Model of orthogonal two-wave mixing in photorefractive crystal of cubic symmetry with optical activity

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Abstract. We developed a physical model describing the process of two-wave vectorial mixing in optically active photorefractive crystal of cubic symmetry for an orthogonal scheme of interaction. Using the model, we calculated the two-wave interaction in a photorefractive crystal of bismuth silicate $Bi_{12}SiO_{20}$ having optical activity. We have determined conditions at that polarization changes don't influence two-wave mixing. It was found that it is possible to define the parameters of the crystal and interacting waves for the quasi-polarization independence mode, when changes of interferometer output signal caused by polarization instability of the signal wave is reduced to a minimum (no more than 3%). We developed a physical model describing vectorial two-wave mixing in optically active, cubic-symmetry photorefractive crystals for an orthogonal interaction geometry. We apply the model to bismuth silicate $(Bi_{12}SiO_{20})$, an optically active photorefractive crystal. We further show that by appropriately selecting crystal and wave parameters, a quasi-polarization-independent regime can be achieved, in which interferometer output fluctuations caused by signal-wave polarization instability are minimized to no more than 3%.

Keywords: photorefractive crystal, adaptive interferometer, dynamic hologram, polarization, optical activity

Модель ортогонального двухволнового взаимодействия в фоторефрактивном кристалле кубической симметрии с оптической активностью

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Аннотация. Разработана физико-математическая модель, описывающая процесс двухволнового векторного взаимодействия в оптически активном фоторефрактивном кристалле кубической симметрии для ортогональной геометрии взаимодействия волн. С использованием модели выполнен расчет двухволнового взаимодействия

в фоторефрактивном кристалле силиката висмута $Bi_{12}SiO_{20}$, обладающем оптической активностью. Определены параметры фоторефрактивного кристалла и взаимодействующих волн для режима квази-поляризационной независимости, при котором изменения выходного сигнала интерферометра, вызванные поляризационной нестабильностью сигнальной волны, минимальны (не более 3%).

Ключевые слова: фоторефрактивный кристалл, адаптивный интерферометр, динамическая голограмма, поляризация, оптическая активность

Introduction

Adaptive holographic interferometers based on dynamic holographic gratings recorded in photorefractive crystals (PRCs) are effective tools for phase demodulation [1−10]. Continuous re-recording of the hologram in the PRC stabilizes the interferometer's operating point, while preserving high sensitivity and providing strong immunity to external noise [2, 4, 6]. However, as in traditional interferometers, the sensitivity of adaptive interferometers depends on the polarization of the light beams. Polarization fluctuations can significantly reduce, or even eliminate the signal, underscoring the need for an interferometer design whose output is insensitive to polarization.

This problem can be addressed by dependencies in combination with an orthogonal wave-mixing geometry. The authors previously developed a physical and mathematical model of vectorial two-wave mixing in photorefractive crystals with optical gyrotropy [11]. The model captures polarization evolution arising not only from the PRC's gyrotropy but also from the two-wave mixing process within the crystal. It enables more precise selection of the interacting waves' polarization parameters and the PRC dimensions, accounting for the crystal's material properties and the light's wavelength, which can yield polarization-independent operation of the adaptive interferometer. This paper presents calculations of two-wave vector interaction in the optically active PRC bismuth silicate $Bi_{12}SiO_{20}$ and identifies parameter regimes in which polarization fluctuations of the interacting waves do not degrade the stability of the adaptive holographic interferometer's output signal.

Model Overview

The developed physical and mathematical model builds on the theory of vectorial wave mixing in cubic photorefractive crystals for transmission geometry described by Sturman et al. [12]. A more detailed description of the model is provided in [11]. Its key elements are summarized here. Unlike collinear wave interaction, the orthogonal geometry has signal and reference waves propagating at right angles, producing spatially varying polarization states throughout the PRC. To address this, the model divides the intersection region of the signal wave *As* and the reference wave *Ar* within the PRC into an $M \times N$ grid of cells. The grid size was selected to ensure grid independence; 100×100 cells were sufficient. Further refinement did not change the results within numerical tolerance but significantly increased computation time. The signal and reference waves entering the cells of the first layer are defined as follows:

$$As_{m1}^{in} = As / N, \; Ar_{1n}^{in} = As / M. \tag{1}$$

Consider a cell (m, n), where m ranges from 0 to M and n from 0 to N (see Fig. 1). The amplitudes of the interacting waves entering this cell are equal to the amplitudes of the waves exiting the preceding cell:

$$As_{mn}^{in} = As_{m-1n}^{out}, \ Ar_{mn}^{in} = Ar_{mn-1}^{out}. \tag{2}$$

Within each cell, vector two-wave mixing is computed. The polarization states of the waves As and Ar change due to both the crystal's optical gyrotropy and the mixing interaction. The waves then propagate to the next cells, $(m+1, n)$ and $(m, n+1)$, respectively, where the procedure is repeated. Because the measured information is encoded in the phase modulation φ of the signal wave As, its complex amplitude is written as $As \cdot e^{i\varphi}$. The amplitude of the signal wave at the output of cell (m, n) is then obtained from the two-wave mixing relations given by [11, 12]:

$$As_{mn}^{out}(\varphi) = (\hat{T}_{+}(y) As_{m-1n}^{out} e^{i\varphi} + e^{\pm i\varphi_0} \hat{T}_{-}(y) Ar_{mn-1}^{out}) e^{-\alpha \Delta L/2}, \tag{3}$$

where y denotes the propagation direction of the signal wave As, ΔL is the cell size, and α denotes the optical absorption coefficient of the PRC. The terms $\hat{T}_{+}$ and $\hat{T}_{-}$ are transformation matrices that account for the diffraction of wave As in the direction of M (zero-order diffraction) and the diffraction of reference wave Ar in the direction of M, respectively. The interferometer's output signal S is defined as the change in the intensity of the signal wave As at the exit of the "final cell layer" (where $m = M$), summed along the N direction:

$$S = \Sigma_{n-1}^{N} \left(As_{mn}^{out}(\varphi) \right)^2 - \left(As_{mn}^{out} \right)^2. \tag{4}$$
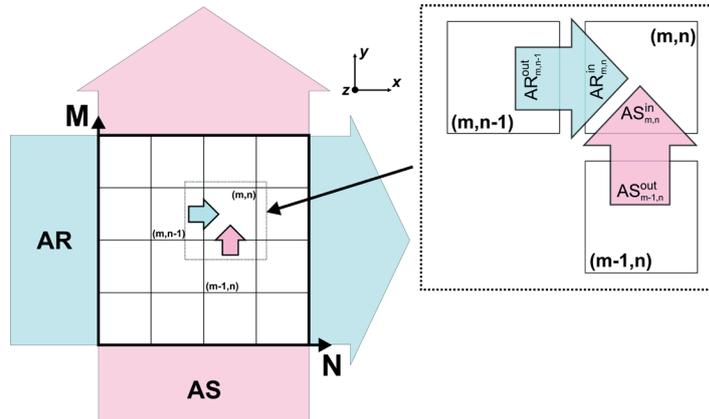

Fig. 1. Scheme of propagation of signal and reference waves in a photorefractive crystal

Results and Discussion

A physical and mathematical model was investigated using a gyrotropic photorefractive crystal of bismuth silicate $Bi_{12}SiO_{20}$ (BSO) of cubic symmetry. Numerical simulations were performed to study vectorial two-wave mixing in the PRC at wavelengths of 532 and 633 nm, for which the optical activity of BSO is 350 and 220 deg/cm, respectively. A linearly polarized signal wave As and a circularly polarized reference wave Ar were launched into the PRC. The developed model allowed for the investigation of the polarization independence of the BSO crystal.

Fig. 2 shows the dependence of the interferometer output signal on the azimuthal angle γ of the signal wave As for BSO crystal of varying lengths (1−30 mm), computed using the developed model Results are given for wavelengths of 532 and 633 nm. The simulations indicate that at certain crystal lengths L_p, the influence of polarization changes of As on the output signal is minimized. Thus, for wavelength 532 nm (Fig. 2, a), L_p occurs at 5.4, 10.7, 16.1 mm, etc. (i.e., multiples of 5.4 mm). For wavelength 633 nm (Fig. 2, b), L_p occurs at 8.7, 17.4, 26.0 mm, etc. (i.e., multiples of 8.7 mm). At these lengths, the variation of the output signal with γ is, on average, about 3% of its maximum possible modulation. Thus, a regime close to polarization independence (PI-mode) is achieved at a crystal length of L_p. However, complete polarization independence − where the
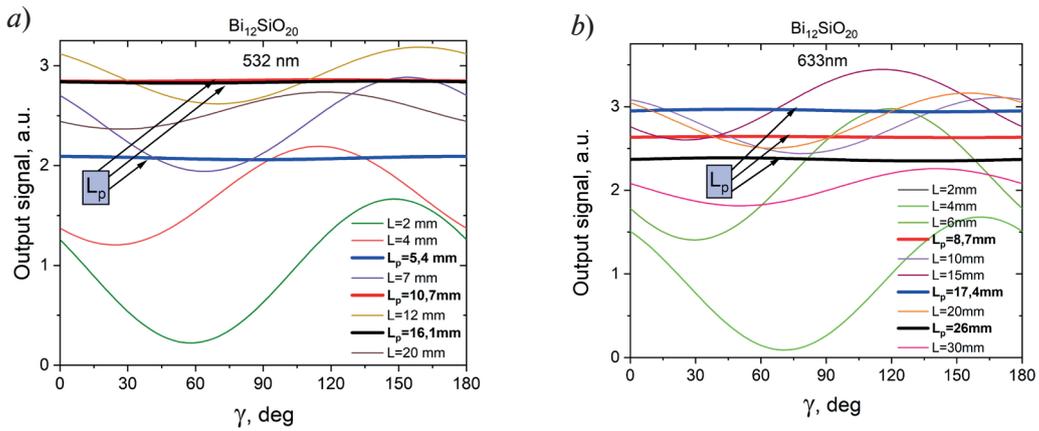
Fig. 2. Dependences of the output signal on the azimuthal angle of the signal wave, obtained for the BSO crystal at wavelength $\lambda = 532$ nm (*a*) and $\lambda = 633$ nm (*b*)

output signal is fully independent of the wave and crystal parameters − is not attainable, likely due to the nonlinear nature of light interaction within the PRC.

Fig. 3 shows the dependence of the interferometer output signal on the length of PRC BSO, at various azimuthal angles γ of the signal wave *As*, at wavelengths of 532 and 633 nm. The maximum and minimum achievable signal levels are indicated by red and blue lines, respectively. The plot shows that increasing the crystal length raises both the minimum and maximum signal levels. Thus, the output signal peaks when the crystal length is $L_{opt} = 13.2$ mm for wavelength 532 nm (Fig. 3, *a*) and $L_{opt} = 13$ mm for wavelength 633 nm (Fig. 3, *b*), achieved with a linearly polarized signal wave *As* oriented at $\gamma = 140°$. At crystal lengths of $L_p = 5.4$ mm and 10.7 mm for 532 nm, and $L_p = 8.7$ mm and 17.4 mm for 633 nm, the difference ΔS between the maximum and minimum achievable output signals is minimized to about 3% of the maximum possible signal. This implies that, regardless of the input polarization state of *As*, the output signal varies only within this narrow range.
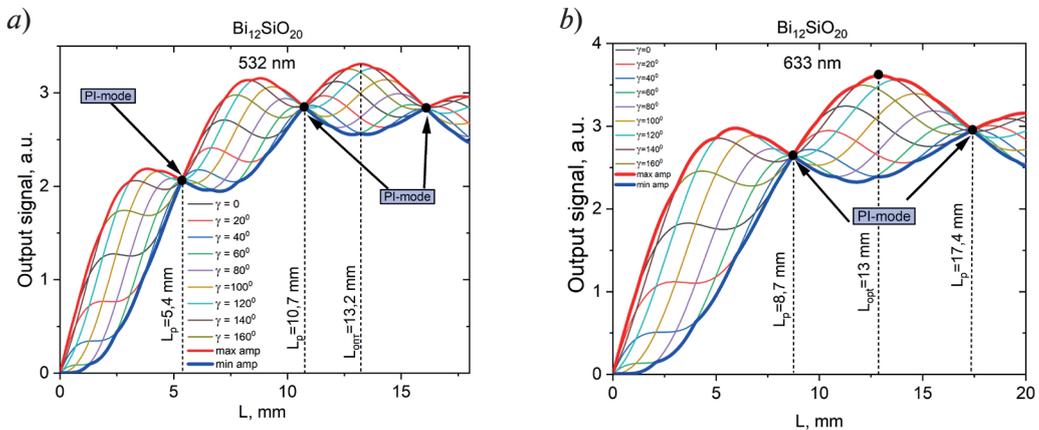

Fig. 3. Dependences of the interferometer output signal on the crystal length, obtained at different azimuthal angles of the signal wave $\lambda = 532$ nm (*a*), $\lambda = 633$ nm (*b*)

Conclusion

This paper presents a physical model for vectorial two-wave mixing in optically active photorefractive crystals with cubic symmetry, applied to an orthogonal interferometer configuration. Using this model, we analyzed two-wave mixing in a gyrotropic bismuth silicate ($Bi_{12}SiO_{20}$, BSO) photorefractive crystal at wavelengths of 532 nm and 633 nm. The results show that complete polarization independence is not achievable; however, by appropriate choice of crystal and wave parameters, signal variations due to polarization instability of the signal wave can be reduced to below 3%. We identify a quasi-polarization-independent operating regime at crystal lengths of 5.4 mm for 532 nm and 8.7 mm for 633 nm. In addition, the optimal crystal length was found to be approximately 13 mm, which maximizes the two-wave mixing efficiency.

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