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THE INFLUENCE OF NONLINEAR PARAMETRIC EXCITATION ON THE INTERACTION OF FORCED, PARAMETRIC AND SELF-OSCILLATIONS

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Abstract. In order to reveal the effect of nonlinear (cubic) parametric excitation (NPE) on the interaction of forced, parametric, and self-oscillation with a limited-power energy source, a widely used computed model of a self-oscillating system receiving energy from such a source was used. Solutions of nonlinear differential equations of the model were constructed using the direct linearization method (DLM), which is distinguished from the known ones by its simplicity its simplicity and low time costs. The friction force characteristic causing self-oscillations was linearized by DLM. Equations for the amplitude, oscillation phase and the velocity of the energy source in nonstationary and stationary motion cases were derived. Using the Routh – Hurwitz criteria, the stability of stationary movements was considered. The influence of NPE on the interaction of forced, parametric and self-oscillations was investigated by calculations. The latter showed NPE to change the shape of the amplitude curves inherent in linear action and to have a significant impact on the motion stability.

Keywords: interaction, forced oscillations, parametric oscillations, self-oscillations, nonlinearity, direct linearization method

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ВЛИЯНИЕ НЕЛИНЕЙНОГО ПАРАМЕТРИЧЕСКОГО ВОЗБУЖДЕНИЯ НА ВЗАИМОДЕЙСТВИЕ ВЫНУЖДЕННЫХ, ПАРАМЕТРИЧЕСКИХ И АВТОКОЛЕБАНИЙ

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Аннотация. С целью выявления действия нелинейного (кубического) параметрического возбуждения (НПВ) на взаимодействие вынужденных, параметрических и автоколебаний при источнике энергии ограниченной мощности использована широко применяемая расчетная модель автоколебательной системы, получающая энергию от такого источника. Решения нелинейных дифференциальных уравнений модели построены методом прямой линеаризации (ПЛ), которому свойственны простота и малые затраты времени. Характеристика силы трения, вызывающая автоколебания, линеаризована методом ПЛ. Выведены уравнения для амплитуды, фазы колебаний и скорости источника энергии в нестационарном и стационарном случаях движения. С использованием критериев

Рауса – Гурвица рассмотрена устойчивость стационарных движений. Влияние НПВ на взаимодействие вынужденных, параметрических и автоколебаний исследовано и расчетным путем; расчеты показали, что НПВ изменяет форму амплитудных кривых, присущих линейному воздействию, оказывает существенное влияние на устойчивость движения.

Ключевые слова: взаимодействие, вынужденные колебания, параметрические колебания, автоколебания, нелинейность, метод прямой линеаризации

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Introduction

The interaction of forced, parametric and self-oscillations is the most complex of the four classes of mixed oscillations according to the classification introduced in monograph [1]; they consist of a combination of oscillation types [2–4]. Of these four classes of mixed oscillations, the interaction of forced and parametric oscillations under cubic nonlinear parametric excitation (NPE) is considered in [5] without taking into account the properties of the energy source.

Taking the specified properties into account gained significance in connection with environmental problems, climate change and the reduction of energy resources. The need to take into account the properties of the energy source, which in the theory of oscillations is associated with the well-known Sommerfeld effect, is consistently stressed in fundamental monograph [6]. Many works have been published in this field all over the world, including books and articles [1, 7–17]. The connection between the level of energy consumed during the operation of parts, the accuracy of their processing and the oscillations is shown [18].

In this paper, we consider mixed forced, parametric and self-oscillations with NPE and limited power of the energy source. The direct linearization method (DLM) was used to solve nonlinear differential equations describing these oscillations [19].

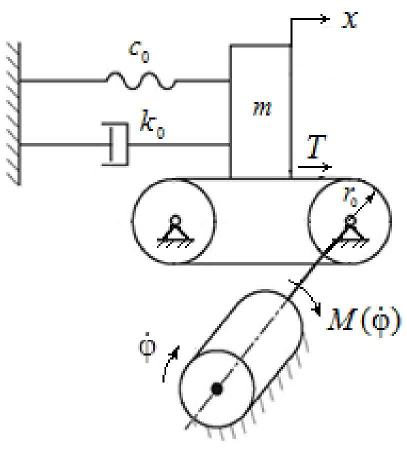

Fig. 1. Model of system considered:
body mass m; angular velocity of motor rotor $\dot{\varphi}$; torque $M(\dot{\varphi})$; friction force T; radius r_0 of the pulley rotating the belt; spring constant c_0 and damping ratio k_0

Calculation model

Under the influence of frictional force, self-oscillations arise in many technical objects [20–24]. The model we consider is widely used to describe them (Fig. 1). A body with mass m is connected to the housing by means of a spring and a damper. It lies on a belt driven by a motor of limited-power with the torque $M(\dot{\varphi})$ and the total moment of inertia I of the rotating parts. A friction force $T(U)$ arises between the body and the belt, depending on the relative velocity U:

$$U = V - \dot{x},\ V = r_0\dot{\varphi},$$

where r_0 is the radius of the pulley rotating the belt, $r_0 = \text{const}$; $\dot{\varphi}$ is the angular velocity of the motor rotor.

Taking into account the external driving force $\lambda\sin(\nu_1 t)$ and NPE $bx^3\cos(\nu t)$ acting on the body, the equations of motion of the system have the following form:

$$m\ddot{x} + k_0\dot{x} + c_0 x = T(U) + \lambda \sin \nu_1 t - bx^3 \cos \nu t \qquad (1)$$

$$I\ddot{\varphi} = M(\dot{\varphi}) - r_0 T(U)$$

where k_0, c_0, λ, b, ν_1, ν are constants.

We take the friction force as a functional dependence, which is widespread in real (including cosmic) conditions [25]:

$$T(U) = T_0(\mathrm{sgn}U - \alpha_1 U + \alpha_3 U^3), \qquad (2)$$

where T_0 is the normal reaction force, T_0 = const; α_1, α_3 are constants; $\mathrm{sgn}U = 1$ at $U > 0$ and $\mathrm{sgn}U = -1$ at $U < 0$. In the case of $U = 0$, i.e., at relative rest, the inequalities $-T_0 \le T(0) \le T_0$ hold true.

Using the DLM [19], we replace the nonlinear components of the friction force $T(U)$ with a linear component:

$$T(U) = T_0(\mathrm{sgn}U + B_T + k_T\dot{x}), \qquad (3)$$

where $B_T = -\alpha_1 u + \alpha_3 u^3 + 3N_2\alpha_3 u a^2 p^2$, $k_T = -\alpha_3 \bar{N}_3(a^2p^2 - h)$, $h = 3(u_0^2 - u^2)/\bar{N}_3$, $u_0^2 = \alpha_1/3\alpha_3$, $N_2 = (2r + 1)/(2r + 3)$, $\bar{N}_3 = (2r+3)/(2r+5)$.

The quantity r, included in the expressions for numerical coefficients N_2 and $\bar{N}_3$, represents a linearization accuracy parameter whose value is not limited, but it is sufficient to select it in the interval (0, 2). The coefficients $\bar{N}_n$ and N_n are compared in [26] with the coefficients obtained by averaging [27–30] for different values of the degree of nonlinearity n, establishing the acceptable agreement for the results obtained by both methods.

In view of substitution (3), Eqs. (1) take the form

$$m\ddot{x} + k\dot{x} + c_0 x = T_0(\mathrm{sgn}U + B_T) + \lambda \sin \nu_1 t - bx^3 \cos \nu t, \qquad (4)$$

$$I\ddot{\varphi} = M(\dot{\varphi}) - r_0 T_0(\mathrm{sgn}U + B_T + k_T\dot{x}),$$

where $k = k_0 - T_0 k_T$.

Solution of the equations

As shown in monograph [1], the solutions of equations with function (2) at $U > 0$ and $U < 0$ are fundamentally different, so they should be considered separately.

We represent these cases with expressions

$$u \ge ap, \; u < ap,$$

where $u = r_0\Omega$ (Ω is the averaged velocity $\dot{\varphi}$).

Let us consider solutions (4) for the main resonances with frequencies $\omega \approx \nu_1$ and $\omega \approx \nu/2$, since they are of primary practical interest.

Using the change of variables with averaging [19], we obtain the expressions

$$x = a\cos\psi, \; \dot{x} = -ap\sin\psi, \; \psi = pt + \xi, \; p = \nu/2, \; \dot{\varphi} = \Omega. \qquad (5)$$

Next, we consider two cases.

i) $u \ge ap$, then

$$\frac{da}{dt} = -\frac{1}{4pm}\left(2aA + 2\lambda\cos\xi - 0.5ba^3\sin 2\xi\right),$$

$$\frac{d\xi}{dt} = \frac{1}{4pma}\left[2am(\omega_0^2 - p^2) + 2\lambda\sin\xi + ba^3\cos 2\xi\right], \qquad (6a)$$

$$\frac{du}{dt} = \frac{r_0}{I}\left[M\left(\frac{u}{r_0}\right) - r_0 T_0(1 + B_T)\right];$$

ii) $u < ap$, then

$$\frac{da}{dt} = -\frac{1}{4pm}\left[2aA + 2\lambda\cos\xi - 0.5ba^3\sin 2\xi - \frac{8T_0}{\pi ap}\sqrt{a^2p^2 - u^2}\right],$$
$$\frac{d\xi}{dt} = \frac{1}{4pma}\left[2am(\omega_0^2 - p^2) + 2\lambda\sin\xi + ba^3\cos 2\xi\right], \tag{6b}$$
$$\frac{du}{dt} = \frac{r_0}{I}\left[M(\frac{u}{r_0}) - r_0T_0(1+B_T) - \frac{r_0T_0}{\pi}(3\pi - 2\psi_*)\right],$$

where $\omega_0^2 = c_0/m$.

The conditions $\dot{a} = 0,\ \dot{\xi} = 0$ in the case $u \geq ap$ provide the following relations for determining the stationary values of a and ξ:

$$(\lambda^2b^2a^6 + b^2a^6L - \lambda^4 + 2\lambda^2L - L^2 - 4b^2a^8A^2)^2 - 4\lambda^2b^4a^{12}L = 0,$$
$$\mathrm{tg}\xi = D(Dba^3 - 4\lambda)/8aA, \tag{7}$$

where $L = \lambda^2 + ba^4[ba^2 + 2m(\omega_0^2 - p^2)]$, $A = p(k_0 - T_0k_T)$, $D = 2(\lambda \pm \sqrt{L})/ba^3$.

To determine the stationary values of the velocity u from the third equation in (6a), we have, provided that $\dot{u} = 0$, an expression of the form

$$M(u/r_0) - S(u) = 0, \tag{8}$$

where $S(u)$ is the load on the energy source from the oscillatory system.

In the case $u < ap$, to determine the amplitude of stationary oscillations, we have an approximate dependence $ap \approx u$ and the stationary values of the velocity u are determined by Eq. (8) taking into account the expression for the load

$$S(u) = r_0T_0[(1 + B_T) + \pi^{-1}(3\pi - 2\psi_*)],$$

following from the third equation in (6b) for $\dot{u} = 0$.

Stability of stationary oscillations

To determine the stability of stationary oscillations of the system, we compose the variational equations for Eqs. (6), for which we use the Routh–Hurwitz criteria. As a result, we obtain the following conditions:

$$D_1 > 0, D_3 > 0, D_1D_2 - D_3 > 0,$$

where $D_1 = -(b_{11} + b_{22} + b_{33})$,

$$D_2 = b_{11}b_{33} + b_{11}b_{22} + b_{22}b_{33} - b_{23}b_{32} - b_{12}b_{21} - b_{13}b_{31},$$
$$D_3 = b_{11}b_{23}b_{32} + b_{12}b_{21}b_{33} + b_{22}b_{13}b_{31} - b_{11}b_{22}b_{33} - b_{12}b_{23}b_{31} - b_{13}b_{21}b_{32}.$$

In the case of velocities $u \geq ap$, we have expressions of the form

$$b_{11} = \frac{r_0}{I}\left(Q - r_0T_0\frac{\partial B_T}{\partial u}\right),\ b_{12} = -\frac{r_0^2T_0}{I}\frac{\partial B_T}{\partial a},\ b_{13} = 0,$$
$$b_{21} = a\frac{T_0}{2m}\frac{\partial k_T}{\partial u},\ b_{22} = -\frac{1}{8pm}\left\{4p\left[k_0 + T_0\bar{N}_3\alpha_3(3a^2p^2 - h)\right] - 3ba^2\sin 2\xi\right\},$$
$$b_{23} = \frac{1}{4pm}\left[2\lambda\sin\xi + ba^3\cos 2\xi\right],\ b_{31} = 0,$$

$$b_{32} = -\frac{1}{2pma^2}(\lambda \sin\xi - ba^3 \cos 2\xi),\ b_{33} = \frac{1}{2pma}\left[\lambda\cos\xi - ba^3 \sin 2\xi\right].$$

In the case $u < ap$, only the coefficients change and the expressions take the form

$$b_{11} = \frac{r_0}{I}\left[Q - r_0 T_0 \frac{\partial B_T}{\partial u} - \frac{2r_0 T_0}{\pi\sqrt{a^2p^2 - u^2}}\right],\ b_{12} = -\frac{r_0^2 T_0}{I}\left[\frac{\partial B_T}{\partial a} + \frac{2u}{\pi a\sqrt{a^2p^2-u^2}}\right],$$

$$b_{21} = \frac{a}{2m}\left[T_0\frac{\partial k_T}{\partial u} + \frac{4uT_0}{\pi a^2p^2\sqrt{a^2p^2-u^2}}\right],$$

$$b_{22} = -\frac{1}{8pm}\left\{4p\left[k_0 + T_0\bar{N}_3\alpha_3(3a^2p^2 - h)\right] - 3ba^2\sin 2\xi\right\} - \frac{2T_0u^2}{\pi m a^2p^2\sqrt{a^2p^2-u^2}},$$

where $\dfrac{\partial B_T}{\partial u} = -(\alpha_1 - 3\alpha_3 u^2 - 3\alpha_3 N_2 a^2p^2)$, $\dfrac{\partial B_T}{\partial a} = 6N_2\alpha_3 uap^2$, $\dfrac{\partial k_T}{\partial u} = -6\alpha_3 u$, $Q = dM(u/r_0)/du$.

Calculations

To obtain information on the effect of cubic NPE on the dynamics of mixed forced, parametric and self-oscillations, calculations were performed using the following parameter values:

$$\omega_0 = 1\ \text{s}^{-1},\ b = 0.686\ \text{N·cm}^{-1},\ \lambda = 0.196\ \text{N},\ k_0 = 0.196\ \text{N·s·cm}^{-1},\ T_0 = 4.9\ \text{N},$$

$$\alpha_1 = 0.84\ \text{s·cm}^{-1},\ \alpha_3 = 0.18\ \text{s}^3\text{·cm}^{-3},\ r_0 = 1\ \text{cm},\ I = 9.8\ \text{N·S·cm}^2.$$

When calculating the amplitude for the linearization coefficient k_T, the number $\bar{N}_3 = 3/4$ was used, which is obtained with the linearization accuracy parameter value $r = 1.50$. For the linearization coefficient B_T, the number $N_2 = 3/5$ ($r = 0.65$) was used. The quantities in the figures and in the text below are normalized.

Fig. 2 shows the frequency response curves $a(\nu)$ obtained by calculations based on Eq. (7). The horizontal lines labeled a_a correspond to the amplitudes of self-oscillations arising at velocities u = 1.14 and 1.20.

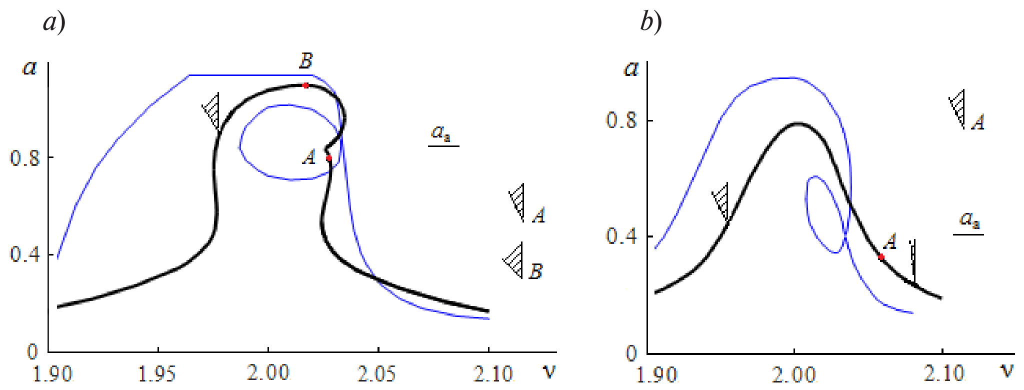

Fig. 2. Frequency response curves at velocities u = 1.14 (*a*) and 1.20 (*b*).
The blue curves are shown for comparison and represent a linear parametric excitation of the form $x\cos(\nu t)$. The horizontal lines labeled a_a show the level of self-oscillation amplitudes. The shaded sectors reflect the slope Q of the energy source characteristic ($Q = dM(u/r_0)/du$) and correspond to stable amplitudes (at point A and in other positions)

The horizontal section on the blue curve (see Fig. 2,*a*) corresponds to the approximate dependence $ap \approx u = 1.14$. The oscillations are stable for the energy source characteristics whose slope Q lies within the shaded sector ($Q = dM(u/r_0)/du$).

The blue curves represent the linear parametric excitation $x\cos(\nu t)$ and are shown for comparison with NPE $x^3\cos(\nu t)$. This comparison shows the difference in the effect of linear and nonlinear parametric excitations on the dynamics of the system. The amplitude levels, shapes of the excitation curves and widths of the resonance regions differ for these excitations. A difference may also be observed in the number of amplitudes at the same frequency. For example, it can be seen from Fig. 2,*a* that the frequencies near point A have three amplitudes under linear excitation, and four under nonlinear excitation. On the other hand, Fig. 2,*b* shows that frequencies with more than one amplitude are absent under NPE but present under linear excitation.

The solution to differential equations (1) was also obtained numerically. The characteristic of the energy source for the solution was taken in the form

$$M(\dot{\varphi}) = M_0 - Q\dot{\varphi}.$$

The slope varied within the range $0 < Q \le 20$, where the value of $Q = 20$ corresponds to an angle of approximately 87°. As an example, Fig. 3 shows a graph for one of the solutions obtained for different values of the parameters M_0, Q, p. The numbers in the frames (in blue) were obtained computationally. In the case of the solution shown in Fig. 3, the values of the parameters $M_0 = 5.9889$, $Q = 5$ were taken, allowing to reach the mode with the average angular velocity of the motor rotor $\dot{\varphi} = 1.1401$ (Y in the frame in the figure); this mode corresponds sufficiently well to the velocity $u = 1.14$. The value $a \approx 0.991$ was obtained for the amplitude (Y in the frames), close to the value of 1.07 obtained by solving Eq. (7).

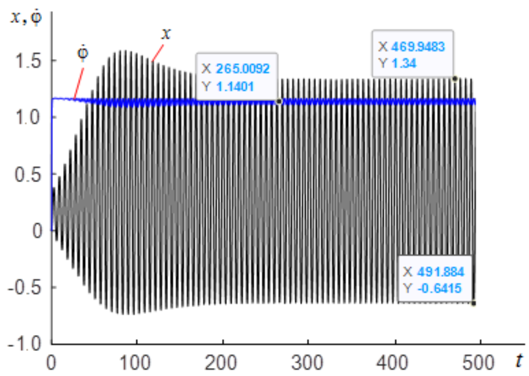

Fig. 3. Dependences x(t) and $\dot{\varphi}(t)$ for the values of the motor parameters $M_0 = 5.9889$, $Q = 5$. The data were obtained at $p = 1$

We also analyzed the influence of the parameters M_0, Q, characterizing the energy source, on dynamic processes. As noted above, the solutions of the equations for the cases $u \ge ap$, $u < ap$ are fundamentally different, and there exists some boundary value of the velocity u_b separating these cases. A transition from one state to another can happen in the system at the boundary value of the velocity u_b. It depends on the slope of the energy source characteristic and its location relative to the curve of the load on the source. The transition takes place at flat characteristics $M(\dot{\varphi})$ and is associated with the load $S(u)$ on the energy source from the oscillatory system, depending on function (2), expressed by Eq. (8).

As an example, Fig. 4 shows a graph of the above-mentioned transition at certain values of motor parameters and energy source characteristic. Given the same slope $Q = 0.4986$ (the angle of inclination is about 26°), a very small ($1 \cdot 10^{-10}$) difference in the values of M_0, equal to 0.

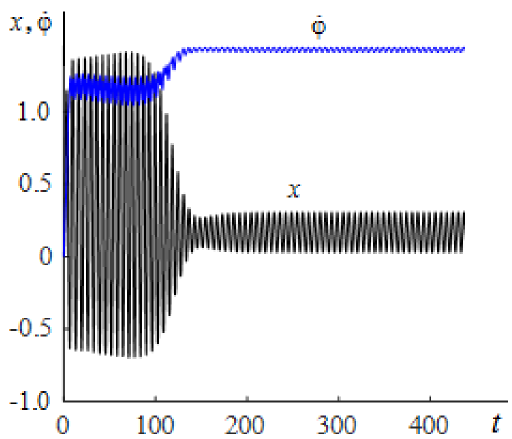

Fig. 4. Time evolution of the x-coordinate and angular velocity of the rotor $\varphi(t)$ for the characteristic $M_0 = 0.8780994695$ (data were obtained at $p = 1$)

As noted above, the amplitude of stationary oscillations at $u < ap$ is determined by the approximate dependence $ap \approx u$. Since the load on the energy source depends on the amplitude of the oscillations, its graphs differ under conditions u

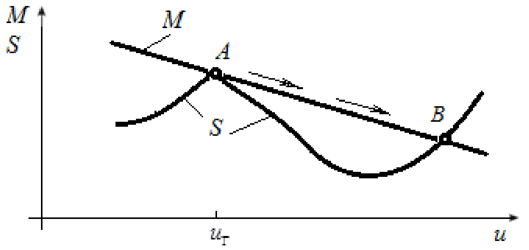

Fig. 5. Dependences of load S and torque M on velocity u in its narrow variation range; u_b is the boundary value of the velocity

$\geq ap$ and $u < ap$. Fig. 5 schematically shows only fragments of the curves for the dependences of the load S and the torque M on the velocity u, over a narrow variation range of u, where the transition from point A to point B takes place (corresponding to the data in Fig. 4) at a fixed position of the energy source characteristic at the boundary velocity u_b corresponding to point A. Different transitions with a description of the full load curves for different classes of oscillations are described in detail in monograph [1], so they are not given here.

In addition to the results presented in this paper, differential equations (6) were also solved numerically. The results are not shown here due to limitations of space. We only note that the results of these solutions are in good agreement with those for Eqs. (7).

Conclusion

As follows from our findings, nonlinear parametric excitation has specific characteristics compared with linear parametric oscillation, namely:

it significantly changes the shape of the amplitude curves;

it has a considerable effect on the stability of oscillations.

Comparing the results for these excitations, we find for the interaction of an oscillatory system with nonlinear parametric excitation (in the case of an energy source with limited power) that a number of effects of the same nature occur as with linear parametric excitation. It seems inappropriate to describe the nature of these effects here, since they have been thoroughly explored in monograph [1].

REFERENCES

1. **Alifov A. A., Frolov K. V.,** Interaction of nonlinear oscillatory systems with energy sources. Hemisphere Publishing Corporation, New York, Washington, Philadelphia, London, 1990.

2. **Andronov A. A., Vitt A. A., Khaikin S. E.,** Theory of vibration, Nauka Publishing, Moscow, 1981 (in Russian).

3. **Migulin V. V., Medvedev V. I., Mustel E. R., Parygin V. N.,** Osnovy teorii kolebaniy [Foundations of vibration theory], 2-nd edition, Nauka Publishing, Moscow, 1988 (in Russian).

4. **Biderman V. L.,** Teoriya mekhanicheskikh kolebaniy [Theory of mechanical vibrations], Vysshaya Shkola [The Higher School Publishing], Moscow, 1980 (in Russian).

5. **Alifov A. A.,** Vliyaniye nelineynogo parametricheskogo vozbuzhdeniya na vzaimodeystviye parametricheskikh i vynuzhdennykh kolebaniy [Effect of nonlinear parametric excitation on the interaction between parametric and forced oscillations], Proc. 15-th All-Russian Sci. & Techn. conf. “Dinamika nelineynykh diskretnykh elektrotekhnicheskikh i elektronnykh system” [Dynamics of nonlinear discrete electrotechnical and electronic systems], June 02, 2023, Cheboksary: Chuvash State University Named after I. N. Ulyanov (2023) 7–8.

6. **Kononenko V. O.,** Vibrating systems with limited power-supply, Iliffe, London, 1969.

7. **Frolov K. V.,** Izbrannyye Trudy v 2 t [Selected works in 2 Vols.], Nauka Publishing, Moscow, 2007 (in Russian).

8. **Rubanik V. P., Starik L. K.,** Ob ustoychivosti avtokolebaniy reztsa v sluchaye neidealnogo istochnika energii [On stability of cutter’s self-oscillations in the case of a nonideal energy source], Nauchnyye trudy vuzov Lit. SSR. Vibrotekhnika [Scientific Transactions of Higher Schools of Lithuanian SSR, Ser. Vibrotechnics] (2(11)) (1971) 205–212 (in Russian).

9. **Abdiyev F. K.,** Avtokolebaniya sistemy s zapazdyvaniyem i s neidealnym istochnikom energii [Self-oscillations of a delayed system with a nonideal energy source], Izv. AN AzSSR. Seriya Fiziko-Tekhnicheskie i Matematicheskie Nauki [News of the Academy of Sciences of the Azerbaijan SSR: Ser. Physical, Technical and Mathematical Sciences]. (4) (1983) 134–139 (in Russian).

10. **Bharti S. K., Bisoi A., Sinha A., et al.,** Sommerfeld effect at forward and backward critical speeds in a rigid rotor shaft system with anisotropic supports, J. Sound Vib. 442 (3 March) (2019) 330–349.

11. **Sinha A., Samantaray A. K., Bhattacharyya R.,** Self-synchronization of two unbalanced DC motor-driven rotors on a common movable platform, In book: Badodkar D., Dwarakanath T. (Eds.) Machines, mechanism and robotics, Proc. iNaCoMM 2017. Book ser. "Lecture Notes in Mechanical Engineering", Springer, Singapore (2018) 207–217.

12. **Jha A. K., Dasgupta S. S.,** Attenuation of Sommerfeld effect in an internally damped eccentric shaft-disk system via active magnetic bearings, Meccanica. 54 (1–2) (2019) 311–320.

13. **Varanis M., Balthazar J.M., Silva A., et al.,** Remarks on the Sommerfeld effect characterization in the wavelet domain, J. Vib. Control. 25 (1) (2019) 98–108.

14. **Rocha R. T., Balthazar J. M., Tusset A. M., et al.,** On a non-ideal magnetic levitation system: nonlinear dynamical behavior and energy harvesting analyses, Nonlinear Dyn. 95 (4) (2019) 3423–3438.

15. **Sinha A., Bharti S. K., Samantaray A. K., Bhattacharyya R.,** Sommerfeld effect in a single-DoF system with base excitation from motor driven mechanism, Mech. Mach. Theory. 148 (June) (2020) 103808.

16. **Chernysheva J., Gorskiy A., Sereda A.,** The effect of Sommerfeld and wagon dynamics, Electronics & Electrical Equipment of Transport. (1) (2019) 8–10 (in Russian).

17. **Eshchin E. K.,** On the Sommerfeld effect and hidden oscillations in the minning machines asynchronous, Russian Electromechanics. 66 (1) (2023) 80–85 (in Russian).

18. **Alifov A. A.,** About calculation of self-oscillatory system delayed and limited excitation, Proc. Int. Sci. Tech. Conf. "Measurement and Quality: Problems, Prospects", Nov. 21–23, 2018, AzTU, Baku, Azerbaijan (2018) 289–293.

19. **Alifov A. A.,** Metody pryamoy linearizatsii dlya rascheta nelineynykh system [Methods of direct linearization for calculation of nonlinear systems], Research Center "Regular and Chaotic Dynamics", Moscow – Izhevsk, 2015 (in Russian).

20. **Klimov D. M.,** On one type of self-excited vibrations in a system with dry friction, Mech. Solids. 38 (3) (2003) 1–7.

21. **Panovko Ya. G.,** Osnovy prikladnoy teorii kolebaniy i udara [Fundamentals of applied theory of oscillations and impact], Mashinostroyeniye Publishing, Leningrad, 1976 (in Russian).

22. **Murashkin L. S., Murashkin S. L.,** Prikladnaya nelineynaya mekhanika stankov [Applied nonlinear mechanics of machine tools], Mashinostroyeniye Publishing, Leningrad, 1977 (in Russian).

23. **Ponomarev A. S., Sokolov V. I., Kozhukhar' A. A.,** Poperechnyye avtokolebaniya silovykh stolov, vyzvannyye silami treniya [Transverse self-oscillations of force tables caused by friction forces], Bulletin of the Kharkiv Polytechnic Institute. Ser. Machine Engineering. 8 (130) (1977) 67–69.

24. **Korityssky Ya. I.,** Krutilnyye avtokolebaniya vytyazhnykh priborov pryadilnykh mashin pri granichnom trenii v oporakh skolzheniya [Torsional self-oscillations of spinning machine drafting devices with boundary friction in sliding bearings], In book: Collected works "Nonlinear vibrations and transition processes in machines", Nauka Publishing, Moscow, 1972.

25. **Bronovets M. A., Zhuravlev V. F.,** On self-excited vibrations in friction source measurement systems, Mech. Solids. 47 (3) (2012) 261–268.

26. **Alifov A. A. O.,** Mixed forced parametric, and self-oscillations with a nonideal energy source and lagging forces, Izvestiya VUZ. Nonlinear Dynamics. 29 (5) (2021) 739–750 (in Russian).

27. **Bogoliubov N. N., Mitropolsky Y. A.,** Asymptotic methods in the theory of non-linear oscillations, CRC Press, Florida, USA, 1961.

28. **Moiseev N. N.,** Asimptoticheskiye metody nelineynoy mekhaniki [Asymptotic methods of nonlinear mechanics], Nauka Publishing, Moscow, 1981 (in Russian).

29. **Butenin N. V., Neymark Yu. I., Fufayev N. A.,** Vvedeniye v teoriyu nelineynykh kolebaniy [Introduction to the theory of nonlinear oscillations], Nauka Publishing, Moscow, 1976 (in Russian).

30. **Chelomey V. N.** (Chairman), Vibratsii v technike: spravochnik [Vibrations in machinery: Handbook], in 6 Vols., Vol. 2. Vibrations of nonlinear mechanical systems, Edited by Blekhman I. I., Publishing House of Mechanical Engineering, Moscow, 1979 (in Russian).

СПИСОК ЛИТЕРАТУРЫ

1. **Alifov A. A., Frolov K. V.** Interaction of nonlinear oscillatory systems with energy sources. New York, Washington, Philadelphia, London: Hemisphere Publishing Corporation, 1990. 327 p.

2. **Андронов А. А., Витт А. А., Хайкин С. Э.** Теория колебаний. М.: Наука. Гл. ред. физ.-мат. лит. 1981. 568 с.

3. **Мигулин В. В., Медведев В. И., Мустель Е. Р., Парыгин В. Н.** Основы теории колебаний. Под ред. В. В. Мигулина. 2-е изд., перераб. М.: Наука. Гл. ред. физ.-мат. лит. 1988. 392 с.

4. **Бидерман В. Л.** Теория механических колебаний. М.: Высшая школа, 1980. 408 с.

5. **Алифов А. А.** Влияние нелинейного параметрического возбуждения на взаимодействие параметрических и вынужденных колебаний // Материалы XV Всероссийской научно-технической конференции «Динамика нелинейных дискретных электротехнических и электронных систем». Чебоксары, 02 июня 2023 года. Чебоксары: Чувашский государственный университет имени И. Н. Ульянова, 2023. С. 7–8.

6. **Кононенко В. О.** Колебательные системы с ограниченным возбуждением. М.: Наука, 1964. 236 с.

7. **Фролов К. В.** Избранные труды. В 2 тт. Т. 1. Вибрация и техника. М.: Наука, 2007. 349 с.

8. **Рубаник В. П., Старик Л. К.** Об устойчивости автоколебаний резца в случае неидеального источника энергии // Научные труды высших учебных заведений Литовской ССР. Серия «Вибротехника». 1971. № 2 (11). С. 205–212.

9. **Абдиев Ф. К.** Автоколебания системы с запаздыванием и с неидеальным источником энергии // Известия АН АзССР. Серия физико-технических и математических наук. 1983. № 4. С. 134–139.

10. **Bharti S. K., Bisoi A., Sinha A., Samantaray A. K., Bhattacharyya R.** Sommerfeld effect at forward and backward critical speeds in a rigid rotor shaft system with anisotropic supports // Journal of Sound and Vibration. 2019. Vol. 442. 3 March. Pp. 330–349.

11. **Sinha A., Samantaray A. K., Bhattacharyya R.** Self-synchronization of two unbalanced DC motor-driven rotors on a common movable platform // Badodkar D., Dwarakanath T. (Eds.). Machines, mechanism and robotics. Proceedings of iNaCoMM 2017. Book Ser. “Lecture Notes in Mechanical Engineering”, Springer, Singapore, 2018. Pp. 207–217.

12. **Jha A. K., Dasgupta S. S.** Attenuation of Sommerfeld effect in an internally damped eccentric shaft-disk system via active magnetic bearings // Meccanica. 2019. Vol. 54. No. 1–2. Pp. 311–320.

13. **Varanis M., Balthazar J. M., Silva A., Mereles A. G., Pederiva R.** Remarks on the Sommerfeld effect characterization in the wavelet domain // Journal of Vibration and Control. 2019. Vol. 25. No. 1. Pp. 98–108.

14. **Rocha R. T., Balthazar J. M., Tusset A. M., Souza S. L. T., Janzen F. C., Arbex H. C.** On a non-ideal magnetic levitation system: nonlinear dynamical behavior and energy harvesting analyses // Nonlinear Dynamics. 2019. Vol. 95. No. 4. Pp. 3423–3438.

15. **Sinha A., Bharti S. K., Samantaray A. K., Bhattacharyya R.** Sommerfeld effect in a single-DoF system with base excitation from motor driven mechanism // Mechanism and Machine Theory. 2020. Vol. 148. June. P. 103808.

16. **Чернышева Ю. В., Горский А. Н., Середа А. Г.** Эффект Зоммерфельда и динамика вагонов // Электроника и электрооборудование транспорта. 2019. № 1. С. 8–10.

17. **Ещин Е. К.** Об эффекте Зоммерфельда и скрытых колебаниях в асинхронном электроприводе горных машин // Известия высших учебных заведений. Электромеханика. 2023. Т. 66. № 1. С. 80–85.

18. **Alifov A. A.** About calculation of self-oscillatory system delayed and limited excitation // Proceedings of the International Scientific and Technical Conference “Measurement and Quality: Problems, Prospects”; November 21–23, 2018, AzTU, Baku, Azerbaijan. Pp. 289–293.

19. **Алифов А. А.** Методы прямой линеаризации для расчета нелинейных систем. Москва–Ижевск: НИЦ «Регулярная и хаотическая динамика». 2015. 74 с.

20. **Климов Д. М.** Об одном виде автоколебаний в системе с сухим трением // Известия Российской академии наук. Механика твердого тела. 2003. № 3. С. 6–12.

21. **Пановко Я. Г.** Основы прикладной теории колебаний и удара. Ленинград: Машиностроение, 1976. 320 с.

22. **Мурашкин Л. С., Мурашкин С. Л.** Прикладная нелинейная механика станков. Ленинград: Машиностроение, 1977. 192 с.

23. **Пономарев А. С., Соколов В. И., Кожухарь А. А.** Поперечные автоколебания силовых столов, вызванные силами трения // Вестник Харьковского политехнического института. Машиностроение. 1977. Т. 8. № 130. С. 67–69.

24. **Коритысский Я. И.** Крутильные автоколебания вытяжных приборов прядильных машин при граничном трении в опорах скольжения // «Нелинейные колебания и переходные процессы в машинах». Сб. статей АН СССР под ред. К. В. Фролова. М.: Наука, 1972. 303 с.

25. **Броновец М. А., Журавлёв В. Ф.** Об автоколебаниях в системах измерения сил трения // Известия Российской академии наук. Механика твердого тела. 2012. № 3. С. 3–11.

26. **Алифов А. А.** Смешанные вынужденные, параметрические и автоколебания при неидеальном источнике энергии и запаздывающих силах // Известия высших учебных заведений. Прикладная нелинейная динамика. 2021. Т. 29. № 5. С. 739–750.

27. **Боголюбов Н. Н., Митропольский Ю.А.** Асимптотические методы в теории нелинейных колебаний. М.: Наука, 1974. 504 с.

28. **Моисеев Н. Н.** Асимптотические методы нелинейной механики. М.: Наука, 1981. 400 с.

29. **Бутенин Н. В., Неймарк Ю. И., Фуфаев Н. А.** Введение в теорию нелинейных колебаний. М.: Наука, 1976. 256 с.

30. Вибрации в технике. Справочник. В 6 тт. Ред. совет: В. Н. Челомей (пред., гл. ред.). Т. 2. Колебания нелинейных механических систем. Под ред. И. И. Блехмана. 1979. 351 с.

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