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AN ELECTRICAL LIFETIME OF POLYMERS IN TERMS OF THE CATASTROPHE THEORY

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Abstract. The present study analyzes the known expressions establishing exponential and power relationships between the lifetime of polymer dielectrics and the strength of the electric field acting on them. The work notes the necessity of mathematical substantiation of the known power expression for the field dependence of electrical lifetime. We obtained the equation of dependence of the electrical lifetime of polymeric dielectrics on the value of the applied electric field strength within the framework of the mathematical catastrophe theory. The study has the performance of quantitative estimation of the parameters of this equation for paper, epoxy and polyethylene terephthalate electrical insulation. We established a good agreement between the literature experimental data and the field dependences of the electrical lifetime of polymers plotted by the proposed equation. The study analyses geometric images of the fold catastrophe function reflecting the nature of the change in the dimensionless parameter of the rate of damage accumulation in a polyethylene terephthalate film under varying electric field strength. The paper shows the prospect of using the mathematical apparatus of catastrophe theory to describe experimental regularities of changes in the electrical strength properties of polymer dielectrics in strong electric fields.

Keywords: polymer dielectric, electrical lifetime, electrical breakdown, electric field strength, catastrophe theory, fold catastrophe.

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ЭЛЕКТРИЧЕСКАЯ ДОЛГОВЕЧНОСТЬ ПОЛИМЕРОВ С ТОЧКИ ЗРЕНИЯ ТЕОРИИ КАТАСТРОФ

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Аннотация. В рамках математической теории катастроф получено уравнение зависимости электрической долговечности полимерных диэлектриков от величины напряженности приложенного электрического поля. Выполнена количественная оценка параметров данного уравнения для бумажной, эпоксидной и полиэтилентерефталатной электроизоляции. Установлено хорошее согласие между опубликованными экспериментальными данными и полевыми зависимостями электрической долговечности полимеров, построенными по предложенному уравнению. Проанализированы геометрические образы функции катастрофы складки, отражающие характер изменения безразмерного параметра скорости накопления повреждений в пленке полиэтилентерефталата при варьировании напряженности электрического поля.

Ключевые слова: полимерный диэлектрик, электрическая долговечность, электрический пробой, напряженность электрического поля, теория катастроф, катастрофа складки.

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Introduction

Insulation of operating high-voltage electrical equipment is exposed to strong electric fields. The destructive effect of the field eventually leads to insulation breakdown, which is one of the main reasons for premature failure of most high-voltage electrical equipment [1–3]. Therefore, it is extremely important to understand the patterns of dielectric breakdown and find ways to prevent [4, 5], especially as increasingly stringent requirements are imposed on high-voltage equipment [6, 7].

It has been established that electrical degradation of polymer materials is kinetic in nature and is a process of gradual accumulation of damage to macromolecules, after which the material completely loses its insulating properties, i.e., electrical breakdown occurs [4, 8]. The main parameter characterizing the electrical strength properties of polymer dielectrics is the electrical lifetime, which is the time period from the beginning of voltage application to the dielectric until its breakdown [9, 10]. Studies on the behavior of dielectrics in strong electric fields focus closely on the variation in electrical lifetime for samples with different thickness [10, 11], ambient temperature [12–14] and applied field strength [3, 15, 16].

The field dependences of electrical lifetime are typically described using exponential [2, 9, 10, 16] and inverse power [1–3, 17] laws. Application of the exponential law to model the field dependence of the electrical lifetime in polymers is directly related to the physical process of damage accumulation in the polymers exposed to electric fields (thermally activated decomposition of molecular ions), since the speed of this process exponentially depends on the strength of this

field [8, 18]. In addition to providing a rigorous theoretical framework, exponential expressions ensure high accuracy of the obtained values of electrical lifetime [4, 10, 16].

Power laws have become widespread for evaluating the lifetime of polymer materials as part of technical tests [16]. However, despite the mathematical simplicity of the power expressions, their use for modeling the field dependences of the electrical lifetime of polymers has no physical justification. Some studies [19, 20] describe the experimental field dependences of electrical lifetime by relations representing a superposition of exponential and power laws. An increased number of significant parameters in such composite relations, on the one hand, contributes to increased accuracy of determining the electrical lifetime. On the other hand, this also complicates the form of these relations [21–23], consequently reducing their practical value.

Approaches from catastrophe theory [24] may provide a solution to this problem, as the phenomenological nature of this theory allows to obtain fairly simple mathematical expressions reflecting the behavior of physical and technical systems of different complexity with variable control parameters characterizing these systems. An important argument in favor of using methods of catastrophe theory to study the electrical properties of polymers is the sudden onset of breakdown in polymer dielectrics and its catastrophic nature [25]. Relatively few studies consider the degradation of polymer materials using the mathematical framework of catastrophe theory [26–29]; as a matter of fact, we have not managed to uncover any works applying this theory to describe the field dependence of electrical lifetime of polymers. We believe that filling this gap is of great scientific and practical importance.

The goal of this study is to better understand the influence of electric field strength on the lifetime of polymer dielectrics.

Catastrophe theory was used to achieve this goal. Our primary objective was to formulate an equation describing the dependence of electrical lifetime in polymer dielectrics on the strength of the applied electric field, determining the parameters of this equation for widely used dielectrics: paper, epoxy and polyethylene terephthalate (PET) insulation.

Problem statement

A key manifestation of the kinetic nature of electrical breakdown in polymers is that their lifetime t_{br} significantly depends on the strength E of the applied electric field [8, 10]. For this reason, a large number of studies on the electrical strength properties of polymer dielectrics are focused on obtaining the field dependence $t_{br} = f(E)$ of electrical lifetime.

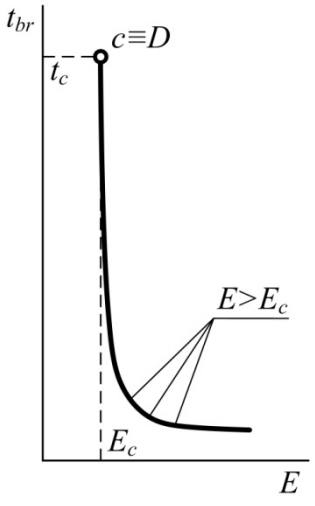

Fig. 1. Characteristic field dependence of electrical lifetime of polymer dielectrics t_{br}; E_c is the value of E at which t_{br} significantly exceeds the experimental period; t_c is the value of t_{br} for $E = E_c$; the identity $c \equiv D$ expresses the correspondence between the physical critical point c and the degenerate mathematical point D (see the explanations in the text)

In practice, the relationship $t_{br} = f(E)$ is established by conducting a series of experiments to measure the lifetime t_{br} at various fixed strengths $E = U/d$, where U is either a DC or AC voltage applied to a dielectric sample with a thickness D. In the case of tests in an AC electric field, the amplitude or RMS value of AC voltage is generally taken as the value of U [3, 4].

The lifetime of polymers placed in an AC electric field is approximately two orders of magnitude less than t_{br} for a DC field [4]. Meanwhile, experimental field dependences of electrical lifetime, measured in DC and AC electric fields, demonstrate almost identical behavior [2–4, 9, 10], which makes it possible to study the general patterns in their variation from a unified perspective. A typical dependence $t_{br} = f(E)$ whose behavior is consistent with known experimental data is shown in Fig. 1.

It can be seen that when the dielectric is in a field with the strength $E > E_c$, there is an inversely proportional relationship between t_{br} and E.

Fig. 1 shows the field strength E_c for which the estimated breakdown time t_{br} (the lifetime of the dielectric) significantly exceeds the duration of the experiment. Experiments to determine the lifetime are usually carried out at field strengths that ensure the detection of dielectric breakdown in the time range from 1 to 10^6 s [30]. In view of this, the value of E_c can range from a few to tens of MV/m, depending on the type and quality of the polymer dielectric sample, as well as its testing conditions [1–3]. The threshold strength of the working electric field [3, 31] corresponding to the given lifetime of the dielectric material can be used as E_c in a first approximation.

For small values of the applied field strength, when $E \approx E_c$ (see Fig. 1), we can observe a stronger field dependence for $t_{br} = f(E)$, so consequently, the lifetime $t_{br} = t_c$ becomes rather long, theoretically tending to infinity. The reason for this behavior of the function $t_{br} = f(E)$ is that the damage accumulation rate for $E \approx E_c$ is so low that a slight destructive effect of the electric field does not cause breakdown of the polymer dielectric at all and it retains its insulating properties for a long time.

Modeling the variation in the lifetime of dielectric materials with varying electric field strength accounting for the physical mechanism of damage accumulation in the dielectric produces an exponential field dependence $t_{br} = f(E)$. For polymer dielectrics, this dependence was obtained in the following form [18, 30]:

$$t_{br} = t_0 \exp\left(\frac{W^+ - el_0qE}{k_{\text{B}}T}\right), \tag{1}$$

where t_0, s, is a pre-exponential factor representing the vibrational frequency of atoms in solids, $t_0 \approx 10^{-13} - 10^{-12}$ s; W^+, J, is the bond dissociation energy of molecular ions; e, C, is the electron charge; l_0, m, is the activation length; q is the overvoltage factor characterizing the enhancement in field strength near irregularities on the electrodes; k_{B}, J/K, is the Boltzmann constant; T, K, is the temperature.

Accumulation of damage that can lead to dielectric breakdown is attributed in [18, 30] to a two-stage process of decomposition of polymer macromolecules in an electric field. At the first stage, field ionization of macromolecules occurs with the formation of positive molecular ions (macro-ions) with reduced bond dissociation energy [8, 18]. At the second stage, decomposition of macro-ions activated by thermal fluctuations occurs, accompanied by the formation of new macro-ions and chemically active free macroradicals [8, 18]. The gradual accumulation of defects (damage) during the decomposition of macromolecules leads to an increase in the concentration of charge carriers (quasi-free electrons and holes); reaching a critical concentration N_c is what causes breakdown [4, 8]. Varying the electric field strength allows to control the kinetics of accumulation of the concentration N_c, and consequently to vary the electrical lifetime of polymers [4, 8].

Exponential equation (1), obtained within the framework of the physical model [18], yields highly accurate theoretical values of the lifetime t_{br} agreeing well with the experimental data for a wide range of polymer dielectrics [9, 10].

Along with analytical expressions similar to Eq. (1), an empirical dependence of the form [2] is often used in practice to reflect the quantitative relationship between t_{br} and E observed in strong electric fields:

$$t_{br} = a(E - E_c)^{-b}, \tag{2}$$

where a, b are parameters depending on the type of insulation.

The simple mathematical form of power expression (2) was the main reason for its widespread use for estimating t_{br} during technical tests of polymer insulation [3, 16], which do not allow for accurate experimental design. It was found in [2, 3] that the use of the power dependence (2), firstly, is not physically justified, and, secondly, leads to a mathematical abstraction $t_{br} \to \infty$ at $E = E_c$, although in reality the period from the time when voltage is applied to the dielectric until the start of breakdown is finite.

Based on this, it is important to obtain an analytical equation for the field dependence $t_{br} = f(E)$, free from the above-mentioned disadvantages of expression (2). To solve it, we use the methods of catastrophe theory [24].

Fundamentals of catastrophe theory

To study physical systems in the theory of catastrophes, potential functions Φ (catastrophe functions) are used, whose type is determined by the number n of state variables x and the number k of control parameters A. The variables x characterizing the state of the system considered depend on the control parameters A. A smooth variation of A can correspond to both continuous and abrupt (catastrophic) variation of x. That is why the system's abrupt response to smooth variation in its control parameters is called a catastrophe.

At $k > 3$, mathematical description of catastrophes becomes rather cumbersome and it is difficult to visually represent the geometric characteristics of potential functions Φ. For many practical applications, it is sufficient to confine the consideration to considering catastrophe functions with one state variable and a number of control parameters not exceeding three. The well-known expression for the catastrophe function Φ with one state variable x has the following form [24]:

$$\Phi(x; A_i) = x^{k+2}/(k+2) + \sum_{i=1}^{k} A_i x^i/i, \tag{3}$$

where i is the number of the control parameter A, $i \in [1, k]$; $k = 1$, 2, and 3 for fold, cusp and swallowtail catastrophes, respectively.

Qualitative changes in the system considered are determined by the number and type of critical points of the catastrophe function. Therefore, the basis of catastrophe theory is analysis of the behavior of the function Φ in the vicinity of critical points. The positions of the critical points and the values of the function Φ at these points are determined by solving the equations obtained by equating its derivatives with respect to the state variable x to zero [24]:

$$\partial\Phi/\partial x = 0;\ \partial^2\Phi/\partial x^2 \neq 0; \tag{4}$$

$$\partial\Phi/\partial x = 0;\ \partial^m\Phi/\partial x^m = 0, \tag{5}$$

where m is the order of the derivative, $m \in [2, k+1]$.

The points found from conditions (4) are called non-degenerate critical points; in physical applications, the mathematical term 'non-degenerate critical point' corresponds to the definition of an equilibrium point. The solutions of Eqs. (5) correspond to degenerate critical points with the degree of degeneracy m, which are identical to physical critical points [24]. Degenerate critical points are more informative than non-degenerate ones, since it is in the vicinity of degenerate points that there is a high probability of a jump in the state of the system considered, i.e., the start of a catastrophe.

Catastrophe theory is concerned with variation in the equilibrium states $x(A_i)$ of the potential function $\Phi(x;A_i)$, given by Eq. (4), with varying control parameters A_i.

Catastrophe theory operates with dimensionless mathematical parameters, i.e., numbers that can take both positive and negative values. Therefore, to use this theory for physical applications, it is necessary to match the dimensionless mathematical parameters included in Eq. (3) with the dimensional quantities characterizing a particular physical process. The transition from dimensional physical quantities x' and A'_i to the corresponding dimensionless mathematical parameters x and A_i is carried out using normalization relations [24]:

$$x = x'/x_D - 1;\ A_i = A'_i/A_{D(i)} - 1, \tag{6}$$

where x' is the physical state variable; A'_i is the ith physical control parameter; x_D, $A_{D(i)}$ are the critical parameters of the catastrophe function, understood as the values of the quantities x' and A'_i at the physical critical point corresponding to the mathematical point D_{k+1}.

The point D_{k+1} is the point with the greatest degeneracy $m_{max} = k + 1$ (for fold, cusp, and swallowtail catastrophes, the degree of degeneracy m_{max} is 2, 3, and 4, respectively).

A bias term equal to unity is included in expressions (6), so consequently, when the physical quantities x' and A'_i reach the critical values x_D and $A_{D(i)}$, the mathematical parameters x and A_i take zero values. Because of this, the degenerate critical points D_2, D_3 and D_4, which are the germs of the fold, cusp and swallowtail catastrophes, are always located at the origin, greatly simplifying the analysis of these catastrophes.

It is advisable to analyze physical systems, processes or phenomena (referred to as systems from now on) in accordance with catastrophe theory by the following algorithm.

Step 1. Set the state variable x' and the control parameters A'_i, describing the physical system in question.

Step 2. Based on the known experimental and theoretical data describing the behavior of variable x', determine the general trends in its variation with varying parameters A'_i.

Step 3. Identify the characteristic attributes, the so-called catastrophe flags [24], which can be used to conclude that a catastrophe has occurred in the given system.

Step 4. Establish the event identified as catastrophe and determine the critical parameters x_D and $A_{D(i)}$, upon reaching which the most characteristic transformations of the system are clearly detected.

Step 5. Based on the relationship between x and A_i resulting from Eqs. (3) and (4) and on analysis of the dependence of x' on A'_i performed in step 2, match the dimensionless parameters x and A_i with the dimensional physical quantities x' and A'_i.

Step 6. Applying relations (6), we reduce the quantities x' and A'_i to a dimensionless form and, starting from general expression (3), write the equation of the function Φ modeling the given system. Based on the correspondence between x, A_i and x', A'_i, adopted in step 5, and considerations of algebraic convenience, multipliers can be introduced into the equation obtained for the catastrophe function to change the scale and reverse the sign of mathematical parameters.

Step 7. Determine the number and type of critical points of the catastrophe function Φ as well as analyze the geometric properties of the function Φ and its derivatives.

Step 8. Using conversion relations (6), represent the equation $\partial\Phi/\partial x = 0$ in physical terms and reduce it to the required dependence $x' = f(A'_i)$.

For consistency with the above algorithm, below we derive the equation describing the field dependence $t_{br} = f(E)$ of the electrical lifetime of polymer dielectrics.

Theoretical analysis of behavior of dielectric in strong electric field

First, we define the physical parameters of state and control that determine the electrical properties of the polymer dielectric at constant temperature. Initially, we take the electrical durability t_{br} as a state variable and the electric field strength E applied to the dielectric as a physical control parameter.

Next, we identify the flags indicating the possibility of a catastrophe in the dielectric exposed to a strong electric field.

The first flag for catastrophe is a critical charge concentration N_c accumulating in a time equivalent to the electrical lifetime t_{br} of the polymer dielectric. Experiments conducted in [4, 8], establishing this dependence found that $N_c \approx 10^{18}$–10^{19} cm^{-3}.

The second flag for catastrophe is the anomalous variance of experimental values of the electrical lifetime t_{br}, observed when low strengths comparable to E_c are applied to the dielectric. Indeed, as noted above, the field dependence $t_{br} = f(E)$ is considerably enhanced for $E \approx E_c$, in fact degenerating into an almost vertical straight line (see Fig. 1). Small variations in E near E_c lead to large variations in the lifetime t_{br}, so consequently, in practice, there is a large variance in the values of t_{br} measured at $E \approx E_c$ [1–3].

An event equivalent to the start of a mathematical catastrophe is the time when a critical concentration of damage is accumulated (which is equivalent to reaching N_c) in the polymer dielectric located in a field with the strength E_c. The point c in Fig. 1, in whose vicinity the field dependence of electrical lifetime becomes degenerate, is considered as the physical critical point. Then, following the above algorithm, we match the physical critical point c with the degenerate mathematical point D. Then the physical parameters t_c and E_c matched with point c become equivalent to the critical parameters of the catastrophe function: $t_c \equiv t_D$ and $E_c \equiv E_D$.

To describe the system considered (with one control parameter), we use equation for the fold catastrophe, which follows from the general expression (3) for $k = 1$ [24]:

$$\Phi(x; A_1) = x^3/3 + A_1 x. \quad (7)$$

It follows from Eq. (7) that the mathematical control parameter A_1 is proportional to the square of the state variable x: $A_1 \sim -x^2$. Therefore, in view of the proportionality $1/t_{br} \sim E^b$, characteristic for physical quantities t_{br} and E, it is convenient to take the field strength E as the state variable instead of the lifetime t_{br}. The quantity $v_{br} = 1/t_{br}$, inverse to electrical lifetime, is taken to be a physical control parameter in further analysis within the framework of catastrophe theory. The critical value v_{br} in this case is determined by the equality $v_D = 1/t_D$. According to the definition in [9], the parameter v_{br} characterizes the damage accumulation rate, i.e., the rate at which the polymer dielectric reaches breakdown.

The physical parameters of state x' and control A' can be interchangeable. As this may be necessary in situations such as ours (parameters t_{br} and E), when the parameters are substituted into the catastrophe equation, it is one of the advantage of the given approach. Moreover, such substitution is valid even when the physical quantity taken as a control parameter, i.e., an independent variable, cannot be directly varied in the real experiment (in this case, it is the electrical lifetime t_{br}), since mathematically, it makes little difference which dependence is considered in practice: $x' = f(A')$ or $A' = f(x')$.

Next, using normalization relations (6), we reduce the dimensional physical quantities E and v_{br} to dimensionless form:

$$F_E = E/E_D - 1;\ v = v_{br}/v_D - 1 = t_D/t_{br} - 1, \quad (8)$$

where F_E is the dimensionless electric field strength (mathematical state variable); v is the dimensionless damage accumulation rate (mathematical control parameter).

Taking $x = F_E$ and $A_1 = -a_t v$, we obtain from Eq. (7) an expression for the fold catastrophe function characterizing the state of the polymer dielectric in a strong electric field:

$$\Phi(F_E; v) = F_E^3/3 - a_t v F_E, \quad (9)$$

where a_t is the positive scale factor introduced for better visual representation of the geometric characteristics of the function Φ. The minus sign for a_t is taken for convenience of further analysis of the function Φ.

Equating the first and second derivatives of function (9) with respect to the variable F_E to zero, we write the equations determining the position of the critical points of this function:

$$\partial\Phi/\partial F_E = F_E^2 - a_t v = 0; \quad (10)$$

$$\partial^2\Phi/\partial F_E^2 = 2F_E = 0. \quad (11)$$

Since $a_t > 0$, Eq. (10) has no solutions for $v < 0$, and therefore, the catastrophe function $\Phi(F_E; v)$ has no critical points. Eq. (10) has one root $F_E = 0$ for $v = 0$ and the function $\Phi(F_E; v)$ has one critical point, a bifurcation point located at the origin. As Eqs. (10) and (11) are fulfilled simultaneously for $v = 0$, this critical point is doubly degenerate. If $v > 0$, Eq. (10) has two opposite roots: $F_E = \pm(a_t v)^{1/2}$, while the function $\Phi(F_E; v)$ has two non-degenerate critical points: a minimum point at $F_E > 0$ and a maximum point at $F_E < 0$.

Using conversion expressions (8), we write the relations that establish the correspondence between the characteristic values of dimensionless mathematical parameters (F_E, v) and dimensional physical quantities (E, t_{br}):

$$\begin{gathered} F_E < 0 \Leftrightarrow E < E_D;\ F_E = 0 \Leftrightarrow E = E_D;\ F_E > 0 \Leftrightarrow E > E_D; \\ v < 0 \Leftrightarrow t_{br} > t_D;\ v = 0 \Leftrightarrow t_{br} = t_D;\ v > 0 \Leftrightarrow t_{br} < t_D. \end{gathered} \quad (12)$$

Accounting for relations (12) and analyzing the results of Eqs. (9)–(11), we reached the following conclusions.

In the case when the electric field with the strength $E < E_D$ is applied to the dielectric, no electrical breakdown occurs, since the damage accumulation rate is low ($v < 0$) and the time to reach the critical concentration N_c exceeds t_D. As the field strength increases to the level $E > E_D$, a noticeable increase in the dimensionless accumulation rate ($v > 0$) is observed, as a result of which electrical breakdown of the dielectric occurs in the time $t_{br} < t_D$. The equality $E = E_D$, for which the field dependence $t_{br} = f(E)$ becomes degenerate, corresponds to the condition for the beginning of a catastrophe, i.e., reaching a critical value of N_c and breakdown of the dielectric during the time $t_{br} = t_D$.

Substituting relations (8) into Eq. (10), we obtain the final expression for the field dependence of electrical lifetime of the polymer dielectric:

$$t_{br} = \frac{t_D}{1 + a_t^{-1}(E/E_D - 1)^2}. \tag{13}$$

Eq. (13) holds true for $E \geq E_D$, i.e., when electrical breakdown of the dielectric occurs in a time not exceeding t_D. An advantage of Eq. (13) is its structure, ensuring a finite value for the electrical lifetime t_{br} of a field with the strength $E_D = E_c$ is applied to the dielectric.

Geometry of fold catastrophe

The profiles $\Phi = f(F_E)$ of the fold catastrophe function corresponding to the cases of variations in the dimensionless parameter v discussed above are shown in Fig. 2. The numbers $l \in [1, 5]$ of the profiles in Fig. 2 correspond to the characteristic values v_l of this parameter. Since $v_1 < v_2 < 0$, curves *1* and *2* have no critical points. Curve *3*, constructed for $v_3 = 0$, has a degenerate bifurcation point D_2 at the origin. Since $v_5 > v_4 > 0$, profiles *4* and *5* of the function Φ have minimum and maximum points characterizing stable and unstable equilibrium states of the system.

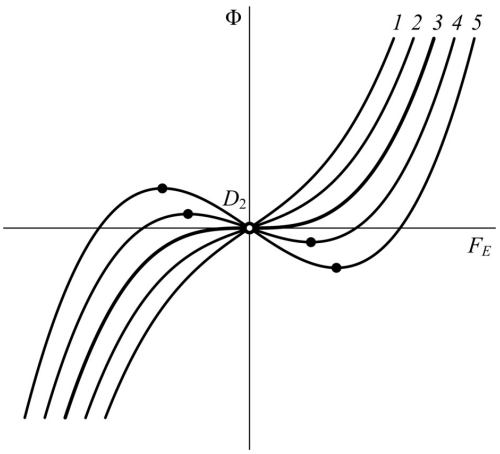

Fig. 2. Profiles of fold catastrophe function for different values of control parameter. Profile numbers correspond to the characteristic values of the parameter v_1–v_5

Black dots mark the positions of minima and maxima, characterizing stable and unstable equilibrium states of the system; D_2 is a degenerate bifurcation point

As the mathematical control parameter increases from v_1 to v_5, the profile of the function Φ gradually transforms, going through the following stages:

curve is flattened near zero,

degenerate point appears,

degenerate point is split into a maximum and a minimum equidistant from it,

amplitudes of these extrema further increase, the extrema keep diverging.

As the control parameter decreases from v_5 до v_1, the maximum and minimum on the profile of the catastrophe function converge, merge, forming a degenerate critical point at the origin, and disappear. The doubly degenerate point D_2 in whose vicinity the bifurcation of solutions is detected, acts as the germ for the fold catastrophe.

The geometric locus of the critical points satisfying Eq. (10) is a parabola whose branches converge at point D_2 (Fig. 3,*a*). The set of critical values Φ_c of the catastrophe function, determined by substituting roots from Eq. (10) into Eq. (9), forms a wedge with the edge at D_2 (Fig. 3,*b*).

There is not a single equilibrium state to the left of the point D_2 and the branches of stable (1*)* and unstable (2) equilibrium states are located to its right, formed respectively by the minima and maxima of the catastrophe function Φ.

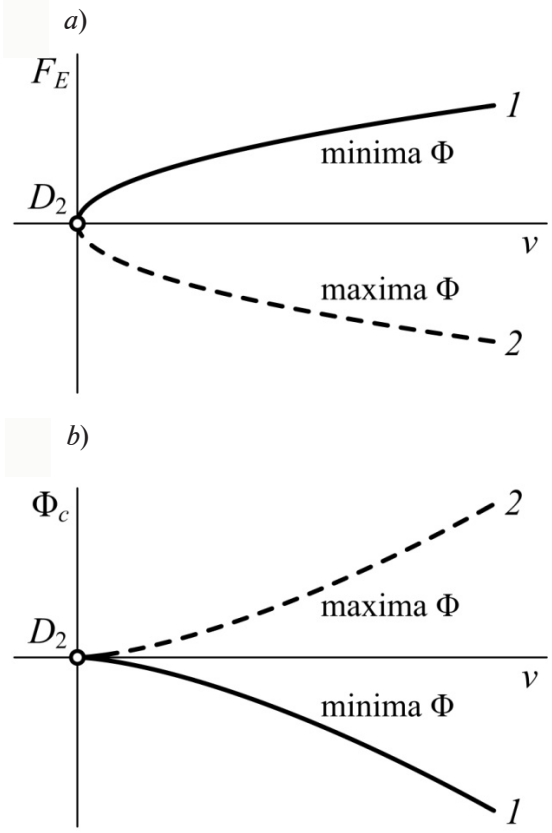

Fig. 3. Geometry of fold catastrophe: function of dimensionless damage accumulation rate (*a*) and critical value function (*b*)
Branches of stable (*1*) and unstable (*2*) equilibrium states are shown, formed respectively by the minima and maxima of the catastrophe function Φ (see Fig. 2)

estimated electrical lifetime at a strength $E = E_D = 2.32$ MV/m turned out to be fairly long: $t_D = 29.23 \cdot 10^7$ s ≈ 9.27 years.

It follows from the profiles $\Phi = f(F_E)$ shown in Fig. 2 that an increase in the parameter v causes a shift in the minimum/maximum of the function Φ to the region of larger positive/negative values of the state variable F_E and an increase in the amplitudes of these extrema. The result of this behavior of Φ is that when v increases on stable branches of the dependences $F_E = f(v)$ and $\Phi_c = f(v)$ (see Fig. 3), F_E increases and Φ_c decreases. Conversely, F_E decreases and Φ_c increases on unstable branches of these dependences.

Results and discussion

The applicability of Eq. (13) was verified by processing the experimental data presented in [1–3, 31–33] for the field dependences of electrical lifetimes of insulating materials. The values of the parameters a_t, E_D and t_D obtained by this treatment are given in Table.

Analyzing the data in Table, we can conclude that insulators made of phenol formaldehyde laminate, oil-impregnated paper and PET film are characterized by much lower t_D values than those made of epoxy dielectrics. The reason for this is that the dependence $t_{br} = f(E)$ for epoxy samples was obtained in experiments performed at lower (compared to E_D) field strengths, resulting in slower defect accumulation in these dielectrics and, accordingly, longer electrical lifetime. For example, the field strength at which the test samples experienced breakdown in a time not exceeding the maximum test time (30,000 hours) was only 2.5 MV/m for the EZK-1 compound [33]. Therefore, the

Table

Parameter values of equation for field dependence of lifetime for several electrical insulators

Electrical insulator	Parameter of Eq. (13)		
	a_t, 10^{-4}	E_D, MV/m	t_D, 10^7 s
Phenol formaldehyde laminate	2.3270	13.05	0.1094
Oil-impregnated paper	14.570	18.58	0.3507
PET film	22.180	24.16	0.0199
Epoxy resin	1.7910	4.59	1.0260
Epoxy compound EZK-1	0.9673	2.32	29.230

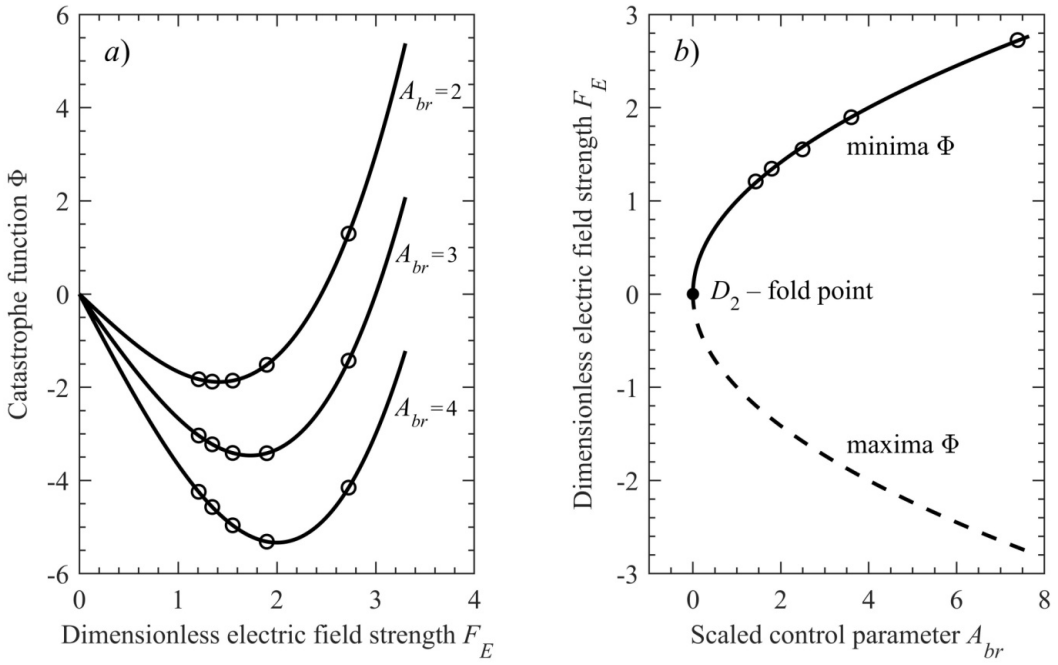

Fig. 4. Field dependences of fold catastrophe function Φ(*a*) and scaled damage accumulation rate A_{br}(*b*) for PET film (experimental data (shown by symbols) are taken from [32]); approximating curves (solid lines) correspond to physically realizable states and the set of unstable states of the function Φ (dashed lines) does not appear in reality

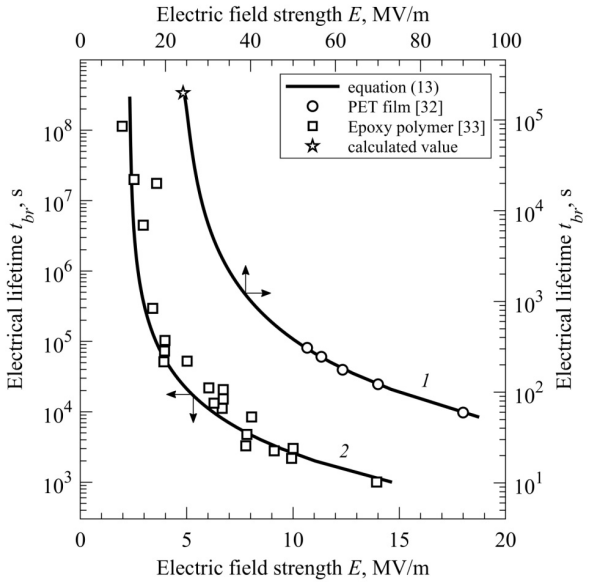

Fig. 5. Comparison of experimental (symbols) and theoretical (lines) field dependences of electrical lifetime of PET film (*1*) and epoxy compound (*2*)
The asterisk indicates the calculated value of t_D for the PET film corresponding to the doubly degenerate mathematical point D_2 (see Fig. 4,*b*)

As an example, Fig. 4 shows the graphical dependences $\Phi = f(F_E)$ и $F_E = f(A_{br})$ for PET film, based on experimental data taken from [32][1]. A_{br} in Fig. 4 denotes a scaled control parameter equal to the product of dimensionless damage accumulation rate v and the scale factor a_t, introduced earlier in Eq. (9): $A_{br} = a_t v$. The graphs constructed confirm an excellent agreement between the experimental data available in the literature and the approximating curve for the scaled damage accumulation rate A_{br}. The dashed line describing a decrease in the parameter A_{br} with an increase in the dimensionless strength F_E corresponds to a set of unstable states of the function Φ that does not occur in reality. The reason for this circumstance is that an increase in the field strength applied to the dielectric during the experiment leads to a decrease in the electrical lifetime t_{br}, equivalent to an increase in A_{br}.

The graph in Fig. 4,*a* clearly shows the trends in the behavior of the catastrophe function discussed above, namely: as the scaled parameter A_{br} increases, the minimum of the function Φ becomes deeper and shifts towards higher values of F_E. The decrease in A_{br} with a decrease in F_E (see Fig. 4,*b*) is consistent with the increase in electrical lifetime t_{br} observed in practice with a decrease in the strength E. The dimensionless parameter A_{br} becomes zero at point D_2 because t_{br} reaches the critical value t_D. Provided that $A_{br} < 0$, which is equivalent to the inequality $t_{br} > t_D$, the processes of electrical breakdown induced by the application of a weak electric field are slowed down and breakdown of the dielectric does not occur, even when the duration of the experiment significantly exceeds the critical value t_D.

The field dependences of electrical lifetime, constructed in accordance with Eq. (13) for PET film (curve *1*) and epoxy compound EZK-1 (curve *2*) in dimensional coordinates, are shown in Fig. 5. It can be seen that the solid curves *1* and *2* agree well with the experimental data [32, 33]. This confirms that Eq. (13) can be used to describe the field dependences $t_{br} = f(E)$ of polymer dielectrics. The asterisk in Fig. 5 indicates the calculated lifetime t_D for PET film corresponding to the doubly degenerate mathematical point D_2.

Some discrepancies between curves *1* and *2* in Fig. 5 and the corresponding experimental points can be explained by the following reasons:

large variance of experimental values of electrical lifetime determined for the same field strength;

complicated procedures to account for the cumulative effect of various factors (for example, temperature, partial discharges, structural inhomogeneity, etc.) on electrical breakdown in real polymer dielectrics.

Finally, using analytical equation (1), we estimate the overvoltage factor q for PET film located in an electric field with the strength $E = E_D$. We take the following values of the physical parameters for the calculation:

$$t_0 = 10^{-13} \text{ s}, W^+ = 1.1 \text{ eV}, l_0 = 1.5 \text{ Å [8]}; T = 293 \text{ K [32]};$$

$$E_D = 24.16 \text{ MV/m}, t_D = 0.0199 \cdot 10^7 \text{ s (see Table)}.$$

As a result, we obtain the value of the factor $q \approx 9.7$, which suggests that the decomposition of molecular ions activated by thermal fluctuations is the mechanism prevailing over all other mechanisms behind electrical breakdown in PET films [32]. Two circumstances can serve to prove this hypothesis. First, according to the information in [18], at $q \approx 10$, favorable conditions are created in the polymer for field ionization of macromolecules, which (as noted above) is the first stage of the two-stage decomposition process of these macromolecules in a strong electric field [8, 18]. Secondly, the field dependence $t_{br} = f(E)$ was measured in [32] at a temperature of 293 K, when thermal breakdown is extremely unlikely to occur [34, 35].

[1] See also: Yemelyanov O. A. Efficiency of metal-film capacitors in accelerated condition: Ph.D. Thesis. St. Petersburg, 2004. 246 p.

Conclusion

We derived Eq. (13) from the standpoint of catastrophe theory to describe the dependence of electrical lifetime of polymer dielectrics on the strength of the applied electric field. A quantitative assessment of the parameters of this equation is given for widely used electrical insulators: phenol formaldehyde laminate, epoxy compounds, oil-impregnated paper and PET film.

PET film was used as an example for analysis of the basic geometric properties determining the behavior of the fold catastrophe function and the scale parameter for damage accumulation rate with varying dimensionless electric field strength. We plotted the the field dependences of electrical lifetime for polyethylene terephthalate and epoxy molding compound EZK-1. The published experimental data and the approximating curves corresponding to Eq. (13) were found to be in excellent agreement.

The results obtained can be used to explain the variations in electrical lifetime based on operational data and technical tests of polymer dielectrics as well as to predict breakdown phenomena.

The advantage of the approach used in the paper is that general expression (3) for the catastrophe function $\Phi(x;A_i)$ can be modified if it is necessary to replace or increase the number of physical control parameters governing the behavior of dielectric materials in strong electric fields.

As a direction for future research, we plan to account for the influence of the operating temperature of insulation on the breakdown strength of polymer dielectrics, which is possible with theoretical consideration of a higher-order catastrophe, a cusp catastrophe.

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