Conference materials
UDC 535.14
DOI: https://doi.org/10.18721/JPM.173.224

Modeling the dynamics and properties of the squeezed state of light in a phase modulator

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Abstract. In this work, we investigate the transformation of a squeezed by photon-number quantum state of a single-mode optical signal during phase modulation process. Within the framework of the semiclassical model of the phase modulator, we obtained estimations for the statistical properties of individual modes and mode sub-ensembles of the signal spectrum. We shown that for the case of modulation of squeezed vacuum, the entire spectrum of the modulated signal evaluates to the entangled state of a number of frequency modes. The results obtained in this work enable the use of squeezed modulated light states in quantum key distribution systems and various interferometric applications.

Keywords: phase modulator, multimode quantum optical signal, SU(1.1) algebra, squeezed state of light

Citation: Tabieva A.V., Trifanov A.I., Tushavin G.V., Matveeva M.V., Modeling the dynamics and properties of the compressed state of light in a phase modulator, St. Petersburg State Polytechnical University Journal. Physics and Mathematics. 17 (3.2) (2024) 125–129. DOI: https://doi.org/10.18721/JPM.173.224

Материалы конференции
УДК 535.14
DOI: https://doi.org/10.18721/JPM.173.224

Моделирование динамики и свойств сжатого состояния света в фазовом модуляторе

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Аннотация. В данной работе мы исследуем преобразование сжатого по числу фотонов квантового состояния оптического сигнала в процессе фазовой модуляции света. В рамках полуклассической модели фазового модулятора мы получили оценки статистических свойств отдельных частотных мод спектра сигнала и их подансамблей. Мы показали, что в случае модуляции сжатого вакуума весь спектр модулированного сигнала представляет собой запутанное состояние ансамбля частотных мод. Полученные в этой работе результаты позволяют использовать сжатые модулированные состояния света в системах квантового распределения ключей и различных интерферометрических приложениях.

Ключевые слова: фазовый модулятор, многомодовый квантовый оптический сигнал, алгебра SU(1,1), сжатое состояние света

Ссылка при цитировании: Табиева А.В., Трифанов А.И., Тушавин Г.В., Матвеева М.В. Моделирование динамики и свойств сжатого состояния света в фазовом модуляторе // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2024. Т. 17. № 3.2. С. 125–129. DOI: https://doi.org/10.18721/JPM.173.224

Introduction

Transformation of the spectral and statistical properties of multimode quantum-optical signals is of great practical importance due to the use of such processes in quantum communication and cryptography systems [1, 2]. This transformation may be implemented in phase modulators [3], which semi-classical model was proposed and studied in the [4–6], where the process of modulating an optical signal in a fully quantum state was considered. The property of a modulator to transform the spectra of optical signals can be utilized to obtain collective states that possess a number of important characteristics necessary for quantum communication systems. In particular, this pertains to the generation of entangled ensembles [7, 8]. Although the modulator is a linear optical device and cannot generate entangled states from classical light on its own, it can be used for this purpose. In this work, we propose a model for transforming a single-mode signal in a squeezed vacuum state into an entangled ensemble of frequency modes. Using numerical simulations, we obtained dependencies of the entanglement characteristics we employed (such as the trace of the square of the reduced density matrix and Shannon entropy) on the degree of squeezing of the initial optical signal.

Materials and Methods

In this work the dynamics of squeezed photon states is considered. A single-tone signal arriving at the input of the electro-optical modulator. The statistical characteristics of the ensemble of modes at the output of the device are analyzed.

A monochromatic signal is fed into the modulator, where upon modulation, it decomposes into a superposition of different modes. At the input of the device, we consider the squeezed state of the central mode, which is expressed through the squeezing operator [9] in the standard manner:

$$\left|\psi_{in}\right| = S(\xi)\left|\psi_0\right\rangle, \tag{1}$$

where S is the squeezing operator, ψ_0 is the vacuum state. After the modulator this state is transformed according to the evolution operator $U(t)$ and has the following form:

$$U(t)\left|\psi_{in}\right\rangle = \left|\psi_{out}\right\rangle = U(t)S(\xi)\left|\psi_0\right\rangle. \tag{2}$$

The action of the operator $U(t)$ on the mode operators a_μ can be written as follows [5, 10, 11]:

$$U(t)aU^\dagger(t) = a(t) = c(t) = \sum_{\nu=-s}^{s} D_{\mu\nu}^{(s)\dagger}(t)a_\nu. \tag{3}$$

Let us denote the value of the operator $a(t)$ at a fixed interaction time with the modulator $t = t_0$. Then, using the operator transformation [12]:

$$\begin{gathered}\left|\psi_{out}\right\rangle = U(t_0)\left|\psi_{in}\right\rangle = U(t_0)S(\xi)\left|\psi_0\right\rangle = \\ = U(t_0)S(\xi)U(t_0)+U(t)\left|\psi_0\right\rangle = S(\xi, t_0)\left|\psi_0\right\rangle,\end{gathered} \tag{4}$$

where the operator $S(\xi, t_0)$ has the form:

$$S(\xi, t_0) = \exp\left\{-i\left(\frac{\overline{\xi}}{2}c^2 - \frac{\xi}{2}c^{\dagger 2}\right)\right\}. \quad (5)$$

Results and Discussion

To investigate the statistical properties of the resulting state $|\psi_{out}\rangle$, let us consider the corresponding density matrix:

$$\rho_{out} = |\psi_{out}\rangle\langle\psi_{out}|. \quad (6)$$

An indicator of the presence of entanglement in a pure state can be represented by the mixed state of its subsystem. For this purpose, let us consider the partial trace over the subspace A, which consists of certain modes of the ensemble:

$$\rho' = Tr_A|\psi_{out}\rangle\langle\psi_{out}|. \quad (7)$$

The mixed state corresponds to such a matrix ρ' for which

$$m = Tr\left(\rho'^2\right) < 1. \quad (8)$$

Fig. 1 presents a graph showing the dependence of the quantity m on the squeezing parameter ξ of the initial state. The following parameters were chosen for numerical modeling: there were the angle of the phase state $\phi = 0$, 5 modes, from which the three middle ones were removed, and then the compression parameter r was changed from 0 to 1 in increments of 0.05.

Another important parameter reflecting the informational capacity of the state is the entropy,

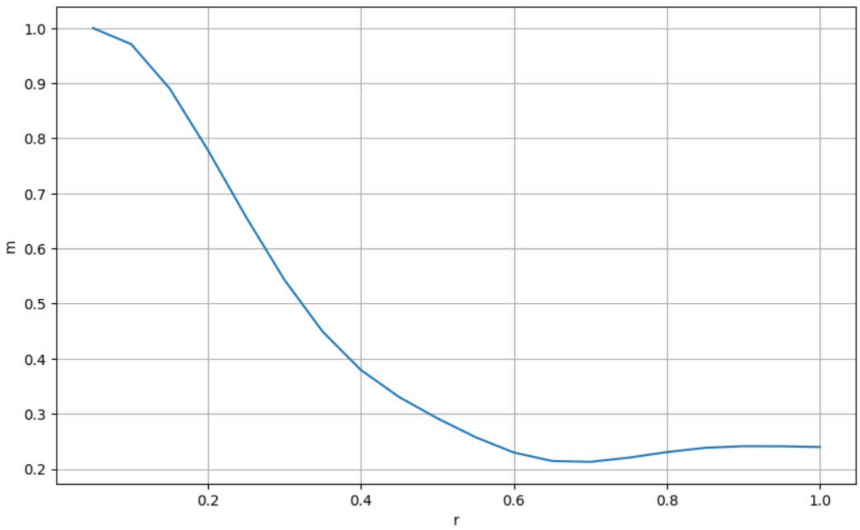

Fig. 1. Dependence of parameter *m* on the squeezing parameter (*r*)

which in the case of a quantum carrier in state ρ can be computed as follows:

$$I_{sh} = -Tr\left(\rho \log_2 \rho\right). \quad (9)$$

In the case of a pure state of the full ensemble, this quantity is equal to zero; however, for subsystem A with density matrix ρ', this quantity should be non-zero if the original full ensemble was in an entangled state [13–14]:

$$I_{sh} = -Tr\left(\rho' \log_2 \rho\right) > 0. \quad (10)$$

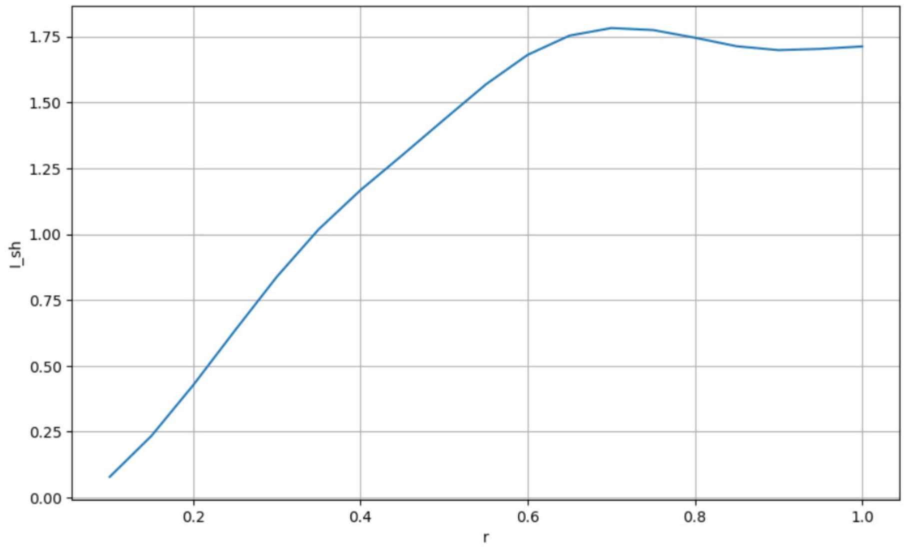

Fig. 2. Dependence of the quantum Shannon entropy as a function of the squeezing parameter ξ with the logarithm of purity $\log_2\rho$

Fig. 2 models the dependence of the quantum Shannon entropy as a function of the squeezing parameter ξ. The graph is constructed with the same parameters as for the first graph.

Conclusion

In this study, we have explored the transformation of a single-mode squeezed vacuum state into an entangled ensemble of frequency modes using an electro-optical modulator. Our model demonstrates that while the modulator itself is a linear optical device and does not generate entangled states from classical light, it can effectively facilitate the generation of entangled ensembles when applied to quantum states. Through numerical simulations, we analyzed the dependence of entanglement characteristics, such as the trace of the square of the reduced density matrix and Shannon entropy, on the degree of squeezing of the input optical signal.

The findings underscore the potential of phase modulators in enhancing the spectral and statistical properties of multimode quantum-optical signals, which are crucial for advancing quantum communication and cryptography systems. By successfully demonstrating the transformation process, this work lays the groundwork for further investigations into optimizing entanglement generation and exploring new applications in quantum technologies. Future research could focus on refining the modulation techniques and investigating their effects on different types of quantum states, ultimately contributing to the development of more robust and efficient quantum communication protocols.

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Received 01.08.2024. Approved after reviewing 17.10.2024. Accepted 21.10.2024.