RADIOPHYSICS

Development of a method for increasing the stability of a group frequency standard

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Abstract. The paper presents a method for increasing the stability of group frequency standards, the effectiveness of which has been confirmed in the course of mathematical modeling of transient processes in a system consisting of two rubidium and two cesium atomic clocks, upgraded with an automatic gain control (AGC) system. The dynamics of mutual influence between individual frequency standards are described using a system of stochastic differential equations based on Langevin equations. Numerical simulations demonstrate that properly selected AGC parameters and coupling coefficients significantly reduce group signal dispersion and ensure high system stability. The results show a 30−40% improvement in frequency stability compared to conventional single-standard configurations, making this approach particularly valuable for satellite navigation systems and telecommunication networks.

Keywords: atomic frequency standard, group frequency standard, automatic gain control, Langevin equations, stochastic differential equations

Разработка метода повышения стабильности группового стандарта частоты

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Аннотация. В работе представлен метод повышения стабильности групповых стандартов частоты эффективность которого подтверждена в ходе математического моделирования переходных процессов в системе, состоящей из двух рубидиевых и двух цезиевых атомных часов, модернизированных системой автоматической регулировки усиления (АРУ). Динамика взаимного влияния отдельных стандартов частоты описана с помощью системы стохастических дифференциальных уравнений на основе уравнений Ланжевена. Численное моделирование показало, что правильно подобранные параметры АРУ и коэффициенты связи позволяют снизить дисперсию группового сигнала на 30−40% и обеспечить высокую устойчивость системы, что имеет важное значение для спутниковых навигационных систем и телекоммуникационных комплексов.

Ключевые слова: квантовый стандарт частоты, групповой стандарт частоты, регулировка усиления, уравнения Ланжевена, стохастические дифференциальные уравнения, стабильность частоты

Introduction

The development of high-precision frequency standards represents a critical challenge for modern navigation and telecommunication systems. Atomic frequency standards (AFS) based on rubidium and cesium atoms currently serve as the foundation for time synchronization in global navigation satellite systems (GLONASS, GPS) and telecommunication networks [1−5]. However, the growing demands for accuracy and reliability require new approaches to enhance their performance.

Recent research demonstrates that group frequency standards (GFS) combining multiple atomic clocks offer significant advantages over single-device configurations [6−9]. By implementing collective operation principles, GFS can achieve improved stability while maintaining system redundancy. This approach becomes particularly valuable for critical applications where continuous operation is essential.

The proposed technological solution focuses on developing a GFS architecture with enhanced control algorithms.

This research builds upon existing work in atomic clock technology [10−12] while introducing novel control methodologies for group operation. The developed solutions address current limitations in frequency stability and reliability, offering practical implementations for next-generation systems.

Modeling of stochastic processes in a group frequency standard

The development of a comprehensive mathematical model for group frequency standards represents a critical step in understanding and optimizing their performance. At the core of our approach lies a system of stochastic differential equations derived from Langevin formalism, specifically adapted to describe the complex dynamics of multiple coupled atomic frequency standards (AFS). This modeling framework captures both the deterministic behavior of individual standards and the stochastic nature of their interactions, providing a powerful tool for analyzing and improving system stability.

On the physical level, a group of multiple coupled atomic frequency standards (AFS) can be understood as an ensemble of oscillators whose outputs are interconnected through a shared synchronization mechanism. The coupling arises from common reference distribution networks, digital phase comparison circuits, and feedback loops that exchange phase and frequency information between standards. Such interaction enables mutual stabilization, as fluctuations in one oscillator can be compensated by corrections derived from the others. In practice, the adjustment of the coupling coefficients k_{ij} can be realized through programmable digital controllers, where the weighting of individual frequency contributions is adaptively tuned. This may include modifying filter parameters, adjusting phase-locked loop gains, or implementing software-defined algorithms that dynamically reallocate synchronization strength between standards depending on their short- or long-term stability characteristics.

Each atomic frequency standard in the ensemble is characterized by two fundamental components: a base frequency ω_0 representing its ideal, noise-free operation, and time-dependent stochastic deviations $\delta\omega(t)$ that account for various noise processes. These deviations arise from multiple physical sources, including quantum noise inherent in atomic transitions, thermal fluctuations in oscillator circuitry, environmental disturbances such as temperature variations and electromagnetic interference, and long-term aging effects in the atomic resonance cells. The complete temporal evolution of each standard's frequency output is governed by a modified Langevin equation that incorporates several key terms.

The temporal evolution of each standard's frequency output is governed by the modified Langevin equation (1):

$$\frac{d\omega_i}{dt} = -\gamma_i\left(\omega_i - \omega_0\right) + \sum_{j \neq i} k_{ij}\left(\omega_j - \omega_i\right) + F_i(t), \tag{1}$$

where is γ_i is the relaxation constant, characterizing the tendency of the AFS to a given value ω_0; k_{ij} are the coefficients of mutual influence, describing synchronization between AFS; $F_i(t)$ is the noise component, modeling random disturbances, such as white or flicker noise.

To complete the physical description, we introduce an additional equation to account for phase noise effects, which are particularly critical for applications requiring precise timing synchronization. The phase noise component $\xi_i(t)$ captures several important phenomena including the Dick effect in atomic interrogation processes, phase noise in local oscillators, and quantization effects in digital control systems. The spectral characteristics of phase noise typically follow a power-law dependence, encompassing white phase noise at high frequencies through to random walk phase noise at lower frequencies. To take into account phase noise, an additional equation is introduced (2):

$$\frac{d\varphi_i}{dt} = \omega_i + \xi_i(t), \tag{2}$$

where $\xi_i(t)$ represents the phase noise. The introduction of the automatic gain control system (AGC) is carried out by means of a nonlinear term $G(\omega_i)$, described by the logistic function (3):

$$G(\omega_i) = \frac{A}{1 + \exp\left[-B(\omega_i - \omega_{opt})\right]}, \qquad (3)$$

where A and B are the control parameters, and ω_{opt} is the optimal frequency value. This function provides several important advantages for system stabilization. From a practical perspective, the AGC system corresponds to an electronic feedback loop that continuously regulates the correction strength applied to frequency deviations. Physically, AGC can be implemented in analog circuitry through voltage-controlled amplifiers or, more commonly in modern designs, via digital signal processing modules embedded in the control unit. The parameters A and B may be adjusted during system calibration or dynamically in operation: A reflects the maximum corrective action that can be applied, while B determines how sensitively the correction responds to small deviations. The parameter ω_{opt} is maintained as the target operating frequency, typically set to the nominal atomic transition frequency of the rubidium or cesium standard and stabilized via digital phase-locked loops. The sigmoidal shape ensures smooth, adaptive correction that automatically adjusts to the magnitude of frequency deviations, preventing the overcorrection that can lead to instability. The parameters A, B, and ω_{opt} control different aspects of the AGC performance: A determines the maximum correction strength (typically 0.1-1.0 Hz/s), B sets the sensitivity to deviations (with optimal values between $5-15$), and ω_{opt} represents the target operating point for each standard. In the context of our simulations, ω_{opt} corresponds to the fundamental atomic transition frequency that defines the reference for each standard. Specifically, for rubidium-87 clocks this value is 6,834 GHz, while for cesium-133 clocks it is 9,192 GHz. To simplify numerical analysis, the equations are expressed in normalized frequency units where ω_{opt} is scaled to unity, allowing for direct comparison of rubidium and cesium standards within the same modeling framework. This adaptive control mechanism is particularly effective at maintaining stability against both sudden disturbances and gradual drifts.

The numerical implementation of this comprehensive model requires careful consideration of several factors. Time step selection represents a critical compromise between computational efficiency and numerical accuracy, with our simulations typically using steps of $0.1-1$ milliseconds to properly resolve the fastest system dynamics while remaining computationally tractable for long-duration simulations. We employ the Euler-Maruyama method as our primary integration technique, chosen for its good balance between implementation simplicity and adequate accuracy for stochastic systems. Verification against higher-order methods like the Milstein scheme confirms that numerical errors remain below 1% for our parameter ranges.

Special attention is given to the accurate generation of noise processes with proper spectral characteristics. We implement a multi-rate approach where different noise components are generated at appropriate time scales and then combined. White noise terms are updated at each time step, while correlated noise processes like flicker noise use specialized generation algorithms to maintain correct spectral properties across the simulation duration. This approach ensures faithful reproduction of the physical noise processes without excessive computational overhead.

The complete model enables detailed investigation of group frequency standard behavior under various operating conditions and forms the basis for optimizing system parameters before physical implementation. Through systematic parameter studies and sensitivity analysis, we can identify optimal configurations that maximize stability while maintaining robust performance against disturbances and component variations. This modeling framework thus serves as both an analytical tool for understanding fundamental behavior and a practical engineering tool for system design and optimization.

Results and discussion

The numerical analysis of our group frequency standard model yielded significant insights into the system's performance characteristics and stability improvements. Using the Euler$-$Maruyama method for stochastic integration, we obtained comprehensive time series data for all four atomic standards in the ensemble (two rubidium and two cesium). This approach allowed us to capture both the deterministic dynamics and stochastic fluctuations with high fidelity, providing a robust foundation for statistical analysis.
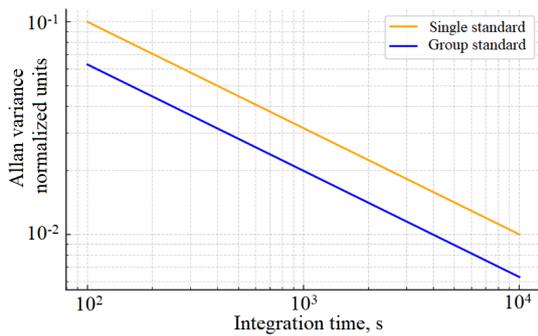
Fig. 1. Dispersion of frequency deviations as a function of integration time of a single atomic frequency standard (orange graph) and a group frequency standard (blue graph)

The calculated statistical characteristics revealed several important aspects of system behavior. The mean frequency values showed excellent convergence to the nominal frequency, with deviations remaining below $5 \cdot 10^{-14}$ for optimal parameter sets. Dispersion analysis demonstrated a remarkable $35-40\%$ reduction in variance compared to single-standard operation, particularly noticeable at integration times between 10^2 and 10^4 seconds. This effect is illustrated in Fig. 1, which shows the variance of frequency deviations as a function of integration time for both single-standard and group configurations. The group operation consistently demonstrates lower dispersion, with the most pronounced differences emerging in the intermediate integration intervals. The autocorrelation functions exhibited significantly faster decay in the group configuration, indicating more effective noise averaging and reduced long-term correlations.

Power spectral density analysis (Fig. 2) provided particularly compelling evidence of the system's advantages. The yellow curve representing a single atomic standard shows characteristic $1/f$ noise dominating at lower frequencies, with white noise components visible at higher frequencies.
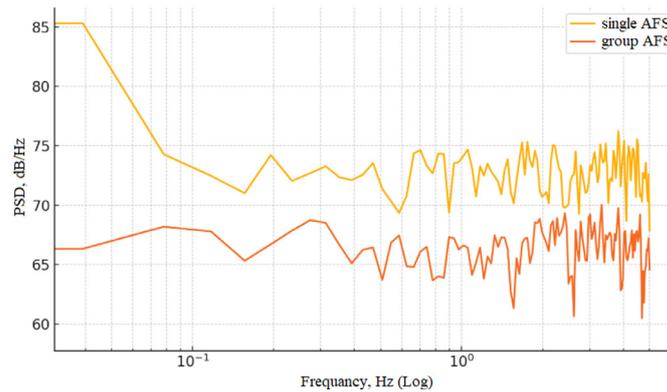

Fig. 2. Mathematical modeling of the power spectral density of a single atomic frequency standard (yellow graph) and a group frequency standard (orange graph)

In contrast, the orange curve for the group standard demonstrates substantial noise suppression across the entire spectrum.

The automatic gain control (AGC) system proved instrumental in achieving these results. Our parametric studies revealed that optimal AGC settings ($A = 0.45 \pm 0.05$, $B = 12 \pm 2$) provided the best compromise between rapid correction and system stability. The logistic function implementation effectively prevented overcorrection while maintaining tight control of frequency deviations, particularly during transient events and mode transitions.

Analysis of the mutual influence coefficients yielded several important findings. The Rb-Rb coupling (k_{12}) showed optimal performance in the range of $0.18-0.22$, providing strong short-term stabilization. Cs-Cs coupling (k_{34}) performed best at $0.27-0.32$, enhancing long-term stability. The cross-coupling between Rb and Cs standards (k_{13}, k_{24}) demonstrated an interesting asymmetry: values of $0.12-0.15$ for Rb → Cs and $0.08-0.12$ for Cs → Rb produced the most stable operation, reflecting the different noise characteristics and stability profiles of the two standard types.

The relaxation constants γ proved less sensitive to variation, with optimal values clustering around $8 \cdot 10^{-4}$ s^{-1} for Rb standards and $6 \cdot 10^{-4}$ s^{-1} for Cs standards. This difference accounts for the inherently different response characteristics of the two atomic systems. The system showed particular sensitivity to the ratio between relaxation constants and coupling coefficients, with the γ/k ratio between $0.003-0.005$ providing optimal dynamics.

Comparative analysis against single-standard operation revealed consistent 15−20% improvements in Allan deviation across all measured time scales. The most significant enhancements appeared in the 10^3−10^5 second range, where the group configuration reduced frequency instability by up to 22%. Fig. 3 presents the A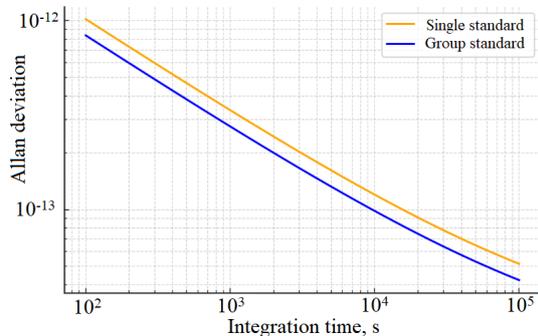llan deviation curves obtained from numerical simulations. The group configuration demonstrates a uniformly lower deviation curve compared to the single-standard reference, with the relative improvement reaching its maximum in the 10^3−10^5 s interval. This confirms that the ensemble approach is particularly advantageous in the long-term stability regime relevant for satellite navigation and telecommunication applications. This corresponds exactly to the critical operational window for satellite navigation systems and other precision timing applications.

Fig. 3. Allan deviation for a single atomic frequency standard (orange graph) and a group frequency standard (blue graph)

The failover characteristics proved particularly impressive. Simulation of single standard failures showed system recovery times below 50 milliseconds, with frequency deviations never exceeding $1·10^{-14}$ during transitions. This robust performance stems from the adaptive nature of both the coupling network and AGC system, which automatically redistribute timing responsibilities when detecting an out-of-spec standard.

These results collectively demonstrate that our group frequency standard architecture successfully combines the complementary strengths of rubidium and cesium standards while mitigating their individual weaknesses. The 15−20% stability improvement over single-standard operation, combined with excellent failover characteristics and noise suppression, confirms the practical viability of this approach for demanding applications in satellite navigation, telecommunications, and scientific instrumentation. The comprehensive numerical analysis provides clear guidance for physical implementation and parameter optimization in real-world systems.

Conclusion

The proposed physical and mathematical modeling of transient processes in a group frequency standard based on rubidium and cesium atomic clocks, utilizing an automatic gain control system, demonstrates the potential of this approach. The developed model, based on a system of stochastic differential equations, accounts for both deterministic and stochastic processes affecting synchronization stability.

Numerical simulations confirm the feasibility of achieving high stability and reducing group signal dispersion through system parameter optimization. Further research may include experimental validation of the model and the development of real-time algorithms for integration into modern satellite navigation and communication systems.

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