THEORETICAL PHYSICS

Conference materials
UDC 530.145.83.
DOI: https://doi.org/10.18721/JPM.173.175

Thermal entanglement in the three-qubit Tavis-Cummings model with many-photon transitions

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Abstract. In this paper, we consider the model consisting of an isolated qubit and two qubits trapped in a lossless cavity and interacting with cavity thermal field via many-photon transitions. We obtain the exact solution of the model under consideration. On its basis we calculate the negativity as a measure of pair qubits entanglement. It is shown that, for many-photons processes entanglement is stronger than for that in the linear one-photon processes and can suppress the sudden death of qubit-qubit entanglement. The pairwise entanglement transfer between qubits pairs are also observed.

Keywords: qubits, thermal field, entanglement, many-photon transitions, sudden death of entanglement, cavity

Citation: Bagrov A.R., Bashkirov E.K., Thermal entanglement in the three-qubit Tavis-Cummings model with many-photon transitions, St. Petersburg State Polytechnical University Journal. Physics and Mathematics. 17 (3.1) (2024) 367–371. DOI: https://doi.org/10.18721/JPM.173.175

Материалы конференции
УДК 530.145.83.
DOI: https://doi.org/10.18721/JPM.173.175

Тепловая перепутанность в трехкубитной модели Тависа-Каммингса с многофотонными переходами

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Аннотация. В этой статье мы рассматриваем модель, состоящую из изолированного кубита и двух кубитов, заключенных в резонатор без потерь и взаимодействующих с тепловым полем резонатора посредством многофотонных переходов. Мы получаем точное решение рассматриваемой модели. На его основе мы вычисляем отрицательность как меру перепутанности пар кубитов. Показано, что для многофотонных процессов перепутанность сильнее, чем для линейных однофотонных процессов, и может подавлять мгновенную смерть кубит-кубитной перепутанности. Также наблюдается попарный переход перепутанности между парами кубитов.

Ключевые слова: кубиты, тепловое поле, перепутанность, многофотонные процессы, мгновенная смерть перепутывания, резонатор

Citation: Багров А.Р., Башкиров Е.К., Тепловая перепутанность в трехкубитной модели Тависа-Каммингса с многофотонными переходами // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2024. Т. 17. № 3.1. С. 367–371. DOI: https://doi.org/10.18721/JPM.173.175

Introduction

In recent years, much attention has been paid to the development of efficient quantum information processing (QIP) systems capable of performing quantum computing and quantum communications tasks [1–3]. The use of natural atoms (Rydberg atoms, ions in magnetic trap and molecules) and artificial atoms (superconducting circuits, spins in solid, quantum dots etc.) has allowed to realize such unique quantum devices as quantum computers and quantum networks [4]. A significant breakthrough in the role of artificial atomic systems in the field of QIP was achieved when these qubits, especially superconducting qubits and spins in solids, were embedded in superconducting microwave cavities [4, 5]. In contrast to natural atoms, artificial atoms have much stronger coupling with cavity fields. It has been shown that such an architecture can be effectively used to control, entangle, and read out the states of qubits. The quantum entanglement is now seen as a necessary resource to help with QIP tasks. Two-qubit and multi-qubit entangled states enable for quantum computation and other QIP [6]. Therefore, the investigations of entanglement dynamics of the qubits interacting with selected modes of the cavity electromagnetic fields is the topical problem of the QIP.

At present there are a number of theoretical and experimental papers devoted to investigation of the properties of entangled states. But the quantitative entanglement criteria are currently only defined for two-qubit states. For many-qubit system, the situation is more involved as there exist several inequivalent classes of entanglement. For the simplest case of three-qubit system, all pure and mixed states are classified into three types. They are separable, biseparable and genuine entangled GHZ- and W-states [7]. The genuine entangled GHZ- and W-states are not equivalent under the stochastic local operations and classical communication (SLOCC). As a universal alternative to two-qubit gates, three-qubit gates based on three-qubit systems, such as Toffoli or Fredkin gates are possible produce the QIP. Three-qubit entangled states types were subsequently realized experimentally for superconducting qubits as well as for spins in solids and ions in traps (see references in [8–10]).

The QIP require maximally entangled states. However, through interaction with environment the decoherence usually occurs. However, it has recently been shown that the opposite effect can also occur: the environment can produce entanglement. In particular, the thermal noise of the cavity can act as such an environment. Kim and co-author showed that a chaotic field of the cavity can entangle two qubits [11]. Later a number of authors investigated the entanglement induced by a thermal cavity in two-qubit (see refs. in [12]) and three-qubit systems [13]. In [14] the authors considering the dynamics of two-qubit two-photon Tavis-Cummings model (TCM) showed that the entanglement induced by nonlinear interaction is larger than that induced by linear one-photon interaction. This result motivated us to focus our attention on studying the thermal three-qubit JCM with many-photon transitions. Since it is not possible to simply generalize the useful tools used in the bipartite case to the multipartite case, it is of particular interest to study the dynamics of three-qubit systems in a cavity for pure biseparable initial qubit states. In this case the three-qubit state is separable under some bipartition of the three qubits.

In this paper we will consider three partite system consisting of an isolated qubit and two qubits trapped in a lossless cavity and interacting with one-mode cavity field via many-photon transitions based on many-photon TCM (see refs. in [15]). We will investigate the dynamics of pairwise entanglement for biseparable initial qubits states and thermal state of the cavity field. As the measure of the pairwise qubits entanglement we will use the negativity. In the following, we shall pay our main attention to the influence of the photon transition multiples on the occurrence conditions of entanglement sudden death (ESD) phenomenon and entanglement transfer between pair of qubits in the process of the system evolution.

Model and its solution

We consider a system consisting of three identical qubits A, B and C. Two qubits B and C resonantly interact with the quantized electromagnetic field of the cavity via many-photon transition. The qubit A can move freely outside the cavity. The Hamiltonian of the interaction of such a system in the dipole approximation and the rotating wave approximation can be written as

$$\hat{H}_{Int} = \sum_{j=B}^{C} \hbar\gamma(\hat{R}_j^+ \hat{b}^k + \hat{R}_j^- \hat{b}^{+k}), \qquad (1)$$

where $\hat{R}_j^+ = |2\rangle_{jj}\langle 1|$ and $\hat{R}_j^- = |1\rangle_{jj}\langle 2|$ are the transition operators between the excited $|2\rangle_j$ and the ground $|1\rangle_j$ states in the j-th qubit (j = B, C), $\hat{b}^+$ and $\hat{b}$ are the creation and the annihilation operators of the photons, γ is the qubit-photon coupling and k is the photon transition multiple.

The initial qubits state is assumed to be biseparable such as

$$|\Psi_1(0)\rangle_{ABC} = \cos\vartheta\,|2,1,1\rangle + \sin\vartheta\,|1,2,1\rangle \qquad (2)$$

or

$$|\Psi_2(0)\rangle_{ABC} = \cos\vartheta\,|2,1,2\rangle + \sin\vartheta\,|1,2,2\rangle. \qquad (3)$$

The initial cavity field state is assumed to be thermal with density matrix

$$\wp_{Field}(0) = \sum\nolimits_p \lambda_p\,|p\rangle\langle p|,$$

where $\lambda_p = \bar{p}^p/(1+\bar{p})^{p+1}$ and $\bar{p}$ is the mean thermal photon number $\bar{p} = \left(\exp[\hbar\omega/k_B T] - 1\right)^{-1}$, k_B is the Boltzmann constant and T is the cavity temperature.

We obtained the exact formula for unitary operator of the system (1) under consideration. On it basis we derived the exact solution of the quantum Liouville equation for whole density matrix $\wp_{A,B,C,Field}(t)$. To obtain the pairwise negativity selected pair of qubits one can obtain the reduced qubit-qubit density matrixes. These can be produce by averaging the whole density matrix over the field and third qubit variables

$$\wp_{ij} = Tr_k Tr_{Field} \wp_{A,B,C,Field}\left(i \neq j \neq k, i, j, k = A, B, C\right).$$

We found the exact formulas for reduced qubit-qubit density matrix for all pairs of qubits and calculated on its basis the pairwise negativities which was defined by standard manner

$$\xi_{ij} = -2\sum_l \mu_{ijl}^-,$$

where μ_{ijl}^- is the negative eigenvalues of partial transpose of a reduced atomic density matrix $\wp_{ij}^{T_1}$. The exact expressions for negativities are too cumbersome to present in this article.

Results and Discussion

In Fig. 1 we plot the time-dependence of negativities ξ_{AB}, ξ_{AC} and ξ_{BC} for initial qubits state (2) with $\vartheta = \pi/4$, fixed values of mean photon number and different values of photon multiple. One can see from Fig. 1 that at some times the entanglement disappears abruptly ($\xi_{ij} \leq 0$) for small values of photon multiple k and remains zero for a finite time before being revived. This means that there is an sudden death of entanglement effect. The sudden death of entanglement is the disappearance of entanglement of qubits at times less than the time of energy dissipation, phase, etc. The entanglement sudden death (ESD) phenomenon disappears for large values of the mentioned parameter k. At initial time qubits 1 and 2 are in maximally entangled Bell type state. The initial entanglement between qubits 1 and 3 and 2 and 3 are absent. One can see that for small multiples k the negativity ξ_{AB} in process of evolution reach zero at such interval of moment for which firstly qubits 2 and 3 become entangled. At subsequent time instants, the states of qubits 2 and 3 are disentangled (with the states of qubits 1 and 2 still being separable), but qubits 1 and 3 are entangled. At subsequent time interval, the states of qubits 2 and 3 are disentangled (the states of qubits 1 and 2 still being separable), but qubits 1 and 3 are entangled. Further, the processes of entanglement and disentanglement of qubit states are repeated. This result can be

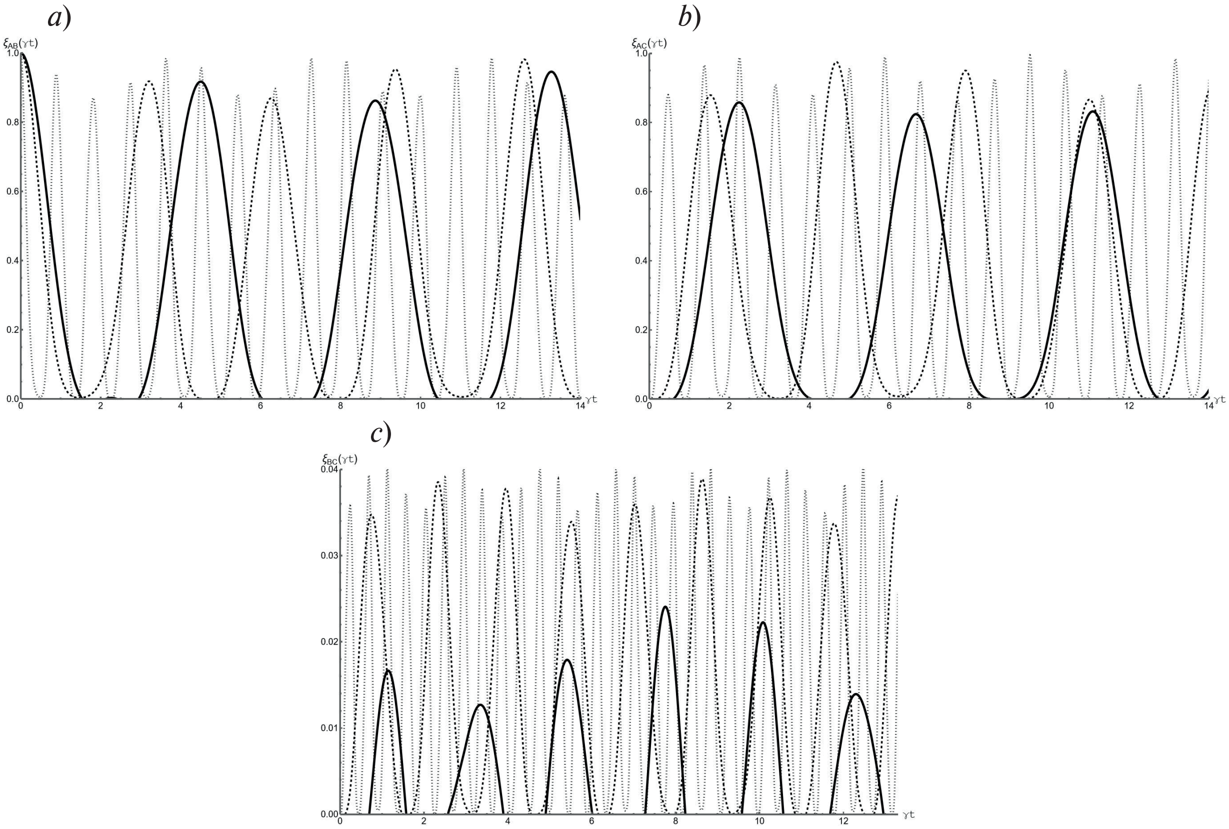

Fig. 1. The negativity criterion ξ_{AB} (*a*), ξ_{AC} (*b*) and ξ_{BC} (*c*) vs scaled time γt for initial biseparable qubits state $\cos\vartheta\,|2,1,1\rangle + \sin\vartheta\,|1,2,1\rangle$ with $\vartheta = \pi/4$. The photon multiple $k = 1$ (solid), $k = 2$ (dashed), $k = 4$ (dotted). The mean photon number $\bar{p} = 0.1$

interpreted as transition of entanglement from one pair of qubits to another pair of qubits. The numerical calculations of pairwise negativity for another biseparable initial state (3) showed that the time behavior of and ξ_{AC} is similar to that for bisparable state (2). But the behavior of pairwise negativity of qubits 2 and 3 ξ_{BC} is quite unexpected. The thermal field (or the vacuum field when $\bar{p} \to 0$) does not induce entanglement between qubits 2 and 3 for all values of model parameter *k*.

Conclusion

In this paper, we studied the dynamics of a system of three identical qubits, one of which is isolated, and the other two are trapped in an ideal cavity and interact resonantly with the one mode of the electromagnetic field of this cavity through many-photon transitions. The biseparable states of qubits and thermal state of field were in the focus of our attention. We derived the exact solution of the model under consideration. To investigate the entanglement of the pair of qubits we calculated the negativity. For biseparable states ESD effect takes place for small values of multiples and disappears for large values of mentioned parameter. The transition of entanglement from one pair of qubits to other pairs of qubits in the evolution process are also observed.

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Received 04.07.2024. Approved after reviewing 31.07.2024. Accepted 31.07.2024.