a) $u \ge ap$:

$$\begin{aligned} \frac{da}{dt} &= -\frac{1}{4pm}(2aG - 0.5ba^3\sin 2\xi), \\ \frac{d\xi}{dt} &= \frac{1}{4pma}(2aW + ba^3\cos 2\xi), \\ \frac{du}{dt} &= \frac{r_0}{I}\left[M\left(\frac{u}{r}\right) - r_0 T_0(1 + B_{\dot{O}})\right]; \end{aligned} \tag{6a}$$

b) $u < ap$:

$$\frac{da}{dt} = -\frac{1}{4pm}\left[2aG - 0.5ba^3 \sin 2\xi - \frac{8T_0}{\pi ap}\sqrt{a^2p^2 - u^2}\right],$$

$$\frac{d\xi}{dt} = \frac{1}{4pma}(2aW + ba^3 \cos 2\xi), \tag{6b}$$

$$\frac{du}{dt} = \frac{r_0}{I}\left[M(\frac{u}{r}) - r_0T_0(1 + B_{\dot{O}}) - \frac{r_0T_0}{\pi}(3\pi - 2\psi_*)\right],$$

where $\omega_0^2 = c_0/m$, $G = p[k_0 - T_0k_T \cos(p\Delta)]$, $W = m(\omega_0^2 - p^2) + c_{\dot{O}} - pT_0k \sin p\Delta$, $\psi^* = 2\pi - \arcsin(u/ap)$.

The stationary values of the amplitude and phase are determined by the conditions $\dot{a} = 0$, $\dot{\xi} = 0$. For the velocity values $u \geq ap$, they are found from the relations

$$4(4G^2 + W^2) = b^2a^4, \text{ tg}(2\xi) = -2G/W,$$

and for $u < ap$, the stationary amplitude satisfies the relation $a \approx u/p$.

The load on the energy source from the oscillatory system is determined by the condition $\dot{u} = 0$, yielding the relation

$$M(u/r) - S(a, u) = 0,$$

where

a) $u \geq ap$: $S(a,u) = r_0T_0(1 + B_T)$;

b) $u < ap$: $S(a,u) = r_0T_0[(1 + B_T) + \pi^{-1}(3\pi - 2\psi)]$.

The stationary values of the velocity u can be found from the intersection points of the curves $M(u/r)$ and $S(a,u)$.

To derive the stability conditions for stationary oscillations, we compose equations in variations for Eqs. (6) and, using the Routh–Hurwitz criteria, we obtain

$$B_1 > 0, B_3 > 0, B_1B_2 - B_3 > 0, \tag{7}$$

where $B_1 = -(b_{11} + b_{22} + b_{33})$, $B_2 = b_{11}b_{33} + b_{11}b_{22} + b_{22}b_{33} - b_{23}b_{32} - b_{12}b_{21} - b_{13}b_{31}$, $B_3 = b_{11}b_{23}b_{32} + b_{12}b_{21}b_{33} + b_{22}b_{13}b_{31} - b_{11}b_{22}b_{33} - b_{12}b_{23}b_{31} - b_{13}b_{21}b_{32}$.

For $u \geq ap$, we obtain the following expressions:

$$b_{11} = \frac{r_0}{I}\left(Q - r_0T_0\frac{\partial B_{\dot{O}}}{\partial u}\right), \; b_{12} = -6I^{-1}r_0^2T_0N_2\alpha_3uap^2, \; b_{13} = 0,$$

$$b_{21} = -3m^{-1}T_0a\,\alpha_3u\cdot\cos(p\Delta),$$

$$b_{22} = -\frac{1}{4pm}\left[2p(k_0 + T_0\bar{N}_3\alpha_3(3a^2p^2 - h)\cos p\Delta) - \frac{3}{2}ba^2 \sin 2\xi\right],$$

$$b_{23} = \frac{ba^3}{4pm}\cos 2\xi, \; b_{31} = -3m^{-1}T_0\alpha_3u\cdot\sin(p\Delta),$$

$$b_{32} = \frac{1}{2pm}\left[\bar{N}_3a(\gamma + 2T_0\alpha_3p^3 \sin p\Delta) + ba\cos 2\xi\right], \; b_{33} = -\frac{ba^3}{2pma}\sin 2\xi,$$

$$\frac{\partial B_{\dot{O}}}{\partial u} = -(\alpha_1 - 3\alpha_3u^2 - 3\alpha_3N_2a^2p^2), \; \frac{\partial k_{\dot{O}}}{\partial a} = -2\bar{N}_3\alpha_3ap^2,$$

where $Q = \frac{d}{du}M(\frac{u}{r_0})$.

If $u < ap$, we obtain the following coefficients:

$$b_{11} = \frac{r_0}{I}\left[Q - r_0 T_0 \frac{\partial B_{\dot{O}}}{\partial u} - \frac{2r_0 T_0}{\pi\sqrt{a^2 p^2 - u^2}}\right],$$

$$b_{12} = -\frac{r_0^2 T_0}{I}\left[\frac{\partial B_{\dot{O}}}{\partial a} + \frac{2u}{\pi a\sqrt{a^2 p^2 - u^2}}\right],$$

$$b_{21} = -\frac{auT_0}{m}\left[3\alpha_3 \cos p\Delta - \frac{2}{\pi a^2 p^2\sqrt{a^2 p^2 - u^2}}\right],$$

$$b_{22} = -\frac{1}{4pm}\left[2p(k_0 + T_0\bar{N}_3\alpha_3(3a^2 p^2 - h)\cos p\Delta) - \frac{3}{2}ba^2 \sin 2\xi\right] - \frac{2T_0 u^2}{\pi m a^2 p^2\sqrt{a^2 p^2 - u^2}},$$

while the coefficients b_{13}, b_{23}, b_{31}, b_{33} are unchanged.

Calculations of stationary oscillation regimes

The following parameter values were chosen for the calculations:

$$m = 9.8\ \text{N}\cdot\text{s}^2\cdot\text{cm}^{-1},\ k_0 = 0.196\ \text{N}\cdot\text{s}\cdot\text{cm}^{-1},\ c_0 = \text{N}\cdot\text{cm}^{-1},\ b = 0.686\ \text{N}\cdot\text{cm}^{-1},$$

$$\gamma = \pm 1.96\ \text{N}\cdot\text{cm}^{-3},\ T_0 = 4.9\ \text{N},\ \alpha_1 = 0.84\ \text{s}\cdot\text{cm}^{-1},\ \alpha_3 = 0.18\ \text{s}^3\cdot\text{cm}^{-3},$$

$$r_0 = 1\ \text{cm},\ I = 9.8\ \text{N}\cdot\text{s}\cdot\text{cm}^2,\ p\Delta = \pi/2,\ \pi,\ 3\pi/2.$$

Some calculated results for $u = 1.2\ \text{cm}\cdot\text{s}^{-1}$ and different characteristics of the spring force ($\gamma > 0$, $\gamma < 0$) are shown in Fig. 1. Solid lines correspond to delay cases $p\Delta = \pi/2$, dashed lines to $p\Delta = \pi$, and dotted lines to $p\Delta = 3\pi/2$.

The shaded sectors correspond to values of the energy source characteristic steepness Q for which oscillations are stable. It would be more accurate to plot these sectors on the load curve $S(a,u)$, but for the sake of saving space, they are shown on the amplitude curves.

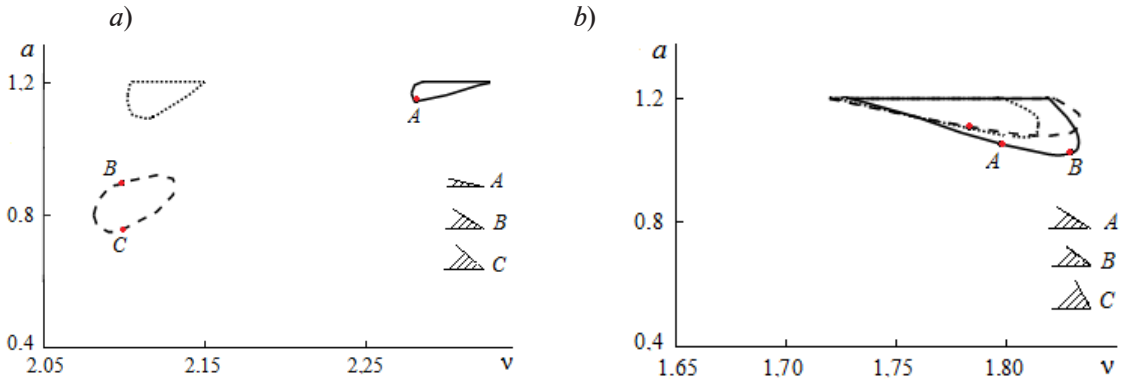


Fig. 1. Results of numerical calculations of the amplitude-frequency characteristics of the system under consideration for elastic forces with

Three cases are shown for the delay $p\Delta$:$\pi/2$: $\pi/2$ (solid lines), π (dashed lines), and $3\pi/2$ (dotted lines). The red points A, B, C correspond to the shaded sectors representing the ranges of the slope Q of the energy source characteristic within which oscillations are stable

Conclusion

We considered dynamics of mixed parametric oscillations and self-oscillations under nonlinear parametric excitation and time delay in friction that causes self-oscillations. A model of a frictional self-oscillating system interacting with a limited-power energy source was used to gain insight into the properties of these oscillations; calculations of stationary oscillations were performed.

Analysis of the obtained data led to the following conclusions.

First, the amplitude curve shifts significantly in the amplitude-frequency plane compared with the no-delay case, and the resonance region expands.

Second, stability regions depending on the slope of the energy source characteristic appear in some sections of the lower branches of the amplitude curves.

A number of effects that occur in the interaction of an oscillatory system with an energy source under linear parametric excitation are described in detail in monograph [1]. As the present study has shown, similar effects also occur under nonlinear parametric excitation.

REFERENCES

1. **Alifov A. A., Frolov K. V.,** Interaction of nonlinear oscillatory systems with energy sources; Hemisphere Publishing Corporation, New York, Washington, Philadelphia, London, 1990.

2. **Chelomey V. N.** (Chairman), Vibratsii v technike: spravochnik [Vibrations in machinery: Handbook], in 6 Vols., Vol. 2; Ed. by Blekhman I. I., Publishing House of Mechanical Engineering, Moscow, 1979 (in Russian).

3. **Andronov A. A., Vitt A. A., Khaikin S. E.,** Theory of vibration, Nauka Publishing, Moscow, 1981 (in Russian).

4. **Krasnoselskiĭ M., Pokrovskiĭ A.,** Systems with hysteresis, Springer-Verlag, Berlin, Heidelberg, New-York, 1989.

5. **Tkhan V. Z., Dementev Yu. N., Goncharov V. I.,** Improving the accuracy calculation of time delay automatic control, Software & Systems. 31 (3) (2018) 521–526 (in Russian).

6. **Keller A. A.,** Time-delay systems with application to mechanical engineering process dynamics and control, Math. Comput. Simul. 12 (2018) 64–73.

7. **Sykora H. T., Sadeghpour M., Ge J. I., et al.,** On the moment dynamics of stochastically delayed linear control systems, Int. J. Robust Nonlin. 30 (18) (2020) 8074–8097.

8. **Gupta S. K., Wang J., Barry O. R.,** Nonlinear vibration analysis in precision motion stage with PID and time-delayed feedback controls, Nonlinear Dyn. 101 (2) (2020) 439–464.

9. **Tsykunov A. M.,** Robust control of object with distributed delay and unknown order of mathematical model, Mekhatronika, Avtomatizatsiya, Upravlenie [Mechatronics., Automatization, Control]. 17 (4) (2016) 219–227 (in Russian).

10. **Rubanik V. P.,** Kolebaniya kvazilineynykh sistem s zapazdyvaniyem [Oscillations of quasilinear systems having delay], Nauka Publishing, Moscow, 1969 (in Russian).

11. **Schmidt G., Schulz R.,** Parametererregte Schwingungen, Deutscher Verlag d. Wiss., VEB, Berlin, 1975.

12. **Bogolyubov N. N., Mitropolsky Y. A.,** Asymptotic methods in the theory of non-linear oscillations, CRC Press, Florida, USA, 1961.

13. **Moiseev N. N.**, Asimptoticheskiye metody nelineynoy mekhaniki [Asymptotic methods of nonlinear mechanics], Nauka, Moscow, 1981 (in Russian).

14. **Butenin N. V., Neymark Yu. I., Fufayev N. A.,** Vvedeniye v teoriyu nelineynykh kolebaniy [Introduction to the theory of nonlinear oscillations], Nauka, Moscow, 1976) (in Russian).

15. **Klimov D. M.,** On one type of self-excited vibrations in a system with dry friction, Mech. Solids. 38 (3) (2003) 1–7.

16. **Bronovets M. A., Zhuravlev V. F.,** On self-excited vibrations in friction source measurement systems, Mech. Solids. 47 (3) (2012) 261–268.

17. **Alifov A. A.,** Metody pryamoy linearizatsii dlya rascheta nelineynykh system [Methods of direct linearization for calculation of nonlinear systems], Research Center "Regular and Chaotic Dynamics", Moscow, 2015 (in Russian).

18. **Alifov A. A.,** Method of the direct linearization of mixed nonlinearities, J. Mach. Manuf. Reliab. 46 (2) (2017) 128–131.

19. **Alifov A. A.,** About direct linearization methods for nonlinearity, Hu Z., Petoukhov S., He M. (Eds.); Conf. proc.: “Advances in Artificial Systems for Medicine and Education III (AIMEE 2019)” (Book Ser. “Advances in Intelligent Systems and Computing”. Vol. 1126). Springer, Cham, Switzerland (2020) 105–114.

20. **Alifov A. A.,** On the calculation by the method of direct linearization of mixed oscillations in a system with limited power-supply, Hu Z., Petoukhov S., Dychka I., He M. (Eds.); Conf. proc.: “Advances in Computer Science for Engineering and Education II (ICCSEEA 2019)” (Book Ser. “Advances in Intelligent Systems and Computing”. Vol. 938). Springer, Cham, Switzerland (2020) 23–31.

СПИСОК ЛИТЕРАТУРЫ

1. **Alifov A. A., Frolov K. V.** Interaction of nonlinear oscillatory systems with energy sources. New York, Washington, Philadelphia, London: Hemisphere Publishing Corporation, 1990. 327 p.

2. Вибрации в технике. Справочник: В 6-ти т. Ред. совет: В. Н. Челомей (пред., гл. ред.) и др. Т. 2. Колебания нелинейных механических систем. Под ред. И. И. Блехмана. М.: Машиностроение, 1979. 351 с.

3. **Андронов А. А., Витт А. А., Хайкин С. Э.** Теория колебаний. М.: Наука. Гл. ред. физ.-мат. лит. 1981. 568 с.

4. **Красносельский М. А., Покровский А. В.** Системы с гистерезисом. М.: Наука, 1983. 272 с.

5. **Тхан В. З., Дементьев Ю. Н., Гончаров В. И.** Повышение точности расчета систем автоматического управления с запаздыванием // Программные продукты и системы. 2018. Т. 31. № 3. С. 521–526.

6. **Keller A. A.** Time-delay systems with application to mechanical engineering process dynamics and control // International Journal of Mathematics and Computers in Simulation. 2018. Vol. 12. Pp. 64–73.

7. **Sykora H. T., Sadeghpour M., Ge J. I., Bachrathy D., Orosz G.** On the moment dynamics of stochastically delayed linear control systems // International Journal of Robust and Nonlinear Control. 2020. Vol. 30. No. 18. Pp. 8074–8097.

8. **Gupta S. K., Wang J., Barry O. R.** Nonlinear vibration analysis in precision motion stage with PID and time-delayed feedback controls // Nonlinear Dynamics. 2020. Vol. 101. No. 2. Pp. 439–464.

9. **Цыкунов А. М.** Робастное управление объектом с распределенным запаздыванием и неизвестным порядком математической модели // Мехатроника, автоматизация, управление. 2016. Т. 17. № 4. С. 219–227.

10. **Рубаник В. П.** Колебания квазилинейных систем с запаздыванием. М.: Наука, 1969. 288 с.

11. **Шмидт Г.** Параметрические колебания. М.: Мир, 1978. 336 с.

12. **Боголюбов Н. Н., Митропольский Ю. А.** Асимптотические методы в теории нелинейных колебаний. М.: Наука, 1974. 504 с.

13. **Моисеев Н. Н.** Асимптотические методы нелинейной механики. М.: Наука, 1981. 400 с.

14. **Бутенин Н. В., Неймарк Ю. И., Фуфаев Н. А.** Введение в теорию нелинейных колебаний. М.: Наука, 1976. 256 с.

15. **Климов Д. М.** Об одном виде автоколебаний в системе с сухим трением // Известия Российской академии наук. Механика твердого тела. 2003. № 3. С. 6–12.

16. **Броновец М. А., Журавлёв В. Ф.** Об автоколебаниях в системах измерения сил трения // Известия Российской академии наук. Механика твердого тела. 2012. № 3. С. 3–11.

17. **Алифов А. А.** Методы прямой линеаризации для расчета нелинейных систем. Москва–Ижевск: НИЦ «Регулярная и хаотическая динамика». 74 .2015 с.

18. **Alifov A. A.** Method of the direct linearization of mixed nonlinearities // Journal of Machinery Manufacture and Reliability. 2017. Vol. 46. No. 2. Pp. 128–131.

19. **Alifov A. A.** About direct linearization methods for nonlinearity // Hu Z., Petoukhov S., He M. (Eds.) Conference proceedings: “Advances in Artificial Systems for Medicine and Education III (AIMEE 2019)” (Book Series “Advances in Intelligent Systems and Computing”. Vol. 1126). Cham, Switzerland: Springer, 2020. Pp. 105–114.

20. **Alifov A. A.** On the calculation by the method of direct linearization of mixed oscillations in a system with limited power-supply // Hu Z., Petoukhov S., Dychka I., He M. (Eds.). Conference proceedings: “Advances in Computer Science for Engineering and Education II (ICCSEEA 2019)” (Book Series “Advances in Intelligent Systems and Computing”. Vol. 938). Cham, Switzerland: Springer, 2020. Pp. 23–31.

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