MECHANICS

Original article
DOI: https://doi.org/10.18721/JPM.18112

TRANSIENT DYNAMICS OF AN ANISOTROPIC CHOW PLATE MOUNTED ON AN ELASTIC-INERTIAL BASE

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Abstract. In the paper, the unsteady dynamics of a thin elastic anisotropic unbounded Chow plate mounted on an elastic-inertial base has been studied. An anisotropy model where the elastic medium had one plane of symmetry geometrically coinciding with the median plane of the plate was considered. The base was specified by its stiffness coefficient and mass one. The problem statement included equations of motion in displacements, initial conditions and those of infinity. The solution of the problem was constructed using the method of fundamental solutions and integral Fourier transforms in spatial coordinates and Laplace ones in time. The Fourier originals were found numerically using the method of integrating rapidly oscillating functions. An example of the calculation was given.

Keywords: fundamental solution, anisotropic material, Chow plate, transient waves, integral transformation

Funding: The reported study was funded by Russian Science Foundation (Project No. 23-21-00189).

Citation: Serdyuk D. O., Zemskov A. V., Ershova A. Yu., Transient dynamics of an anisotropic Chow plate mounted on an elastic-inertial base, St. Petersburg State Polytechnical University Journal. Physics and Mathematics. 18 (1) (2025) 149–162. DOI: https://doi.org/10.18721/JPM.18112

Научная статья
УДК 539.3
DOI: https://doi.org/10.18721/JPM.18112

НЕСТАЦИОНАРНАЯ ДИНАМИКА АНИЗОТРОПНОЙ ПЛАСТИНЫ ЧОУ, СВЯЗАННОЙ С УПРУГО-ИНЕРЦИОННЫМ ОСНОВАНИЕМ

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Аннотация. В работе исследована нестационарная динамика тонкой упругой анизотропной неограниченной пластины Чоу, связанной с упруго-инерционным основанием. Рассмотрена модель анизотропии, в которой упругая среда имеет одну плоскость симметрии, геометрически совпадающую со срединной плоскостью пластины. Основание пластины характеризуется его коэффициентом жесткости и массовым коэффициентом. Постановка задачи включает уравнения движения в перемещениях, начальные условия и условия на бесконечности. Решение задачи построено с применением метода фундаментальных решений и интегральных преобразований Фурье по пространственным координатам и Лапласа по времени. Оригиналы по Фурье найдены численно с применением метода интегрирования быстро осциллирующих функций. Приведен пример расчета.

Ключевые слова: фундаментальное решение, анизотропный материал, пластина Чоу, нестационарное возмущение, интегральное преобразование

Финансирование: Работа выполнена при финансовой поддержке Российского научного фонда (проект № 00189-21-23).

Ссылка для цитирования: Сердюк Д. О., Земсков А. В., Ершова А. Ю. Нестационарная динамика анизотропной пластины Чоу, связанной с упруго-инерционным основанием // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2025. Т. 18. № 1. С. 149–162. DOI: https://doi.org/10.18721/JPM.18112

Introduction

Thin-walled structures, including plates, are among the structural elements that are widely used in diverse industries.

Such structures are generally compact, efficient and based on complex technologies. The already well-proven sheet rolling technology is often used for their production [1, 2]. Additive technologies are also developed [3–5]. Embedding of threads with complex structure to create composite materials was considered in [6], in particular for production of materials with spatial reinforcement. Adopting such technologies necessitates either purposeful or, as in the case of sheet metal rolling, secondary production of plates with anisotropy (including structural) of material properties.

These solutions are in demand in high-tech industries, for example, in the aerospace industry. This, in turn, stimulates the development of mathematical models and approaches to their solution in design of new machinery. Even though loads in technical systems are steady (cyclic, static), methods of transient dynamics should be used for simulations of pulse or shock effects.

Solving the related initial boundary value problems is often accompanied with considerable mathematical difficulties, since the required results depend not only on spatial coordinates, but also on time. However, it is not always possible to apply the variable separation procedure, which significantly complicates the Laplace transform as the main tool for solving transient problems.

The most comprehensive research in the field of transient plate dynamics began after the 1950s, which was greatly accelerated by the development of computer technologies. Chow [7, 8] obtained dynamic equations for orthotropic multilayer plates based on the Timoshenko beam theory taking into account the effects of transverse shear and rotatory inertia. Fallström and Lindblom conducted experimental and theoretical studies on the propagation of transient waves of normal displacements in isotropic, orthotropic, and anisotropic plates for the case of impact loading [9–11]. Miklowitz [12, 13] investigated the propagation of transient bending waves in unbounded isotropic plates and longitudinal waves in anisotropic plates. Daros [14] presented a dynamic fundamental solution for anisotropic Kirchhoff plates. The finite element method was used in [15–21] to study the transient dynamics of three-layer, and multilayer anisotropic plates under impact and contact loading.

This brief overview covers a wide range of approaches to study of transient processes in anisotropic plates. There is a noticeable trend in modern research towards predominantly using numerical procedures based on finite difference methods, finite and boundary elements. On the other hand, there are scarce works developing analytical methods for solving problems of transient plate dynamics. However, it is analytical solutions that make it possible to perform adequate parametric studies of the stress-strain state of the plate, depending on the nature of the load and the physical properties of the material.

Notably, there is a fairly wide variety of existing plate models, from the simplest ones based on Kirchhoff's hypotheses to the more complex ones, such as the Timoshenko model, which unlike the former, take into account shear deformation.

Chow's plate theory is an intermediate version that combines the simplicity of the Kirchhoff model and the ability to account for shear deformations.

In this paper, a new transient problem of elasticity theory is solved analytically, namely, new fundamental solutions are found for normal displacement of an anisotropic Chow plate [7]. The validity of the Chow plate theory was verified experimentally [9].

Problem statement

The rectangular coordinate system $Ox_1x_2x_3$ is taken to analyze the transient dynamics of an unbounded elastic Chow plate mounted on an elastic inertial base exposed to transient pressure $p(x_1,x_2,t)$ (Fig. 1). The midplane of the plate coincides with the Ox_1x_2 plane.

The plate has a constant thickness. The base is characterized by two parameters: stiffness c and mass coefficient m_f.

The equations of motion of the anisotropic Chow plate with a distributed surface load $p(x_1,x_2,t)$ applied have the following form [7, 8]:

$$\rho h\frac{\partial^2}{\partial t^2}\left(w_b+w_s\right)=hK_2\left(w_s\right)+p-c\left(w_b+w_s\right)-m_f\frac{\partial^2}{\partial t^2}\left(w_b+w_s\right),$$
$$\rho I\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_b}{\partial x_1^2}+\frac{\partial^2 w_b}{\partial x_2^2}\right)=IK_1\left(w_b\right)+hK_2\left(w_s\right), \tag{1}$$

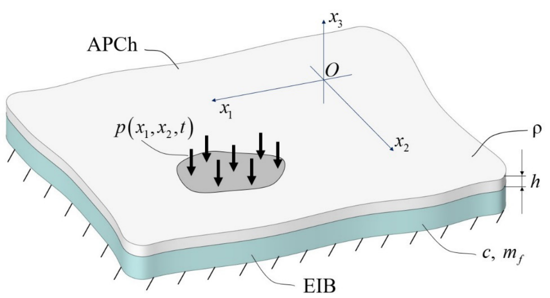

Fig. 1. Problem statement:
unbounded elastic anisotropic Chow plate (APCh) with thickness h mounted on an elastic inertial base (EIB) is subjected to transient pressure $p(x_1,x_2,t)$; ρ is the density of the APCh material; c, m_f are the stiffness and the mass coefficient of EIB

where p, Pa, is the load; ρ, kg/m³, *is* the density of the plate material; h, m, is its thickness; c, Pa/m, is the stiffness of the base; m_f, kg/m², is its mass coefficient; w_b, w_s, m, are the normal displacements induced by bending and shear of the plate, respectively; I, m³, is the linear moment of inertia, $I = h^3/12$; K_1, K_2 are differential operators of the equations of motion.

An elastic medium with one symmetry plane is considered. This plane geometrically coincides with the midplane of the plate Ox_1x_2. In this case, the physical characteristics of the elastic medium are characterized by eight components of the elasticity tensor:

$$c_{11},\ c_{12},\ c_{16},\ c_{22},\ c_{26},\ c_{44},\ c_{55},\ c_{66},$$

included in the expressions for K_1 and K_2:

$$K_1=c_{11}\frac{\partial^4}{\partial x_1^4}+c_{22}\frac{\partial^4}{\partial x_2^4}+2\left(c_{12}+2c_{66}\right)\frac{\partial^4}{\partial x_1^2\partial x_2^2}+4c_{16}\frac{\partial^4}{\partial x_1^3\partial x_2}+4c_{26}\frac{\partial^4}{\partial x_1\partial x_2^3},$$
$$K_2=c_{55}\frac{\partial^2}{\partial x_1^2}+c_{44}\frac{\partial^2}{\partial x_2^2}. \tag{2}$$

The total normal displacement of the Chow plate is determined by the sum of the normal displacements induced by both its bending and its displacement:

$$w\left(x_1,x_2,t\right)=w_b\left(x_1,x_2,t\right)+w_s\left(x_1,x_2,t\right);$$

This is characteristic of this model is what distinguishes it, in particular, from the models of Kirchhoff and Timoshenko plates.

Assuming that the plate is unbounded, Eqs. (1) are supplemented by conditions for the boundedness of the required fields at infinity and zero initial conditions, which mean that the plate is in an unperturbed state at the initial time:

$$w_b\big|_{t=0} = \frac{\partial w_b}{\partial t}\bigg|_{t=0} = w_s\big|_{t=0} = \frac{\partial w_s}{\partial t}\bigg|_{t=0} = 0,$$
$$\lim_{r\to\infty} w_b\left(x_1, x_2, t\right) = 0,\ \lim_{r\to\infty} w_s\left(x_1, x_2, t\right) = 0,\ r = \sqrt{x_1^2 + x_2^2}. \quad (3)$$

Fundamental solutions

The solution of problem (1)–(3) is constructed using the method of fundamental solutions (Green's functions, influence functions):

$$w\left(x_1, x_2, t\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{t} G_b\left(x_1 - \xi, x_2 - \zeta, t - \tau\right) p\left(\xi, \zeta, \tau\right) d\tau d\zeta d\xi +$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{t} G_s\left(x_1 - \xi, x_2 - \zeta, t - \tau\right) p\left(\xi, \zeta, \tau\right) d\tau d\zeta d\xi, \quad (4)$$

where $G_b(x_1, x_2, t)$, $G_s(x_1, x_2, t)$, s/kg, are the fundamental solutions (Green's functions) for normal displacements induced by bending and shear of the plate, respectively.

The formulation of the problem of fundamental solutions follows from the equations of motion, the initial conditions, and the conditions for the boundedness of the solution at infinity (1)–(3), where the Dirac delta function $\delta(x_1, x_2)\delta(t)$ acts as the load:

$$\rho h \frac{\partial^2}{\partial t^2}\left(G_b + G_s\right) = hK_2\left(G_s\right) + \ddot{a}\left(x_1, x_2\right)\ddot{a}\left(t\right) - c\left(G_b + G_s\right) - m_f \frac{\partial^2}{\partial t^2}\left(G_b + G_s\right),$$
$$\rho I \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 G_b}{\partial x_1^2} + \frac{\partial^2 G_b}{\partial x_2^2}\right) = IK_1\left(G_b\right) + hK_2\left(G_s\right), \quad (5)$$

$$G_b\big|_{t=0} = \frac{\partial G_b}{\partial t}\bigg|_{t=0} = G_s\big|_{t=0} = \frac{\partial G_s}{\partial t}\bigg|_{t=0} = 0,$$
$$\lim_{r\to\infty} G_b\left(x_1, x_2, t\right) = 0,\ \lim_{r\to\infty} G_s\left(x_1, x_2, t\right) = 0,\ r = \sqrt{x_1^2 + x_2^2}. \quad (6)$$

The solution of the problem is constructed using forward and inverse Fourier transforms in spatial coordinates x_1, x_2 and Laplace transforms in time t [22]:

$$\mathrm{A}\mathrm{g}^{FL} = \mathrm{q}^{FL},$$
$$\mathrm{g}^{FL} = \left(G_b^{LF}, G_s^{LF}\right)^T,\ \mathrm{q}^{FL} = \left(q^{FL}, 0\right)^T,\ q^{FL} = 1\big/\left(\rho h + m_f\right),$$
$$\mathrm{A} = \begin{pmatrix} s^2 + \dfrac{c}{\rho h + m_f} & s^2 + \dfrac{hQ_1 + c}{\rho h + m_f} \\ -s^2 Q_2 - \dfrac{Q_3}{\rho} & \dfrac{12 Q_1}{\rho h^2} \end{pmatrix}, \quad (7)$$
$$Q_1 = Q_1\left(q_1, q_2\right) = c_{55} q_1^2 + c_{44} q_2^2,\ Q_2 = Q_2\left(q_1, q_2\right) = q_1^2 + q_2^2,$$
$$Q_3 = Q_3\left(q_1, q_2\right) = c_{11} q_1^4 + c_{22} q_2^4 + 2\left(c_{12} + 2c_{66}\right) q_1^2 q_2^2 + 4c_{16} q_1^3 q_2 + 4c_{26} q_1 q_2^3,$$

where

$$\begin{Bmatrix} G_b^{LF}(q_1,q_2,s) \\ G_s^{LF}(q_1,q_2,s) \end{Bmatrix} = \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \begin{Bmatrix} G_b(x_1,x_2,t) \\ G_s(x_1,x_2,t) \end{Bmatrix} e^{-i(q_1x_1+q_2x_2)-s\tau} dx_1 dx_2 dt.$$

The solution of system (7) has the form

$$G_b^{LF} = \frac{1}{\rho h + m_f} \frac{R_1}{s^4 + R_3 s^2 + R_4}, \ G_s^{LF} = \frac{1}{\rho h + m_f} \frac{s^2 + R_2}{s^4 + R_3 s^2 + R_4}; \tag{8}$$

$$R_1 = R_1(q_1,q_2) = \frac{12 Q_1}{Q_2 \rho h^2}, \ R_2 = R_2(q_1,q_2) = \frac{Q_3}{Q_2 \rho},$$
$$R_3 = R_3(q_1,q_2) = R_1 + R_2 + \frac{Q_1 h + c}{\rho h + m_f}, \ R_4 = R_4(q_1,q_2) = \frac{c(R_1+R_2) + Q_1 h R_2}{\rho h + m_f}. \tag{9}$$

The roots of the denominator in solution (8) are found by the formulas

$$s_{1,2} = \pm i\alpha, \ s_{3,4} = \pm i\beta, \ \alpha = \alpha(q_1,q_2) = \sqrt{\frac{R_3 - \sqrt{D}}{2}}, \ \beta = \beta(q_1,q_2) = \sqrt{\frac{R_3 + \sqrt{D}}{2}},$$
$$D = R_3^2 - 4R_4, \ \sqrt{D} < R_3,$$

The initial values of the Laplace transform for fundamental solutions (8) are constructed analytically using deductions:

$$\begin{Bmatrix} G_b^F(q_1,q_2,t) \\ G_s^F(q_1,q_2,t) \end{Bmatrix} = \sum_{k=1}^{4} \lim_{s\to s_k} \left[(s - s_k) \begin{Bmatrix} G_b^{LF}(q_1,q_2,s_k) \\ G_s^{LF}(q_1,q_2,s_k) \end{Bmatrix} \right] e^{s_k t};$$

this leads to the expressions

$$G_b^F(q_1,q_2,t) = -\frac{1}{\rho h + m_f}\left(f_1 \sin(\alpha t) + f_2 \sin(\beta t) \right),$$
$$G_s^F(q_1,q_2,t) = \frac{1}{\rho h + m_f}\left(g_1 \sin(\alpha t) + g_2 \sin(\beta t) \right), \tag{10}$$

where

$$f_1 = f_1(q_1,q_2) = \frac{R_1}{\alpha^3 - \alpha\beta^2}, \ f_2 = f_2(q_1,q_2) = \frac{R_1}{\beta^3 - \beta\alpha^2},$$
$$g_1 = g_1(q_1,q_2) = \frac{\alpha^2 - R_2}{\alpha^3 - \alpha\beta^2}, \ g_2 = g_2(q_1,q_2) = \frac{\beta^2 - R_2}{\beta^3 - \beta\alpha^2}.$$

Fourier inversion is performed numerically, based on integration of rapidly oscillating functions [23]:

$$G_b(x_1,x_2,t) = \frac{\Delta_1^2}{16\pi^2} \sum_{k=0}^{R-1} \sum_{f=0}^{R-1} e^{\frac{i}{2}\left(\left(q_{1_{k+1}} + q_{1_k}\right)x_1 + \left(q_{2_{f+1}} + q_{2_f}\right)x_2 \right)} L(x_1,x_2,t),$$
$$L_{kf}(x_1,x_2,t) = e^{-i(m_1+m_2)} G_b^F\left(q_{1_k}, q_{2_f}, t\right) + e^{i(-m_1+m_2)} G_b^F\left(q_{1_k}, q_{2_{f+1}}, t\right) +$$
$$+ e^{i(m_1-m_2)} G_b^F\left(q_{1_{k+1}}, q_{2_f}, t\right) + e^{i(m_1+m_2)} G_b^F\left(q_{1_{k+1}}, q_{2_{f+1}}, t\right), \tag{11}$$

$$G_s\left(x_1,x_2,t\right)=\frac{\Delta_2^2}{16\pi^2}\sum_{p=0}^{F-1}\sum_{c=0}^{F-1}e^{\frac{i}{2}\left(\left(q_{1_{p+1}}+q_{1_p}\right)x_1+\left(q_{2_{c+1}}+q_{2_c}\right)x_2\right)}J\left(x_1,x_2,t\right),$$

$$J\left(x_1,x_2,t\right)=e^{-i\left(j_1+j_2\right)}G_s^F\left(q_{1_k},q_{2_f},t\right)+e^{i\left(-j_1+j_2\right)}G_s^F\left(q_{1_k},q_{2_{f+1}},t\right)+$$
$$+e^{i\left(j_1-j_2\right)}G_s^F\left(q_{1_{k+1}},q_{2_f},t\right)+e^{i\left(j_1+j_2\right)}G_s^F\left(q_{1_{k+1}},q_{2_{f+1}},t\right), \quad (12)$$

where

$$m_1=\frac{\Delta_1}{2}x_1,\ m_2=\frac{\Delta_1}{2}x_2,\ \Delta_1=\frac{2E}{R},$$

$$q_{1_k}=-E+k\Delta_1,\ q_{2_f}=-E+f\Delta_1,\ q_{1_{k+1}}=-E+\left(k+1\right)\Delta_1,\ q_{2_{f+1}}=-E+\left(f+1\right)\Delta_1,$$

$$j_1=\frac{\Delta_2}{2}x_1,\ j_2=\frac{\Delta_2}{2}x_2,\ \Delta_2=\frac{2Q}{F},$$

$$q_{1_p}=-Q+p\Delta_2,\ q_{2_c}=-Q+c\Delta_2,\ q_{1_{p+1}}=-Q+\left(p+1\right)\Delta_2,\ q_{2_{c+1}}=-Q+\left(c+1\right)\Delta_2.$$

The numerical integration parameters E and Q (integration limits) as well as R and F (integration steps) in Eqs. (11), (12), are calculated via a double-loop iterative scheme similar to the method described in [24]. This makes it possible to construct fundamental solutions with a given accuracy.

The new fundamental solutions G_b(11) and G_s (12) allow to study not only the transient dynamics of anisotropic plates but also plates whose material is orthotropic, transversally isotropic, or isotropic.

Example of fundamental solution analysis

Problem conditions and analysis of results**.** As an example, consider two plates made of orthotropic (E-glass fiber [25]) and anisotropic (carbon fiber [26]) materials. The physical properties of the plate materials are given in Table. Both plates have a thickness h = 0.003 m and are mounted on an elastic inertial base with a stiffness c = 100 Pa/m and a mass coefficient m_f = 30 kg/m^2.

Fig. 2 shows the spatial dependencies of the fundamental solutions $G_b(x_1,x_2,t)$ and $G_s(x_1,x_2,t)$ at a selected time t = 8 ms (see Eqs. (11) and (12), respectively). Analyzing these data, we detected

Table

Physical properties of plate materials selected as examples

Material	Elasticity tensor components, GPa	ρ, kg/m^3
E-glass fiber (orthotropic)	c_{11} = 48.210, c_{12} = 5.357, c_{16} = 0, c_{22} = 12.500, c_{26} = 0, c_{44} = 3.850, c_{55} = c_{66} = 5.000	2000
Carbon fiber (anisotropic)	c_{11} = 95.50, c_{12} = 28.90, c_{16} = 44.70, c_{22} = 25.90, c_{26} = 15.60, c_{44} = 4.40, c_{55} = 6.45, c_{66} = 32.70	1957

Note. The plates have the same thickness h = 0.003 m.

the characteristics of the propagation of perturbations: two planes of symmetry are clearly visible for the orthotropic medium, while asymmetric nature of this propagation is observed for the anisotropic medium. Notably, the amplitudes of the fundamental solutions G_s turned out to be four orders of magnitude smaller than the amplitudes of the function G_b. Consequently, the fundamental solution G_s makes an insignificant contribution to the normal displacement of the plate.

The new fundamental solutions G_b (11) and G_s(12) also make it possible to analyze plates without taking into account the elastic inertial base or to consider the case of a purely elastic base.

Using the example of the transient function (11), let us assess the degree to which the parameters of the inertial base c and m_f affect the behavior of the fundamental solution. The initial geometric data and parameters of the anisotropic carbon fiber plate material are as follows [9]:

Thickness, m 0.0041,

Material density, kg/m^3. 1957,

Elasticity tensor components, GPa. . .c_{11} = 23.51, c_{12} = 1.74, c_{16} = 0.88, c_{22} = 14.63, c_{26} = −3.54, c_{44} = 4.44, c_{55} = 4.22, c_{66} = 8.88

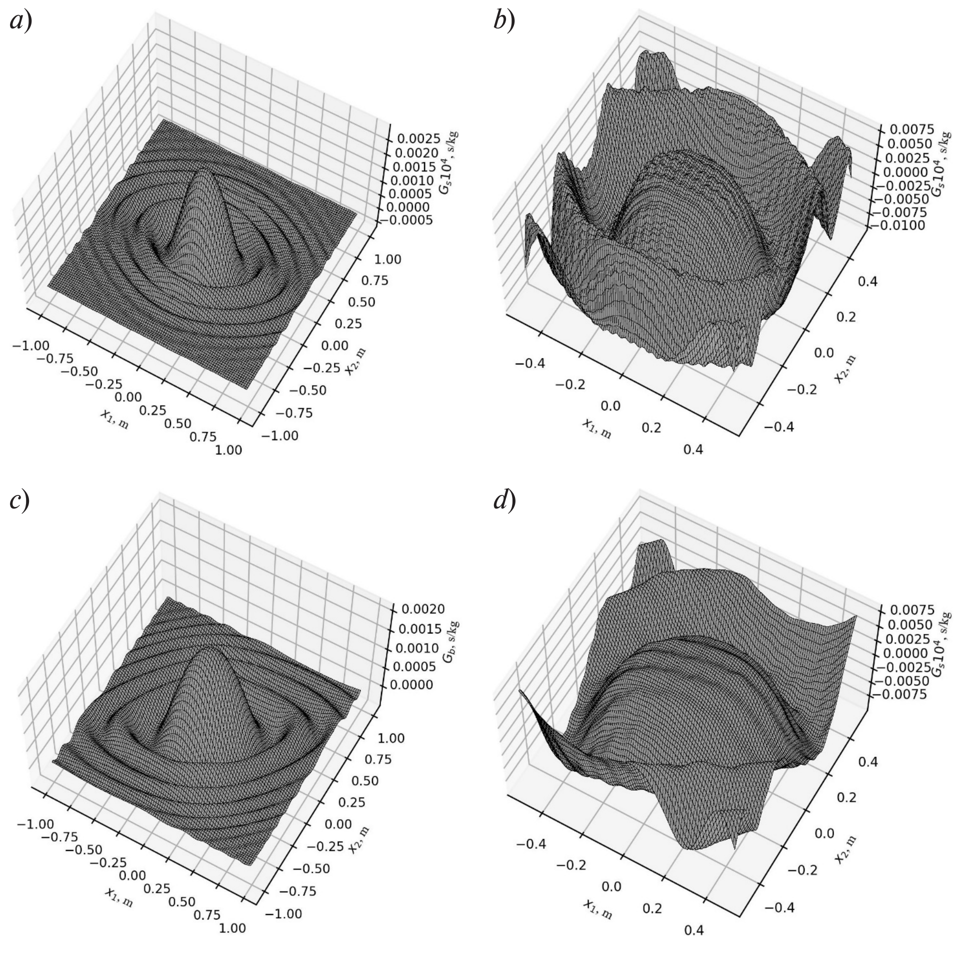

Fig. 2. 3D dependences of fundamental solutions $G_b(x_1, x_2, t)$ (*a*, *c*) and $G_s(x_1, x_2, t)$ (*b*, *d*) for normal displacements induced by bending (*a*, *c*) and shear (*b*, *d*) in plates made of E-glass fiber (*a*, *b*) and carbon fiber (*c*, *d*)

Time point t = 8 ms was taken

Fig. 3, *a*, *b* shows the behavior of the fundamental solution $G_b(x_1)$ at time points t = 8 and 10 ms with varying values of the base stiffness c, where the mass coefficient of the base is constant: m_f = 0 kg/m2. Here, curves *1–4* correspond to the following values of c, MPa/m: 0, 1, 2 and 3, respectively. It can be seen from Fig. 3, *a*, *b* that the maximum amplitude of the fundamental solution decreases with an increase in the stiffness of the base. Furthermore, as the time t increases from 8 to 10 ms, the amplitude decay rate increases with an increase in the stiffness of the base.

Fig. 3, *c*, *d* illustrates the behavior of the fundamental solution $G_b(x_1)$ at time points 8 and 10 ms with varying values of the mass coefficient m_f of the base but a constant value of c = 100 Pa/m. Here curves *5–8* correspond to the following values of m_f, kg/m^2: 0, 30, 60 and 90, respectively. It should be noted that the increase in the mass coefficient m_f not only reduces the maximum amplitude of the fundamental solution but also slows down the propagation of perturbations over time (see Fig. 3, *c*, *d*).

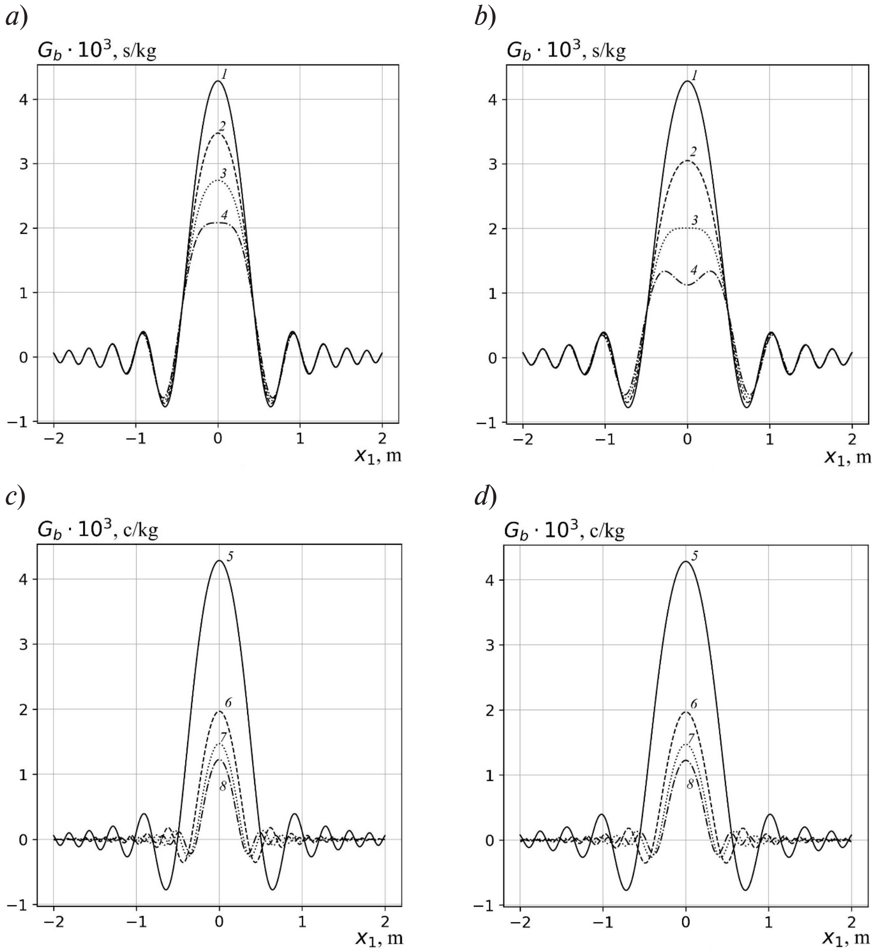

Fig. 3. Influence of stiffness (a, b) and mass coefficient (c, d) of Chow plate base on behavior of function G_b at time points t, ms: 8 (a, c) and 10 (b, d)
Values of stiffness c, MPa/m: 0 (curve *1*), 1 (curve *2*), 2 (*3*), 3 (*4*) and c = 100 Pa/m (curves *5*–*8*); values of mass coefficient m_f, kg/m²: 0 (curves *1*–*5*), 30 (curve *6*), 60 (*7*), 90 (*8*)

The constructed new fundamental solutions (11), (12) make it possible to analyze the transient dynamics of anisotropic, orthotropic, or isotropic Chow plates on the elastic inertial base or without it at all in solutions of direct, inverse, or contact unsteady problems. Below is an example solution to a direct problem.

Example of solution for direct problem. Consider the effect of two loads distributed over rectangular areas, taking the following form:

$$p(x_1, x_2, t) = \sum_{i=1}^{2} A_i(t) H(t) H\left(\frac{c_i}{2} - |x_1 - a_i|\right) H\left(\frac{d_i}{2} - |x_2 - b_i|\right), \tag{13}$$

where

$$a_1 = -0.3,\ b_1 = 0,\ c_1 = 0.03,\ d_1 = 0.03,\ a_2 = 0.3,\ b_2 = 0,\ c_2 = 0.03,\ d_2 = 0.03,$$

$$A_1(t) = -120000 \sin(200\pi t) e^{-1000t}\, \mathrm{H}(0.01 - t),$$

$$A_2(t) = -20000(1 - \cos(200\pi t))\, \mathrm{H}(0.01 - t),$$

on the Chow plate mounted on the elastic inertial base. In this expression, $A_i(t)$ corresponds to variation in the amplitude of the ith pressure area; H(t) is the Heaviside function; a_i, b_i are the coordinates along the axes x_1, x_2 where the ith resultant pressure area is applied at the initial point in time; c_i, d_i are the dimensions of the ith rectangular pressure area.

The material of the plate is carbon fiber (the initial calculation data, including elastic constants, are given above). The plate thickness and base parameters are taken as follows: h = 0.006 m, c = 0.4 MPa/m, m_f = 10 kg/m^2.

The loading conditions and the law for the amplitude of the transient load are shown in Fig. 4.

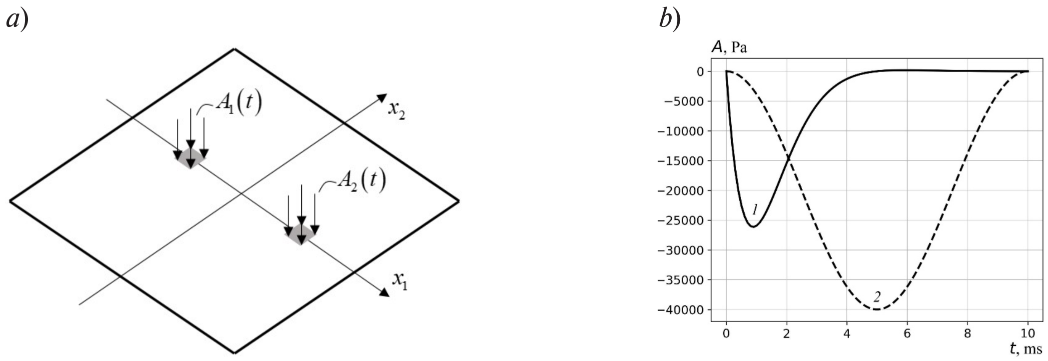

Fig. 4. Graphical representation for conditions of transient perturbation problem: loading conditions (a); dynamics of load amplitudes $A_1(t)$ (curve 1) and $A_2(t)$ (curve 2) (b)

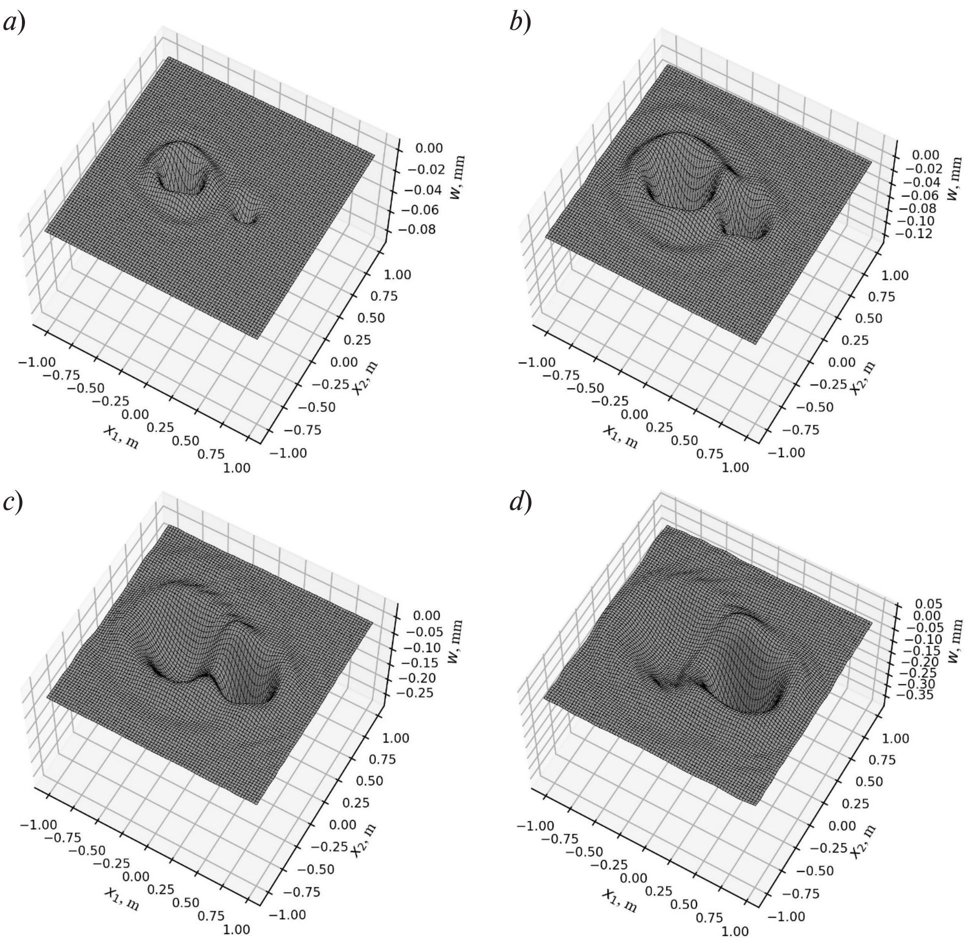

Fig. 5. Transient normal displacements of anisotropic Chow plate on elastic inertial base at different time points t, ms: 2 (a), 4 (b), 6 (c), 8 (d)

Substituting expression (13) into conditions (4), taking into account fundamental solutions (11) and (12), produces the results shown in Fig. 5: normal displacements of the anisotropic Chow plate on the elastic inertial base.

Three-dimensional images of normal displacements at fixed points in time (see Fig. 5) demonstrate their complex dependence on both distance and time. At time t = 2 ms, the asymmetric nature of the perturbations, induced by the anisotropic properties of the plate material, is clearly visible.

After that we can observe how the perturbations from the two pressure sources gradually reach each other (t = 4 ms), then merging (t = 6 ms) and mutually influencing each other (t = 8 ms).

Conclusion

In this paper, new solutions were obtained for the transient problem of elasticity theory posed for a thin elastic anisotropic unbounded Chow plate mounted on an elastic inertial base. These new fundamental solutions represent a significant contribution to the study of transient processes not only in anisotropic plates, but also in isotropic, transversally isotropic, and orthotropic structures.

This opens up avenues for new fundamental solutions in diverse fields, including transient dynamics of plates with initial perturbations, solutions of contact problems, problems with adhesion, as well as in inverse retrospective studies to determine the loading law based on normal displacements.

The practical significance of the results obtained lies in the possibility of developing engineering recommendations for solving applied problems related to transient normal displacements in plates in the period when perturbations have not yet reached the supports. The relationship of the normal displacement function with the known physical equations for linear bending moments, as well as with normal and tangential stresses, and, moreover (via Hooke's inverse law), with deformations, makes it possible to significantly improve the results based on the fundamental solutions obtained in the paper.

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Received 02.07.2024. Approved after reviewing 26.11.2024. Accepted 26.11.2024.
Статья поступила в редакцию 02.07.2024. Одобрена после рецензирования 26.11.2024. Принята 26.11.2024.