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Investigation of entangled states of a three-mode electro-optical modulator

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Abstract. We study the dynamic properties and algebraic invariants of the model of phase modulation of quantum light by microwave radiation. The quasi-energy levels of the system are described by the eigenvalues of the effective Hamiltonian, which generators obbey to su(2) algebra and have a nontrivial internal structure. The dynamics of states is studied in Fock space. Invariant Hamiltonian spaces are associated with irreducible representations of the su(2) algebra, within which the Hamiltonian matrix is partitioned into finite blocks. Within this model we investigate the generating process of two-mode entangled states using the single-tone Fock state at the input of phase modulator.

Keywords: Phase modulator, Hamiltonian, entangled states, ladder operators

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Исследование запутанных состояний трехмодового электрооптического модулятора

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Аннотация. Рассматриваются динамические свойства и алгебраические инварианты модели фазовой модуляции квантового света микроволновым излучением. Уровни квазиэнергии системы описываются собственными значениями эффективного гамильтониана, генераторы которого подчиняются алгебре su(2) и имеют нетривиальную внутреннюю структуру. Динамика состояний исследуется в пространстве Фока. Инвариантные гамильтоновы пространства связаны с неприводимыми представлениями алгебры su(2),где матрица гамильтониана разбивается на конечные блоки. В рамках этой модели исследуется процесс генерации двухмодовых запутанных состояний с использованием однотонального фоковского состояния на входе фазового модулятора.

Ключевые слова: Фазовый модулятор, Гамильтониан, запутанные состояния, лестничные операторы

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Introduction

The work [1] presented an explicitly solvable algebraic model of the process of electro-optic modulation of light by a microwave field based on the linear electro-optic effect. To describe the inter-mode interaction of light, the corresponding operators were obtained by applying the Jordan-Schwinger mapping [2] to generators of the matrix algebra su(2). It was shown that in the high-intensity approximation of the modulating microwave radiation, the model reduces to a semiclassical one, and the dynamic problem becomes exactly solvable. However, when the number of interacting optical modes exceeds three, the irreducible representations of the su(2) algebra related to invariant spaces turn out to be degenerate— the induced classification by the su(2) algebra is incomplete. To address this issue, a method of ladder operators for the Casimir operator image of the su(2) algebra was proposed in [3]. The obtained algebra of ladder operators generates the spectrum and eigenvectors of the Casimir operator, allowing for a description of all invariant spaces within the problem with three interacting frequency modes.

Materials and Methods

In this work, within the model described above, we investigate the generating process of two-mode entangled state using the single-tone optical signal in two-photon Fock state at the input of phase modulator. To the end we consider the image of the su(2) algebra represented by operators J_z, J_+, J_-. These operators through Jordan-Schwinger mapping are expressed by polynomials of bosonic operators a_μ, a_μ^+ of of frequency modes, associated with the eigenvalue s (non-negative integer or non-negative half-integer), $n = 2s + 1$.

$$J_z \sum_{\mu=-s}^{s} \mu a_\mu^+ a_\mu, \tag{1}$$

$$J_+ \sum_{\mu=-s}^{s-1} \sqrt{(s+\mu+1)(s-\mu)}\, a_{\mu+1}^+ a_\mu = (J_-)^+. \tag{2}$$

The operators J_z, J_+, J_- satisfy the the following commutation relations:

$$[J_z, J_+] = \pm J_+, [J_+, J_-] = 2J_z. \tag{3}$$

In order to obtain the basis of the invariant subspace associated with the irreducible representation of the su(2) algebra, we act on the vacuum vector with the ladder operator $\tau_1^+ = p_0^+ (j+1) + 2p_1^+$, resulting in the Fock state $\sqrt{\frac{2}{3}}|020\rangle + \frac{1}{\sqrt{3}}|101\rangle$, which is a coherent state of the su(2) algebra corresponding to state $|n = 2, j = 2, j_z = 0\rangle$ Acting on this obtained basis element with the operators J_+ and J_- yields the other Fock states $|200\rangle$, $|110\rangle$, $|011\rangle$, $|002\rangle$.These states form a basis for a five-dimensional invariant space (with “total moment” $j = 2$). The corresponding two-photon Fock space is six-dimensional thus, there exists an additional one-dimensional invariant space corresponding to $j = 0$, with its basis vector being the orthogonal vector $\frac{1}{\sqrt{3}}|020\rangle - \sqrt{\frac{2}{3}}|101\rangle$.

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In the following we consider the Hamiltonian of the phase modulator:

$$H = \omega m_{opt} N + \omega J_z + \gamma \cdot \left(J_+ e^{-i\varphi} + J_- e^{i\varphi} \right), \tag{4}$$

the obtained basis of the five-dimensional invariant space, when we have $H = \gamma \cdot V$:

$$V = \begin{pmatrix} 0 & 2e^{i\varphi} & 0 & 0 & 0 \\ 2e^{-i\varphi} & 0 & \sqrt{6}e^{i\varphi} & 0 & 0 \\ 0 & \sqrt{6}e^{-i\varphi} & 0 & \sqrt{6}e^{i\varphi} & 0 \\ 0 & 0 & \sqrt{6}e^{-i\varphi} & 0 & 2e^{i\varphi} \\ 0 & 0 & 0 & 2e^{-i\varphi} & 0 \end{pmatrix}. \tag{5}$$

Then we can analyze the spectral properties of matrix V and find the eigenvectors corresponding to the stationary states of the system:

$$\sigma_V = \{-4\gamma_0, -2\gamma_0, 0, 2\gamma_0, 4\gamma_0\}, \tag{6}$$

$$v_{-4} = \frac{1}{4}\begin{pmatrix} e^{2i\varphi} \\ -2e^{i\varphi} \\ \sqrt{6} \\ -2e^{-i\varphi} \\ e^{-2i\varphi} \end{pmatrix}, v_{-2} = \frac{1}{2}\begin{pmatrix} e^{2i\varphi} \\ -e^{-i\varphi} \\ 0 \\ -e^{-i\varphi} \\ -e^{-2i\varphi} \end{pmatrix}, v_0 = \frac{1}{2}\begin{pmatrix} -\sqrt{\frac{3}{2}}e^{2i\varphi} \\ 0 \\ 1 \\ 0 \\ -\sqrt{\frac{3}{2}}e^{-2i\varphi} \end{pmatrix},$$
$$v_2 = \frac{1}{2}\begin{pmatrix} e^{2i\varphi} \\ e^{i\varphi} \\ 0 \\ -e^{-i\varphi} \\ -e^{-2i\varphi} \end{pmatrix}, v_4 = \frac{1}{4}\begin{pmatrix} e^{2i\varphi} \\ 2e^{i\varphi} \\ \sqrt{6} \\ 2e^{-i\varphi} \\ e^{-2i\varphi} \end{pmatrix}. \tag{7}$$

Each eigenvector can be expressed in terms of a Fock basis $|200\rangle$, $|110\rangle$, $|011\rangle$, $|002\rangle$.

Results and Discussion

Here we use eigenstates of the Hamiltonian to model initial state evolution during the phase modulation process.

The figure illustrates the action of the evolution operator on the initial state $|020\rangle$. This allows us to determine the time dependence of the probability amplitudes of the Fock state components. The green curve corresponds to the dynamics of the components $|200\rangle$ and $|002\rangle$ of the Fock state vector $\sqrt{\frac{2}{3}}|020\rangle + \frac{1}{\sqrt{3}}|101\rangle$. It is evident from this that there is a specific moment in time when no other components are present in the expansion of this state vector.

To construct the dynamics of the state $|020\rangle$, we will act with the evolution operator on the initial state, as a result of which we obtain this expression with coefficients $\alpha = \frac{\sqrt{6}}{4}$, $\beta = \frac{1}{2}$.

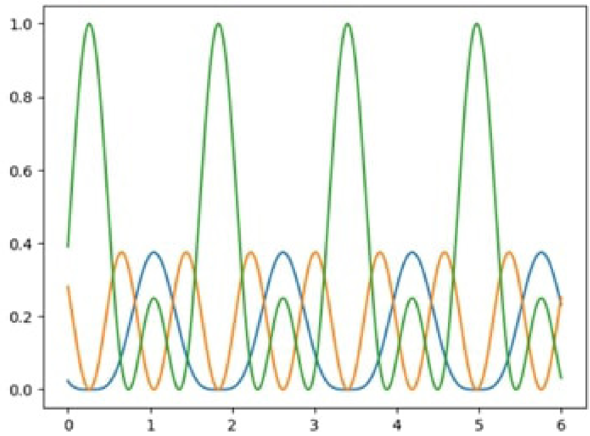

Figure. Probability amplitudes of state $|020\rangle$ in dynamics

$$\frac{\alpha}{4}\begin{pmatrix} e^{2i\varphi} \\ -2e^{i\varphi} \\ \sqrt{6} \\ -2e^{-i\varphi} \\ e^{-2i\varphi} \end{pmatrix} e^{-4it} + \frac{\beta}{2}\begin{pmatrix} -\sqrt{\frac{3}{2}}e^{2i\varphi} \\ 0 \\ 1 \\ 0 \\ -\sqrt{\frac{3}{2}}e^{-2i\varphi} \end{pmatrix} + \frac{\alpha}{4}\begin{pmatrix} e^{2i\varphi} \\ -2e^{i\varphi} \\ \sqrt{6} \\ -2e^{-i\varphi} \\ e^{-2i\varphi} \end{pmatrix} e^{4it} = 0. \tag{8}$$

Passing to the system of equations and expressing the components, we obtain

$$\begin{gathered} -\frac{\alpha}{2}e^{i\varphi}e^{-4it} + \frac{\alpha}{2}e^{i\varphi}e^{4it} = 0, \\ -\frac{\alpha}{2}e^{-i\varphi}e^{-4it} + \frac{\alpha}{2}e^{-i\varphi}e^{4it} = 0, \\ -\frac{\alpha}{4}\sqrt{6}e^{-4it} + \frac{\beta}{2} + \frac{\alpha}{4}\sqrt{6}e^{4it} = 0. \end{gathered} \tag{9}$$

Hence, from here we can conclude that a required moment of time is approximately $t \approx \pi/5$ That is, in the process of evolution quantum multimode states of the following form can arise

$$\alpha|200\rangle + \beta|020\rangle + \alpha|002\rangle. \tag{10}$$

In which the states $|011\rangle$ and $|110\rangle$. At the same time, we have determined a point in time at which the component $|020\rangle$ is absent in expression (10).

Conclusion

In this work, we have shown that a phase modulator performs electromagnetic field quantum state transformations that transcend the boundaries of local operations in quantum communication (LOCC) systems, inducing an essentially quantum channel for state transformation.

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