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ACTIVE CONTROL OF BENDING VIBRATIONS OF TIMOSHENKO BEAMS USING STATE OBSERVERS

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Abstract. When describing bending vibrations of elastic beams, the transition from the Bernoulli – Euler model to the Timoshenko model leads to a complication in the dynamic behavior of the beam and to the emergence of new dynamic effects and a new spectrum of vibration modes. The aim of this study is to test control approaches developed for Bernoulli – Euler beams, for thicker beams described by the Timoshenko model, and to study the influence of beam thickness on the efficiency of such approaches. For this purpose, the problem of active damping of forced bending vibrations of simply supported metal beams has been studied numerically using control systems with state observers, where point forces or moments serve as control inputs. It was shown that the proposed approach remained effective for the vibration control problem of Timoshenko beams at lower modes over a wide thickness range of the considered beams.

Keywords: Bernoulli – Euler beam, Timoshenko beam, active vibration control, state observer

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АКТИВНОЕ ГАШЕНИЕ ИЗГИБНЫХ КОЛЕБАНИЙ БАЛОК ТИМОШЕНКО С ИСПОЛЬЗОВАНИЕМ НАБЛЮДАТЕЛЕЙ СОСТОЯНИЯ

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Аннотация. При описании изгибных колебаний упругих балок переход от модели Бернулли – Эйлера к модели Тимошенко ведет к усложнению динамического поведения балки, появлению новых эффектов и новых форм колебаний. Цель работы – протестировать подходы к управлению, разработанные для балок Бернулли – Эйлера, в применении к более толстым балкам, которые описываются моделью Тимошенко, и исследовать влияние толщины балок на эффективность таких подходов. Для этого проведен численный анализ задачи активного гашения вынужденных колебаний шарнирно-опертых металлических балок с помощью систем управления с наблюдателями состояния, где в качестве управляющих воздействий выступают сосредоточенные силы или моменты. Показано, что для задачи гашения колебаний балки Тимошенко по низшим формам предложенный подход остается эффективным для широкого диапазона значений толщины рассматриваемых балок.

Ключевые слова: балка Бернулли – Эйлера, балка Тимошенко, активное гашение колебаний, наблюдатель состояния

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Introduction

While the Bernoulli–Euler model well describes the bending vibrations in thin beams, more complex models are required to describe the dynamics of relatively thick beams. The Timoshenko model differs from the Bernoulli–Euler model by accounting for the shear in beam cross-sections: this means that these cross-sections are not necessarily perpendicular to the beam centerline (the cross-sections remain flat in both models). Additionally, the Timoshenko model accounts for rotational inertia of the beam cross-sections. There are also intermediate models accounting for only one of these two effects (shear or rotational inertia of the cross-sections), but they are inferior to the Timoshenko model in accuracy and are used much less frequently.

The transition from the Bernoulli–Euler model to the Timoshenko model in the problem of free bending vibrations in beams expectedly transforms the solution, complicating it. First, the existing eigenmodes and eigenfrequencies of vibrations are slightly modified: the eigenmodes gain a shear component, while the frequencies are shifted to the left (toward decreasing). Secondly, a completely new part of the spectrum of eigenfrequencies and eigenmodes appears: these are the modes in which shear deformations prevail over bending ones. Thus, the behavior of the beams changes not only quantitatively, but also qualitatively.

Evidently, the influence of these factors becomes more pronounced with increasing thickness of the given beam; also growing with increasing vibration frequency, since the Timoshenko model specifically provides a better description of high-frequency dynamics compared to the Bernoulli–Euler model. Therefore, it is more correct to use the Timoshenko model in problems on high-frequency dynamics of beams, for example, in simulations of damping of elastic waves, [1, 2].

The Bernoulli–Euler model is often used to describe the behavior of thin beams in problems where active vibration suppression is considered not as blocking of elastic waves but as damping of certain vibration modes [3] (in particular, this model was selected in several of our earlier studies [4, 5]). The natural questions, then, are how an increase in beam thickness influences the efficiency of control in such problems and at what point this influence becomes significant.

To find answers to these questions, it is necessary to make a transition to the Timoshenko model, considering several cases with beams of different thicknesses.

Several studies confirmed that effective active damping of Timoshenko beam vibrations can be achieved with piezoelectric sensors and actuators using both the proportional control law [6] and state observers [7]. However, these studies did not compare the efficiency of control systems (CS) for beams of different thicknesses. It was shown in [8] that using the control law synthesized for the Bernoulli–Euler beam to dampen the vibrations of the Timoshenko beam can lead to instability of a closed system.

Thus, the influence of the factors inherent to the Timoshenko model on the operation of systems for controlling beam vibrations remains an important issue.

This paper continues the research carried out in [4, 5], reporting on the control of vibrations in elastic beams using observers was.

The goal of this study consists in numerical analysis of the influence of contributors in the Timoshenko model appearing with an increase in beam thickness on the efficiency of control systems for active damping of beam vibrations using observers.

In this study, the thickness of the beam varies over a wide range of values: from 1/250 to 1/10 of the length of the beam. The control inputs considered are either concentrated forces (in this case, the transverse displacements of the beam in the corresponding cross-sections are fed to the input of the control system) or concentrated moments (in this case, the rotation angles of the corresponding cross-sections of the beam are measured).

Optimization methods involving formulation and solution of matrix Riccati equations are widely used to synthesize optimal control systems with an observer. However, this approach to synthesis of CS cannot be directly applied in this study, since the size of the elements of the observation and control matrices in the problem considered is limited not by the factors adopted within the framework of this approach, but by others.

These limiting factors in the standard formulation of the optimization problem are the dependence of the quality functional on the magnitude of the control inputs (for the control matrix) and the measurement noise (for the observation matrix). On the contrary, the limiting factor in this study is the excitation of higher modes by the CS which are not controlled, because of the phase shift in control inputs due to a delay in the control loop (the so-called spillover effect). Accounting for this particular factor as the main one is dictated by our experience in cnducting experimental studies on active damping of metal beam vibrations in a metal beam [9].

An alternative approach to optimal synthesis of CS is the Linear Matrix Inequalities (LMI) method, however, the given problems are not directly reduced to LMI for the case of control with an observer, so this method is also ineffective in this study. In view of this circumstance, the synthesis of CS in the study was carried out by the Nelder—Mead numerical optimization method.

Peculiarities of the Timoshenko beam model

The Bernoulli—Euler beam model is widely used to describe the bending vibrations in thin beams. This model is based on the assumption that the cross-sections of the beam always remain perpendicular to its centerline (i.e., there is no cross-section shear). Thus, the state of the beam under bending deformation is fully described by the transverse displacement function of the beam centerline depending on the longitudinal coordinate. Furthermore, this model does not account for the rotational inertia of the beam cross-sections.

A model describing bending vibrations in thicker beams was proposed by Stepan Timoshenko in 1921 [10]. This model accounts for both the shear deformation and the rotational inertia of the beam cross-sections. Thus, the Timoshenko model uses two functions of the longitudinal coordinate to fully describe the dynamics of the beam: the transverse displacement w and the rotation angle of the cross-section φ. The equations of bending vibrations of the beam for the model under consideration can be written as follows (for simplicity, we assume that the beam has a constant cross-section):

$$\begin{cases} \rho A\ddot{w} + \kappa AG\left(\varphi' - w''\right) = q, \\ \rho I\ddot{\varphi} + \kappa AG\left(\varphi - w'\right) - EI\varphi'' = m, \end{cases} \tag{1}$$

where ρ is the bulk density of the beam material; A and I are the area and moment of inertia of the beam cross-sections; E and G are Young's modulus and shear modulus of the beam material; κ is the dimensionless Timoshenko shear coefficient (depends on the shape of the beam section and is commonly taken equal to 5/6 for a rectangular cross-section); q, m are the distributed transverse force and the bending moment applied to the beam.

The two second-order equations above can be combined into one fourth-order equation for the beam's transverse displacement w:

$$EI\frac{d^4w}{dx^4} + \rho A\ddot{w} - \rho I\left(1 + \frac{E}{\kappa G}\right)\ddot{w}'' + \frac{\rho^2 I}{\kappa G}\frac{d^4 w}{dt^4} = q + \frac{\rho I}{\kappa AG}\ddot{q} - \frac{EI}{\kappa AG}q'' - m'. \tag{2}$$

If the shear stiffness of the beam κAG tends to infinity and we neglect the rotational inertia of the beam cross-sections ρI, the distinctive effects of the Timoshenko model disappear, and Eq. (2) is reduced to an equation describing the bending vibrations of the Bernoulli—Euler beam.

The solution to the problem of free beam vibrations is significantly transformed with the transition from the Bernoulli—Euler beam model to the Timoshenko model. Details of the solution to this problem for the Timoshenko model are discussed in [11, 12]; in particular, a simply supported beam and its set of eigenfrequencies and eigenmodes are considered.

A similar formulation of the problem is used in this study. The complete solution is rather cumbersome, so here we will briefly provide only basic information about it.

The solution to the problem of free vibrations in a Bernoulli–Euler beam is an infinite series (spectrum) of eigenfrequencies and their corresponding eigenmodes. The frequency spectrum has a lower bound (the first eigenfrequency), but there is no upper bound. The Timoshenko model is the more complex form of the Bernoulli–Euler model, so the solution for it contains all the same elements, i.e., frequencies and modes, but with a certain correction: the frequencies decrease slightly, and the modes acquire a shear component; in addition, the model adds new solutions: frequencies and their corresponding modes.

Thus, a new frequency spectrum is added, and this spectrum also has a lower bound, the transition frequency:

$$\Omega^* = \sqrt{\frac{\kappa AG}{\rho I}}. \tag{3}$$

The presence of an eigenmode corresponding to the given frequency depends on the boundary conditions. In the case of a simply supported beam, this mode exists and represents the rotation of all beam cross-sections by the same angle (i.e., constant shear deformation along the length of the beam) in the absence of transverse displacement. The new part of the spectrum originating from the transition frequency includes vibration modes for which shear deformations prevail over bending ones. The frequencies in this region of the spectrum also grow to infinity with an increase in the sequence number.

Problem statement

The goal of the study is to synthesize CS with observers and numerically compare their efficiency for active damping of forced bending vibrations in Timoshenko beams with different thicknesses.

A schematic of the beam with applied input for two configurations of control elements considered is shown in Fig. 1. A 1 m long simply supported aluminum beam with a rectangular cross-section is subjected to an external perturbation, which is a concentrated bending moment M_0 applied to the beam at a distance of 0.4 m from the left end. The study considers three cases of beams with different cross-sections: 4 × 25 mm, 20 × 25 mm and 10 × 10 cm.

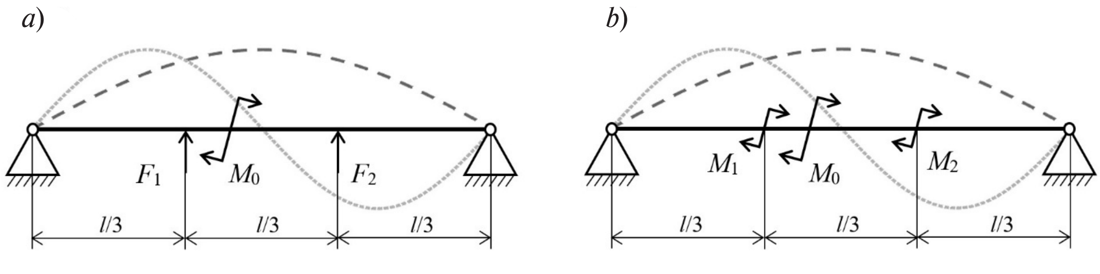

Fig. 1. Schematics of control object with applied control inputs: concentrated forces F_1 and F_2 (*a*), concentrated moments M_1 and M_2 (*b*); beam length l, external bending moment M_0

The paper considers two configurations of the CS.

Case A. The sensors measure the transverse displacement in two cross-sections of the beam, dividing it into three equal parts, and the control input consists of concentrated transverse forces applied in the same cross-sections.

Case B. The control system is designed differently from case *A* in that the it is the rotation angles of these cross-sections that are measured and the input is applied as concentrated bending moments.

Control is aimed at dampening forced vibrations at the two lowest resonant frequencies of the beam. The vibration modes corresponding to these frequencies (more precisely, their components in the form of transverse displacement, since there are also other components in the form of rotation angles) are also shown in Fig. 1.

In this paper, we consider CS with observers, widely used for feedback control in the case when the state of an object cannot be directly measured and an adequate model of the object is known. A detailed mathematical description of this method with respect to the problems considered in this paper can be found in our earlier works [4, 5]. To set each such CS, it is necessary to define the observation and control matrices; the Nelder–Mead optimization method built into the computing package used was selected for this purpose in our study. Then the results generated by the synthesized systems for beams of different thicknesses are compared.

Modeling of object without control

Before constructing CS, it is necessary to model the object itself. This section is dedicated to modeling of beams without control. It consists of the expansion in terms of the beam's vibration eigenmodes, calculated together with the eigenfrequencies using the Timoshenko beam model.

First of all, we should verify that the mathematical model of beam behavior used is correct, subsequently determining the number of eigenmodes in the model sufficient to adequately describe the required dynamics. For this purpose, the frequency response obtained for this model as an eigenmode expansion were compared with the frequency response obtained for beams with the same parameters by finite element analysis.

Three-node beam elements were used in all FE models. The beam was divided into 200 elements with a length of 5 mm, each model contained a total of 401 nodes.

Fig. 2 shows the frequency response of beam III (the thickest one, with the thickness of 10 cm), calculated by applying the first (left) concentrated control moment to the beam and measuring the rotation angle in the same cross-section (obtained by both methods). Two solutions are given for two numbers of eigenmodes taken into account for the second of these models:

configuration *A* consists of 25 traditional modes (their equivalents are included in the Bernoulli–Euler beam model), one transitional mode, and 15 shear-dominated forms from the new spectrum of the Timoshenko model;

configuration *B* consists of 45 traditional modes, one transitional mode and 25 shear mode.

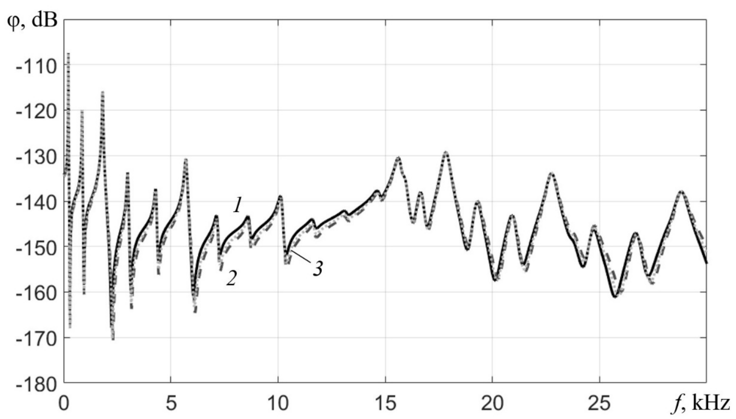

Fig. 2. Comparison of the frequency response for moment versus rotation angle in beam III (10 cm thick) obtained for two models: FE (curve *1*) and models with eigenmode expansion (curves *2*, *3*), taking into account 41 (*2*) and 71 (*3*) modes

The ordinate axis on the frequency response curves here and below has a logarithmic scale, therefore, the quantities plotted along this axis (transverse displacement or rotation angle) are measured in decibels (dB).

As evident from Fig. 2, there is a good agreement between the two models considered, and the proposed number of modes taken into account (45 + 1 + 25) turns out to be sufficient for an adequate description of beam dynamics within the framework of the problem solved in a wide frequency range. A similar comparison was carried out for two other beam configurations (I and II), with the following numbers of modes taken into account in the simulations selected for them:

45 + 1 + 25 for beam II (same as for beam III),

45 for beam I (traditional vibration modes).

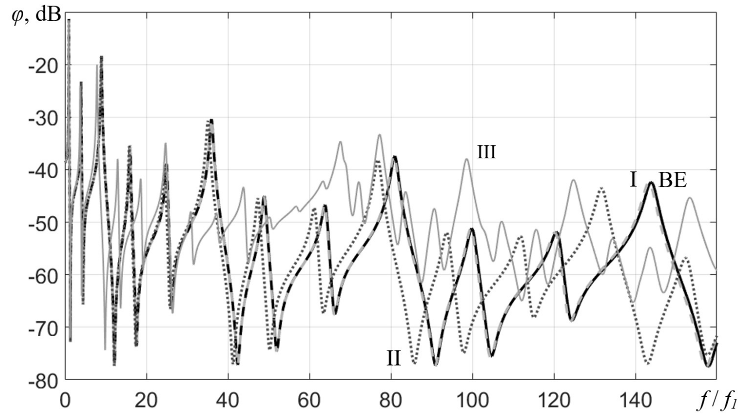

Fig. 3. Comparison of frequency response for moment versus rotation angle φ in Timoshenko beams I–III taking into account scaling (curve numbers correspond to beam numbers): beam I taking into account 45 modes; beams II and III both taking into account 71 modes; Bernoulli–Euler beam model taking into account 45 modes (BE curve)

The frequency f is normalized by the frequency of the first resonance f_1 for each of the beams

The comparison shows that the eigenmode expansion of the Timoshenko beam model used in the study is correct.

It should be noted here that with the beam dimensions adopted (length of 1 m, thickness of 10 cm), the finite element beam model itself, like the Timoshenko model, may not be entirely accurate due to the warping of the beam cross-sections. Investigation of this circumstance is beyond the scope of this work, since our goal was to compare the Timoshenko and Bernoulli–Euler models, and the thickness-to-length ratio (1:10) of beam III was chosen as the limit for the correct use of the Timoshenko model.

When an adequate model of the object is selected, it becomes possible to compare the frequency response for beams of different thicknesses. Such a comparison is of particular interest, since it is the differences between these frequencies that will have the key influence on the synthesis process and the efficiency of CS constructed for each of the beams.

Fig. 3 shows a comparison of the frequency characteristics obtained for the first control moment acting on the beam, measuring the rotation angle in the same section for each of the beams. To compare these characteristics in the same graph, it is necessary to coordinate their scale on both axes. The frequencies along the abscissa axis were normalized by the frequency of the first resonance of each beam, since this value increases exactly by n times with an increase in the thickness of the Bernoulli–Euler beam by n times. The rotation angles of the cross-section (ordinate axis) were also normalized, since the vibration amplitude of the Bernoulli–Euler beam decreases by n^3 times with an increase in its thickness of beam by n times and by n times with an increase in width. Moreover, the vibration amplitude of the beam decreases inversely with an increase in beam width, and the width of beam III, as mentioned earlier, is 4 times the width of beams I and II. For this reason, the values of the rotation angles in this graph were multiplied by 125 for beam II, and by 62,500 for beam III (the ordinate axis in Fig. 3 also has a logarithmic scale, and the logarithm of the angle was calculated after multiplying it by a multiplying factor).

Fig. 3 also shows the reference characteristic corresponding to the Bernoulli–Euler beam model (its BE curve nearly coincides with the curve for beam I).

Importantly, if this particular model were used for beams of different thicknesses, then all the curves on the graph would have coincided with the reference one. Furthermore, all synthesis results and efficiency indicators of CS for different beams would also have been the same with an accuracy up to the similarity coefficients. Therefore, the graph shown in Fig. 3 illustrates precisely the influence of the Timoshenko beam model, becoming more pronounced with increasing beam thickness.

Let us analyze how the influence of the Tymoshenko model manifests on the graph. A comparison of the curves for beams I and II shows that the resonant frequencies are shifted to the left (towards a decrease) and this shift increases with increasing frequency. This is even more pronounced for beam III, where a new factor is also noticeable: a significant change in the structure of the solution to the right of a certain boundary. This boundary is located in the vicinity of the transition frequency (mentioned earlier in the section on the characteristics of the Timoshenko model): for beam III, it exceeds the first eigenfrequency by about 68 times. A new part is added to the spectrum present in the Bernoulli–Euler model starting from this transition frequency: modes with a predominant shear component. This feature is not observed on the graph for beams I and II, since for them the value of the transition frequency significantly (by several orders of magnitude) exceeds the value of the first eigenfrequency, therefore not falling within the considered range.

Synthesis of control systems

Consider the operation of CS with state observers used to actively dampen beam vibrations in this study. Only the basic equations are given here; a more detailed description of this method as applied to the problems considered in this paper can be found in [4, 5].

We assume that n lower vibration modes must be taken into account for satisfactory description of the dynamics of an object. We introduce the state vector q^n:

$$q^n = \begin{pmatrix} q_1 & \dots & q_n & \dot{q}_1 & \dots & \dot{q}_n \end{pmatrix}^T, \tag{4}$$

where q_i is the generalized coordinate corresponding to the ith mode of the beam's vibrations; $\dot{q}_i$ is the generalized velocity.

In this case, the object's behavior can be described by the following equations:

$$\dot{q}^n = Aq^n + Bu + Dd, \tag{5}$$

$$y = Cq^n, \tag{6}$$

where d is the vector of external perturbations; y, u are the vectors of measured signals and control inputs; A, B, C are the matrices defining the behavior of the object and its interaction with drives (actuators) and sensors.

We assume that observation and control are carried out for k lowest vibration modes of the object ($k \leq n$). The observer's objective is to evaluate the state vector q^k corresponding to these modes:

$$q^k = \begin{pmatrix} q_1 & \dots & q_k & \dot{q}_1 & \dots & \dot{q}_k \end{pmatrix}^T. \tag{7}$$

The observer forms an estimate of this vector $\hat{q}$ using the known matrices $A^{(1)}$, $B^{(1)}$, $C^{(1)}$ describing the dynamics of the k lowest modes of the object and this estimate is then used to define the vector of control inputs:

$$\dot{\hat{q}} = A^{(1)}\hat{q} + B^{(1)}u + L\left(y - C^{(1)}\hat{q}\right), \tag{8}$$

$$u = -R\hat{q}, \tag{9}$$

where L, R are the observation and control matrices that must be set correctly (so that the control objective is achieved).

Within framework of this study, we consider the matrices $A^{(1)}$, $B^{(1)}$ and $C^{(1)}$ to be known (they are obtained from matrices A, B and C by eliminating unnecessary columns and rows), so only the matrices L and R are to be determined to define the CS. Observation and control in the given problem are carried out for two modes using two sensors and two actuators. Since observation and control of these modes are carried out separately, the matrices L and R can be set as follows [5]:

$$L_{4\times 2} = \begin{bmatrix} K^L_{2\times 2} \\ K^{Ld}_{2\times 2} \end{bmatrix} T_{2\times 2},\; R_{2\times 4} = F_{2\times 2}\begin{bmatrix} K^R_{2\times 2} & K^{Rd}_{2\times 2} \end{bmatrix}, \tag{10}$$

where K^L, K^{Ld}, K^R, K^{Rd} are 2×2 diagonal matrices; T, F are modal matrices (mode analyzer and synthesizer) [4, 5, 9], determining how each control loop uses each sensor and actuator in its operation.

Since the sensors and actuators are located symmetrically on the beam, and the beam's eigenmodes are symmetrical or antisymmetric, the modal matrices in this study have a specific form, presented below. In the case when the transverse displacement is measured and the control input is applied as concentrated forces, they can be set as follows:

$$T^{(1)} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix}, \; F^{(1)} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{11}$$

In the case when the rotation angles are measured, and the control inputs are concentrated moments, the modal matrices take the form:

$$T^{(2)} = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix}, \; F^{(2)} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}. \tag{12}$$

Thus, taking into account the structure of the matrices K^L, K^{Ld}, K^R and K^{Rd} to be set (they are diagonal and have the size of 2 × 2), it is necessary to optimally select 8 parameters to fully define the CS in the case of simultaneous damping of two vibration modes. If the objective of the CS is to dampen only one of the resonances, then only 4 parameters should be determined.

The optimization problem for the given number of parameters was solved by the Nelder–Meade optimization method. The objective function for damping of a single mode was the height of the corresponding resonant peak on the frequency response of the beam with the control switched on. If both modes were to be damped, the objective function was set as the sum of the heights of both resonant peaks. In addition, a penalty was added to this function in case of violation of stability margins in the closed system.

The Nyquist criterion was used for stability analysis, modified for the case of two inputs and two outputs of the control system (it is described in detail in [13]). The aspects of numerical simulation of a closed system with observers are discussed in [4]. It contains, in particular, a formulation of the problem where a link simulating a delay with the transfer function $R^{del}(s)$ is introduced into the control loop:

$$R^{del}(s) = \frac{1}{1 + \tau s}, \tag{13}$$

where τ is the time constant and s is the complex variable.

This exact formulation is used in this study. To preserve the similarity of the problem formulations for different beams, a specific time constant is selected in each formulation:

$$\tau_{\mathrm{I}} = 0.005\text{ s}, \; \tau_{\mathrm{II}} = \frac{1}{5}\tau_{\mathrm{I}} = 0.001\text{ s}, \; \tau_{\mathrm{III}} = \frac{1}{25}\tau_{\mathrm{I}} = 0.0002\text{ s}, \tag{14}$$

where τ_i is the time constant for controlling the oscillations of the ith beam.

Example. As an example, consider the one of the synthesized systems, namely, the system for beam II (cross-section of 20x25 mm), measuring angles and controlling moments while simultaneously damping the first and second vibration modes of the beam. The diagonal matrices defining observation and control were obtained in the following form:

$$\begin{cases} K^L = \begin{bmatrix} -81.63 & 0 \\ 0 & -63.24 \end{bmatrix}, \quad K^{Ld} = \begin{bmatrix} 76.01 & 0 \\ 0 & 79.78 \end{bmatrix} \cdot 10^3, \\ K^R = \begin{bmatrix} -53.93 & 0 \\ 0 & 13.82 \end{bmatrix} \cdot 10^2, \quad K^{Rd} = \begin{bmatrix} 37.25 & 0 \\ 0 & -149.3 \end{bmatrix}. \end{cases} \tag{15}$$

Fig. 4 shows a Nyquist plot for the resulting system (the general view and an enlarged fragment of the vicinity of the origin). The stability margins for all synthesized CS were set such that the system's Nyquist plot did not intersect a circle with 0.5 radius centered at (– 1.0) (this corresponds to an amplitude margin of 6 dB and a phase margin of 29°). Evidently, all stability requirements are satisfied for this CS.

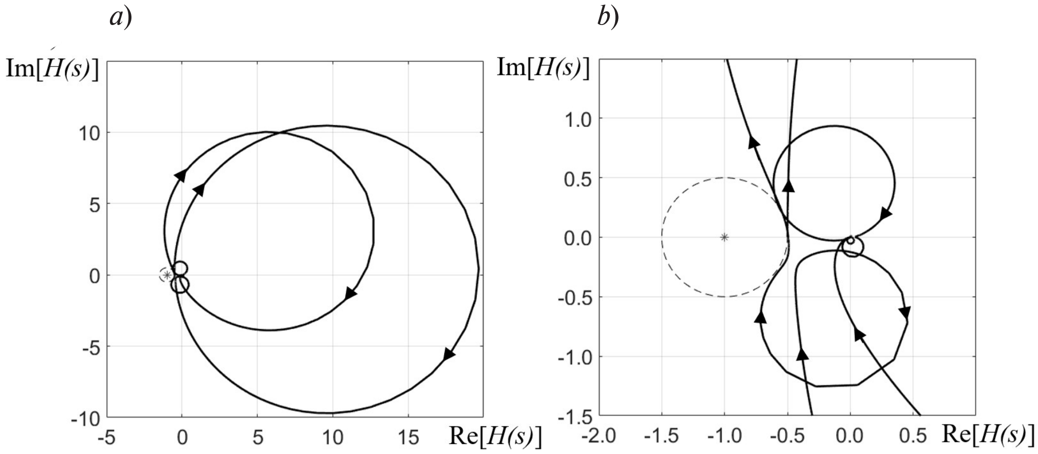

Fig. 4. Example of a Nyquist plot for one of the synthesized control systems: general view (*a*), enlarged fragment (*b*)

Comparison of results obtained from different CS

This section provides an overview of the results obtained from all synthesized CS for beams of different thicknesses.

As already noted, to determine the efficiency of control, the values of the height y of the first and second resonant peaks on the frequency response of the beam with control were analyzed against the height of these peaks in the absence of control. The choice of the frequency response curve itself should be explained: we considered the total energy of steady-state vibrations of the beam exposed to a harmonically varying external perturbation in the form of a concentrated moment, depending on the frequency of the applied input. This quantity was calculated by the following formula for each value of the frequency ω:

$$E(\omega)=\sum_{i=1}^{n}\omega^2 Q_i^2(\omega), \tag{16}$$

where Q_i is the steady-state amplitude of the ith generalized coordinate of the vibration mode, n is the number of generalized coordinates (beam vibration modes) in the model.

As with all frequency characteristics in this study, the amplitude of the quantity considered here was expressed in decibels (dB).

Expression (16) corresponds to the total energy of beam vibrations, since all vibration modes (with transverse displacement $X_i(x)$ and rotation ${}_i(x)$ components) are normalized as:

$$w(x,t)=\sum_{i=1}^{n}X_i(x)q_i(t),\ \varphi(x,t)=\sum_{i=1}^{n}\Phi_i(x)q_i(t) \tag{17}$$

$$\int_0^l\left(\rho A X_i(x)X_j(x)+\rho I\Phi_i(x)\Phi_j(x)\right)dx=\delta_{ij} \tag{18}$$

where δ_{ij} is the Kronecker symbol.

The control results for all systems are summarized in Table. A decrease in amplitude at the first (Res1) and second (Res2) resonances as a result of control is observed for all three beams, cases of control using both concentrated forces and moments are considered. Vibration damping was carried out either for only one of the modes, or simultaneously for both modes for each formulation. The best results for each beam and each of the resonances are highlighted in bold.

Fig. 5 shows the frequency response of beams with and without control for CS simultaneously operating at both resonances. To compare the results obtained for different beams on the same graph, the frequency response was scaled along both axes as done in Fig. 3.

Table

Decrease in beam vibration amplitudes at resonance for different control systems

Beam	Control method	Damping scenario	Amplitude decrease, dB, at resonance	
			Res1 (Δy_1)	Res2 (Δy_2)
I (*1*, *4a*, *4b*)	Forces	Separately	**35.47**	29.4
		Together	34.05	**29.5**
	Moments	Separately	31.18	25.91
		Together	30.16	24.78
II (*2*, *5a*, *5b*)	Forces	Separately	35.44	29.73
		Together	33.79	**29.73**
	Moments	Separately	31.14	25.93
		Together	30.16	24.77
III (*3*, *6a*, *6b*)	Forces	Separately	35.03	27.9
		Together	34.16	**27.9**
	Moments	Separately	30.59	25.21
		Together	29.79	23.97

Note. The numbers of the curves in Fig. 5 corresponding to each beam are given in parentheses in the left column.

Analyzing the data in Table and Fig. 5, we can conclude that the CS obtained make it possible to efficiently dampen the forced vibrations of the beam at both the first and second resonances. The main conclusion is that the efficiency of the synthesized CS only changes insignificantly as the beam thickness increases over a wide range. This means that the considered control method allows to efficiently control the vibrations in not only thin but also relatively thick beams. A noticeable trend is that the damping efficiency decreases slightly with increasing thickness of the beam both at each resonance in the case of separate damping and as the sum of parameters at both resonances in the case of simultaneous damping. This effect is almost unnoticeable for the results for beams I and II compared but manifests clearly for beam III. A possible explanation for this result is that the frequency response curves are virtually identical at low frequencies for beams I and II in the absence of control, while the resonant peaks on the frequency response curve of beam III shift to the left with increasing frequency, compared with the frequency response of

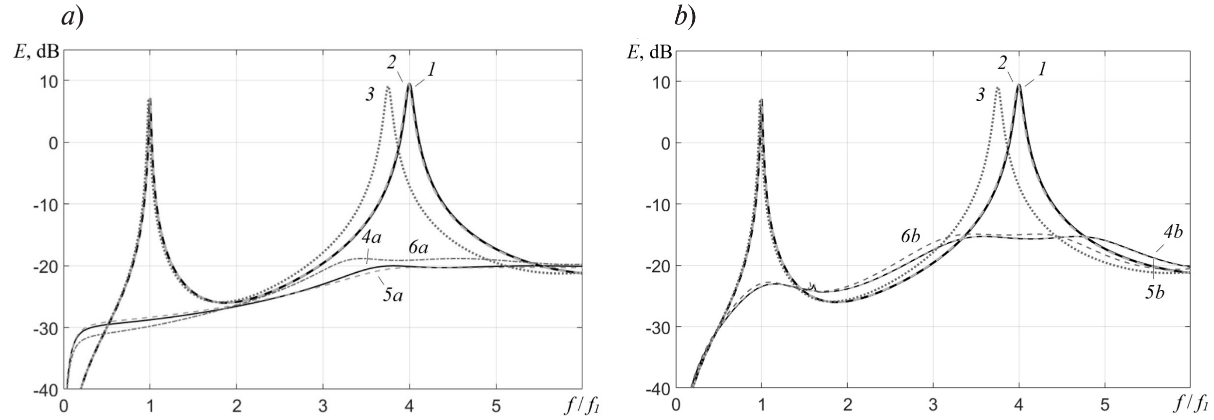

Fig. 5. Frequency response for beams I– III (*a*, *b*) without control (curves *1–3*), with force-based control (curves *4a–6a*) and with moment-based control (curves *4b–6b*) (see Table)

thinner beams; accordingly, the gains in the CS are more noticeably limited by the deterioration in stability at higher frequencies and the efficiency of the CS decreases.

Notably, the observed difference is caused not so much by the influence of the new part of the frequency spectrum and the corresponding shear modes found in the solution to the problem on the Timoshenko beam vibrations, but rather by the 'deformation' of its scaled frequency response near the lower resonant frequencies with increasing beam thickness (this effect can also be observed in Fig. 3). The influence of the new part of the spectrum is small, since the transition frequency bounding this part of the spectrum from below for all the beams considered significantly exceeds the cutoff frequency of the open system for all synthesized CS (even in the case of beam III, the first quantity is about an order of magnitude higher than the second and the difference is even more significant for the remaining beams).

In addition, the presented results show that control using displacements and concentrated forces is carried out far more efficiently than with systems using rotation angles and concentrated moments. This is explained by the fact that the resonant amplitudes on the force—displacement curve of the beam (without control) decrease significantly faster with increasing frequency than on moment—rotation angle curve, i.e., the influence of control for the first systems is more pronounced at lower frequencies than at higher ones, therefore, the damping efficiency at lower frequencies increases, and the decrease in stability at high frequencies decreases.

Another observation concerns the results given in Table: as a rule, the vibrations at the first and second resonances are dampened individually somewhat more efficiently than together, this effect is characteristic of CS both with and without observers, as discussed in our earlier studies [4, 5]. An explanation for this result is that when two loops are switched on simultaneously in the CS, the degree of stability of the closed system decreases, so the gain coefficients must be reduced, consequently decreasing the efficiency of the CS.

Conclusion

The paper considers the problem of active damping of forced vibrations in metal beams using control systems with state observers. We analyzed the influence of beam thickness on control efficiency. For this purpose, the problem was solved numerically using the Timoshenko beam model for different beam thicknesses, varying over a wide range. The study focuses specifically on the effects of the Timoshenko model, since the Bernoulli—Euler model would yield identical results for the dynamics of beams with different thicknesses.

We found that the proposed CS remain efficient for all the beams considered in the case of damping beam vibrations at lower resonances. Only a slight decrease in vibration damping efficiency was observed with a significant increase in beam thickness; a reasonable explanation was given for this effect.

The following directions are of the greatest interest for future studies:

modeling of beam vibration control using specific piezoelectric sensors and actuators,

damping of beam vibrations at higher resonant frequencies,

variation of the object model in the observer, including the expansion of the number of beam eigenmodes included in this model.

Other promising directions are accounting for cross-section warping for thick beams and measurement noise during synthesis of the control system, as well as analysis of various quality functionals for optimization of the parameters of the control system. It is also of interest to solve the problem on damping of elastic waves in the Timoshenko beam.

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