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AN ALGORITHM FOR FINDING THE DAMPING COEFFICIENTS BASED ON VIBRATION SURVEYS USING THE FREQUENCY DOMAIN DECOMPOSITION (FDD) METHOD

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Abstract. The article presents an algorithm and a theoretical justification for the method of finding the damping coefficients on the base of vibration surveys using the frequency domain decomposition (FDD) method. This method is used for monitoring the structures such as buildings, bridges, dams, to determine experimentally their state (under operating conditions) without application of vibroexciters.

Keywords: frequency domain decomposition, damping coefficient, spectral density matrix, natural mode shape and frequency

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АЛГОРИТМ НАХОЖДЕНИЯ КОЭФФИЦИЕНТОВ ДЕМПФИРОВАНИЯ ПО ДАННЫМ ВИБРАЦИОННЫХ ОБСЛЕДОВАНИЙ МЕТОДОМ FDD (ДЕКОМПОЗИЦИИ В ЧАСТОТНОЙ ОБЛАСТИ)

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Аннотация. В статье представлен алгоритм и теоретическое обоснование методики нахождения коэффициентов демпфирования по данным вибрационных обследований при использовании метода декомпозиции в частотной области (FDD). Этот метод применяется при динамическом тестировании сооружений (здания, мосты, плотины) для экспериментального определения их динамических характеристик в условиях нормальной эксплуатации без применения вибровозбудительного оборудования.

Ключевые слова: декомпозиция в частотной области, коэффициент демпфирования, спектральная плотность сигнала, собственные частоты, формы собственных колебаний

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Introduction

Full-scale dynamic testing of building structures remains a crucial problem, especially for unique structures, such as dams. Experimental assessment of dynamic characteristics (natural frequencies, mode shapes, relative damping ratios) and monitoring of these characteristics allows to control the safety, strength, integrity of the structure as well as to identify substantial changes without resorting to specialized devices or visual inspection of each structure.

Operational modal analysis (OMA) comprises an entire group of methods aimed at experimentally determining the dynamic characteristics of structures under normal operating conditions. These methods have become increasingly widespread for diagnostics of dynamic characteristics in various structures as advances are made in measuring and recording systems. A particular popular method of the OMA group is Frequency Domain Decomposition (FDD) [2–4]. The FDD method and the ARTeMIS Modal software based on it have been adopted since 2019 by the scientists of the B.E. Vedeneev All-Russian Research Institute of Hydraulic Engineering (St. Petersburg, Russia) [9].

In addition to determining natural frequencies and mode shapes, the EFDD method expanding the capabilities of FDD also offers an algorithm for determining damping ratios [4–6], however, it is rather complex, often yielding large errors.

A simpler, more accurate algorithm is proposed in this paper for identifying the damping parameters.

Our goal was to formulate and theoretically substantiate a new method for finding damping ratios based on vibration surveys.

The FDD method is described in detail in [2, 3, 7], and its theoretical framework is formulated in [1]. The algorithm of this method consists of the following mandatory steps.

Step 1. A cross-spectral density matrix (CSDM) $\mathbf{G}_y(\omega)$ of simultaneously measured vibration signals is calculated for each frequency ω of a given range.

Step 2. A singular-value decomposition (SVD) of the matrices $\mathbf{G}_y(\omega)$ is performed at each frequency ω, their first singular value $\sigma_1(\omega)$ is determined, and a frequency function of the first singular value is constructed, averaged over all measurements.

Note that the main idea of the FDD algorithm (see, for example, [2, 3, 7]) is that the first singular value $\sigma_1(\omega)$ of the matrix $\mathbf{G}_y(\omega_m)$ has local maxima near modal frequencies. The mathematical justification for this was given in [1]. Alternatively, we use the function $\sigma_1(\omega)$ to determine the logarithmic decrements corresponding to each natural frequency.

Theoretical justification of the procedure for determining logarithmic decrements

The response $\mathbf{y}(t)$ of the system is uniquely decomposed into their linear combination (due to the linear independence of the eigenmodes):

$$\mathbf{y}(t) = \boldsymbol{\varphi}_1 \cdot q_1(t) + \boldsymbol{\varphi}_2 \cdot q_2(t) + \ldots = \boldsymbol{\Phi}\, \mathbf{q}(t). \tag{1}$$

As found in [7], if white noise is considered as external force, and dissipation is assumed to be small, the following expression holds true for CSDM $\mathbf{G}_y(\omega)$:

$$\mathbf{G}_y(\omega) = \sum_{m=1}^{M} \frac{c_m\, \boldsymbol{\varphi}_m \boldsymbol{\varphi}_m^{\mathrm{H}}}{i\omega - \lambda_m} + \frac{c_m\, \boldsymbol{\varphi}_m \boldsymbol{\varphi}_m^{\mathrm{H}}}{-i\omega - \lambda_m^*} = \boldsymbol{\Phi} \cdot \mathrm{diag}\left(2\,\mathrm{Re}\left(\frac{c_m}{i\omega - \lambda_m} \right) \right) \cdot \boldsymbol{\Phi}^{\mathrm{H}}, \tag{2}$$

where λ_m is the pole,

$$\lambda_m = -\gamma_m + i\omega_{dm}, \tag{3}$$

$$(\gamma_m = \omega_{0m} \varsigma_m); \tag{4}$$

$\boldsymbol{\varphi}_m$ is the eigenmode; $\boldsymbol{\Phi}$ is a matrix whose columns are eigenvectors, $\boldsymbol{\Phi} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots \boldsymbol{\varphi}_M]$; M is the number of modes accounted for in decomposition (1); c_m is a positive coefficient; i is the imaginary unit; H is the Hermitian conjugate.

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The component ω_{dm} of expression (3) is the natural frequency accounting for damping. In expression (4), ω_{0m} is the natural frequency without accounting for damping; ς_m is the damping ratio.
Next, we introduce the notation

$$\alpha_m(\omega) = 2\,\mathrm{Re}\left(\frac{c_m}{i\omega - \lambda_m}\right) = \frac{c_m \gamma_m}{(\omega - \omega_{md})^2 + \gamma^2_m}. \tag{5}$$

Notably, modal vectors $\boldsymbol{\varphi}_m$ in expression (2) are assumed to be normalized, since the coefficient α_m contains, according to expression (5), a constant c_m that can be supplemented with a normalization factor.
Then expression (2) can be written as follows:

$$\mathbf{G}_y(\omega) = \mathbf{\Phi} \cdot \mathrm{diag}\left(\alpha_m(\omega)\right) \cdot \mathbf{\Phi}^{\mathrm{H}}, \tag{6}$$

or

$$\mathbf{G}_y(\omega) = \sum_{m=1}^{M} \alpha_m \boldsymbol{\varphi}_m \boldsymbol{\varphi}_m^{\mathrm{H}}. \tag{7}$$

Dynamic testing of the structure can be used to calculate the CSDM of the measured signals over a certain frequency range; next, the SVD of these matrices can be used to obtain the frequency function of the first singular value [10–12].
It was proved in our earlier study [1] that the natural frequencies of the structure considered are located near the local maxima of this function. This paper proposes and substantiates a method for determining the damping ratios based on an experimentally constructed function of the first singular value.
An obvious idea is to compare the analytical expression for the first singular value and the experimentally constructed function [14]. Even though there is no general analytical expression for the first singular value of CSDM, a fairly good analytical approximation can be obtained under certain conditions.
Consider two main cases when it is possible to obtain such an approximation.
Case of single natural frequency. Here, the response $\mathbf{y}(t)$ of the system (see Eq. (1)) in the vicinity of some natural frequency (let us denote it as ω_{ds}) is determined mainly by eigenmode with the same number. Then the following relation holds true:

$$\mathbf{y}(t) \approx \boldsymbol{\varphi}_s \cdot q_s(t), \tag{8}$$

and the expression for the CSDM given by Eq. (4) can be rewritten as:

$$\mathbf{G}_y(\omega) \approx \alpha_s \boldsymbol{\varphi}_s \boldsymbol{\varphi}_s^{\mathrm{H}}. \tag{9}$$

Evidently, Eq. (9) holds true when the values of the functions $\alpha_s(\omega)$ significantly exceed the other values $\alpha_m(\omega)$ in the vicinity of the frequency ω_{ds}. Now let us find the conditions under which this requirement is satisfied.
Consider the properties of functions $\alpha_m(\omega)$. It can be seen from expression (5) that these functions depend on the corresponding natural frequencies and damping ratios. They have one maximum each, reached at the corresponding natural frequency.
Indeed (see our study [1]), determining the extreme values of the functions $\alpha_m(\omega)$, we obtain for $\omega = \omega_{dm}$

$$\alpha_m(\omega_{dm}) = \mathrm{c}_m / \gamma_m. \tag{10}$$

We introduce the notation for the minimum distance d_s on the frequency scale between the frequency ω_{ds} and the rest of the natural frequencies ω_{dm}, namely:

$$d_s = \min_{m \neq s} \left| \omega_{ds} - \omega_{dm} \right|. \quad (11)$$

Then the following relation holds true for the frequency ω_{dm} with all $m \neq s$:

$$\alpha_m(\omega_{ds}) \leq \frac{c_m \gamma_m}{d_s^{\,2} + \gamma_m^{\,2}} = \frac{c_m / \gamma_m}{(d_s / \gamma_m)^2 + 1}. \quad (12)$$

Now let us introduce the notation

$$r_{sm} = d_s / \gamma_m = \frac{d_s}{\omega_{0m} \cdot \varsigma_m}. \quad (13)$$

Comparing relations (12) and (10), we can see that the condition

$$\alpha_s(\omega_{ds}) >> \alpha_m(\omega_{ds}) \quad (14)$$

is satisfied if

$$r_{sm} >> 1, \text{ for all } s \neq m. \quad (15)$$

Therefore, as condition (15) is satisfied, only one term in expression (7) can be taken into account, i.e., the CSDM is calculated by Eq. (9).

An analytical expression can be constructed for the first singular value pf the CSDM $\mathbf{G}_y(\omega)$ described by expression (9). The matrix $\mathbf{G}_y(\omega)$ is evidently square and symmetrical (it is Hermitian for complex modes). The rank of the matrix $\mathbf{G}_y(\omega)$ equals unity (since the rank of the product of the matrices does not exceed the ranks of the multipliers), therefore this matrix has no more than one eigenvalue other than zero.

We find it by defining the eigenvalues. Let u be the eigenvector and λ the eigenvalue of the matrix $\mathbf{G}_y(\omega)$; then, by defining the eigenvector and the eigenvalue, we obtain the following equality:

$$\mathbf{G}_y(\omega) u = \lambda u, \quad (16)$$

then,

$$\alpha_s \boldsymbol{\varphi}_s \boldsymbol{\varphi}_s^{\mathrm{H}} u = \alpha_s \boldsymbol{\varphi}_s (\boldsymbol{\varphi}_s^{\mathrm{H}} u) = \alpha_s (\boldsymbol{\varphi}_s^{\mathrm{H}} u) \boldsymbol{\varphi}_s = \lambda u. \quad (17)$$

An immediate consequence of equality (17) is that the only nonzero eigenvector $u = \boldsymbol{\varphi}_s$, and the eigenvalue $\lambda = \alpha_s \left\| \boldsymbol{\varphi}_s \right\|^2$. Evidently, $\lambda \geq 0$, since the coefficient $c_m \geq 0$. Consequently, the matrix $\mathbf{G}_y(\omega)$ is positive semi-definite, and then (since it is also Hermitian), its singular values coincide with its eigenvalues. Since $\left\| \boldsymbol{\varphi}_s \right\|^2 = 1$, then, apparently, σ_1 coincides with α_s. Therefore, the maximum singular value of the CSDM in the vicinity of natural frequencies can be written as

$$\sigma_1 = \alpha_s = \frac{c_s \gamma_s}{(\omega - \omega_{ds})^2 + \gamma_s^{\,2}} = \frac{c_s \gamma_s}{(\omega - \omega_{ds})^2 + \omega_{0s}^{\,2} \cdot \varsigma_s^{\,2}}. \quad (18)$$

Furthermore, the natural frequencies accounting for damping (ω_{dk}) and without it (ω_{0k}) practically coincide for small damping ratios.

If we compare the function of the first singular value, obtained by processing experimental data, with analytical dependence (18), we can estimate the logarithmic decrements. Let us rewrite Eq. (18) in the following form:

$$\sigma_1 = \frac{A}{(\omega - \omega_{ds})^2 + \omega_{ds}^{\,2} B^2}. \quad (19)$$

Then we use the least squares method (for example), making it possible to determine the coefficients A and B that approximate the analytical function σ_1 as close as possible (see Eq. (19)) to the experimentally obtained dependence in the vicinity of some natural frequency. The value of the parameter B apparently corresponds to the damping decrement.

The case of two converged natural frequencies. Let us now consider the second case, when the values of two natural frequencies numbered k and $k + 1$ (ω_{dk} and ω_{dk+1}) are located close to each other (such frequencies are generally known as converged in the literature), i.e., condition (15) is not satisfied for frequencies with these numbers.

However, if condition (15) is satisfied for all other natural frequencies, except for frequencies numbered k and $k + 1$, then CDSM can be calculated by Eq. (7) with only two terms:

$$\mathbf{G}_y(\omega) = \sum_{m=k}^{k+1} \alpha_m \boldsymbol{\varphi}_m \boldsymbol{\varphi}_m^{\mathrm{H}}, \tag{20}$$

or the matrix has the coordinate form

$$\mathbf{G}_y = \begin{bmatrix} \sum_{m=k}^{k+1} \alpha_m (\varphi_m^{(1)})^2 & \sum_{m=k}^{k+1} \alpha_m \varphi_m^{(1)} \varphi_m^{(2)} & & \sum_{m=k}^{k+1} \alpha_m \varphi_m^{(1)} \varphi_m^{(N)} \\ \sum_{m=k}^{k+1} \alpha_m \varphi_m^{(2)} \varphi_m^{(1)} & \sum_{m=k}^{k+1} \alpha_m (\varphi_m^{(2)})^2 & \dots & \sum_{m=k}^{k+1} \alpha_m \varphi_m^{(2)} \varphi_m^{(N)} \\ \dots & \dots & & \dots \\ \sum_{m=k}^{k+1} \alpha_m \varphi_m^{(N)} \varphi_m^{(1)} & \sum_{m=k}^{k+1} \alpha_m \varphi_m^{(N)} \varphi_m^{(2)} & & \sum_{m=k}^{k+1} \alpha_m (\varphi_m^{(N)})^2 \end{bmatrix}. \tag{21}$$

This matrix has the dimensions $N \times N$, and it is difficult to find its singular values (or eigenvalues) analytically.

To simplify the task, we compose a Gram matrix (denoting it as $\mathbf{K}$) based on the vectors $\sqrt{\alpha_k}\boldsymbol{\varphi}_k$ and $\sqrt{\alpha_{k+1}}\boldsymbol{\varphi}_{k+1}$.

The matrix $\mathbf{K}$ has the following coordinate form:

$$\mathbf{K} = \begin{bmatrix} \alpha_k & \sqrt{\alpha_k \alpha_{k+1}} (\boldsymbol{\varphi}_k, \boldsymbol{\varphi}_{k+1}) \\ \sqrt{\alpha_k \alpha_{k+1}} (\boldsymbol{\varphi}_{k+1}, \boldsymbol{\varphi}_k) & \alpha_{k+1} \end{bmatrix}. \tag{22}$$

This matrix, like the matrix $\mathbf{G}_y$, is Hermitian and positive semi-definite (a property of the Gram matrix). A proof that nonzero eigenvalues of the matrix $\mathbf{G}_y$ coincide with the eigenvalues of the matrix $\mathbf{K}$ (the Gram matrix constructed from the corresponding vectors) was given in [1], and the eigenvalues of the matrices $\mathbf{K}$ and $\mathbf{G}_y$ coincide with their singular values. Thus, the first singular value of the matrix $\mathbf{G}_y$ is equal to the spectral radius of the matrix $\mathbf{K}$.

In this case, the matrix $\mathbf{K}$ has the dimensions of 2×2, and we can easily construct an analytical expression for the value of its spectral radius [16]:

$$\rho = \frac{\mathrm{tr}(\mathbf{K}) + \sqrt{\mathrm{tr}^2(\mathbf{K}) - 4\det(\mathbf{K})}}{2}. \tag{23}$$

Since it follows from the expression for matrix (22) that

$$\det(\mathbf{K}) = \alpha_k \alpha_{k+1} - \alpha_k \alpha_{k+1} \left| (\boldsymbol{\varphi}_k, \boldsymbol{\varphi}_{k+1}) \right|^2, \tag{24}$$

$$\mathrm{tr}(\mathbf{K}) = \alpha_k + \alpha_{k+1}, \tag{25}$$

we obtain the following expression for the spectral radius of the matrix **K** (and therefore, for the first singular value of the matrix $\mathbf{G}_y$):

$$\sigma_1 = \frac{\alpha_k + \alpha_{k+1} + \sqrt{(\alpha_k - \alpha_{k+1})^2 + 4\alpha_k \alpha_{k+1} \left|(\varphi_k, \varphi_{k+1})\right|^2}}{2}. \tag{26}$$

It follows directly from Eq. (26) that if φ_k is orthogonal to φ_{k+1}, then $\sigma_1 = \max\limits_{i=k,k+1} \alpha_i$, regardless of whether condition (15) is satisfied.

Fig. 1 shows graphs of functions σ_1 and α_i calculated for a system with three degrees of freedom. Evidently, in the vicinity of the first natural frequency ω_1, function of the first singular value σ_1 coincides well with the function α_1 in a fairly wide frequency range ($r_{12} = 12.7$ in this example). However, the behavior of the function σ_1 changes for the second and third natural frequencies ($r_{23} = 0.85, r_{32} = 1.7$, respectively), while the greatest difference between the graph of this function and the corresponding graphs of functions α_i is observed in the frequency range corresponding to the interval between the maxima of the curves for the functions α_2 and α_3.

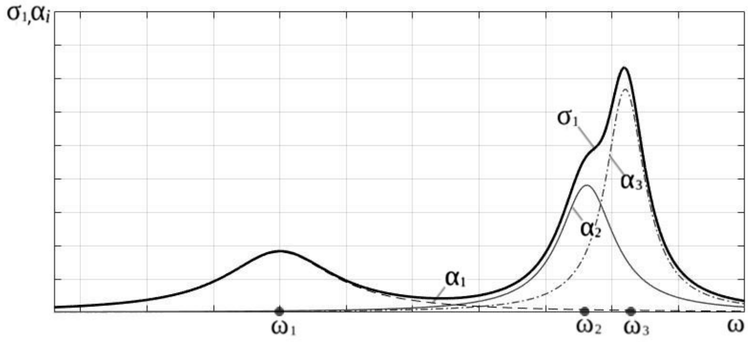

Fig. 1. Behavior of first singular value σ_1 compared with functions $\alpha_i(\omega)$ for system with three degrees of freedom ($\omega_1 - \omega_3$)

Note that expression (26) is simplified at the point of intersection of the curves for α_k and α_{k+1} (α_2 and α_3 in the example):

$$\sigma_1 = \alpha_k (1 + \left|(\varphi_k, \varphi_{k+1})\right|). \tag{27}$$

In other cases, expression (26) as a function of logarithmic decrements is a rather complex expression, so using it to find the necessary parameters turns out to be a difficult task.

We propose a different approach to solving this problem in the case of converged frequencies. It is known from linear algebra that the sum of the eigenvalues of a square matrix is equal to its trace [15–17], and since, as noted above, the eigenvalues and singular values of the matrix **K** coincide in this case, the following formula holds true:

$$\sigma_1 + \sigma_2 = \alpha_k + \alpha_{k+1}. \tag{28}$$

Let us introduce the notation

$$s(\omega) = \sigma_1(\omega) + \sigma_2(\omega). \tag{29}$$

Then expression (28) can be rewritten as follows:

$$s = \alpha_k + \alpha_{k+1} \tag{30}$$

(for brevity, the argument is omitted here).

If we substitute the expressions for α_k and α_{k+1} in Eq. (30), then the following relation holds true:

$$s=\frac{c_k\gamma_k}{(\omega-\omega_{dk})^2+\omega_{0k}\varsigma_k^{\ 2}}+\frac{c_{k+1}\gamma_{k+1}}{(\omega-\omega_{dk+1})^2+\omega_{0k+1}\varsigma_{k+1}^{\ 2}}. \tag{31}$$

Fig. 2 shows a comparison of the sum of the first two singular values s and the sum $\alpha_k + \alpha_{k+1}$ for the previously considered case (see Fig. 1), a system with three degrees of freedom ($\alpha_2 + \alpha_3$ in the example).

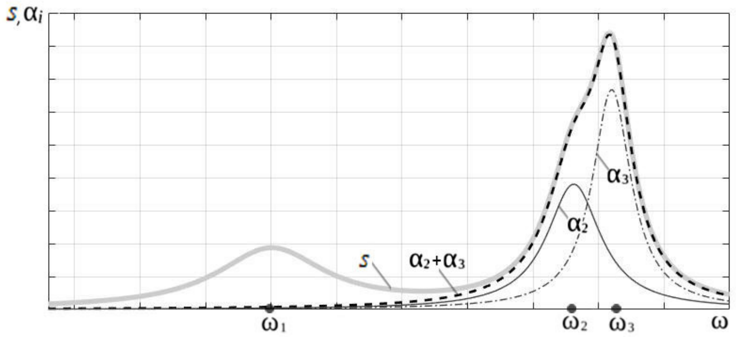

Fig. 2. Sum of first two singular values $s(\omega)$ compared with functions $\alpha_i(\omega)$ and sum $\alpha_k + \alpha_{k+1}$ for system with three degrees of freedom (see Fig. 1)

Singular-value decomposition of the matrix $\mathbf{G}_y(\omega)$ is performed during processing of experimental data obtained by dynamic measurements at each frequency ω of a given range; not only the first singular value $\sigma_1(\omega)$ but also the remaining singular values of the function are determined. Thus, the sum of the first two singular values is known.

Similar to the case of a single natural frequency, the natural frequencies accounting for damping (ω_{dk}) and without it (ω_{0k}) practically coincide for small damping ratios. Therefore, the analytical expression for the sum s of the first two singular values has the form

$$s=\frac{A}{(\omega-\omega_{dk})^2+\omega_{dk}^{\ 2}B^2}+\frac{C}{(\omega-\omega_{dk+1})^2+\omega_{dk+1}^{\ 2}D^2}, \tag{32}$$

where A, B, C, D are unknown parameters.

The least squares method can be used to determine these unknown parameters. Evidently, the coefficient B is an estimate of the damping ratio ς_k, and the coefficient D corresponds to the ratio ς_{k+1}.

Example calculations of damping ratios

The method for determining logarithmic decrements was tested with a mathematical model of a system with 8 degrees of freedom (Fig. 3).

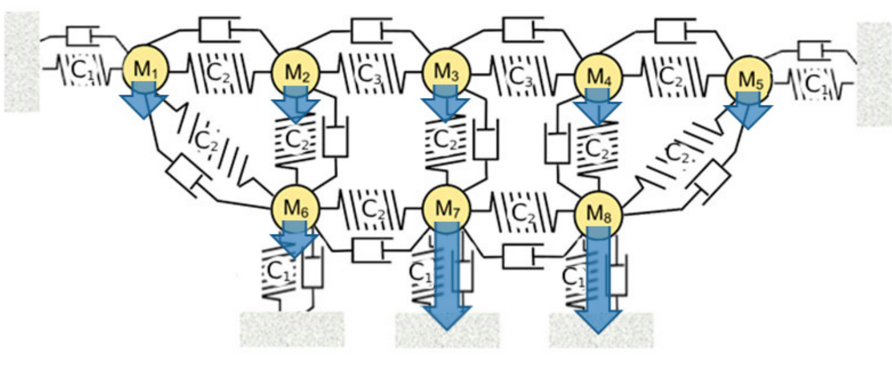

Fig. 3. Model of dynamic system with 8 degrees of freedom

Table 1

Parameters set for model of dynamic system with 8 degrees of freedom and their values (see Fig. 3)

Mass, kg	Stiffness, N/m	Force amplitude, N (applied to mass)
$M_1 = \ldots = M_8 = 25.9$	$C_1 = 770$, $C_2 = 1000$, $C_3 = 950$	$F_1 = \ldots = F_6 = 1$, $F_7 = F_8 = 10$

Note. The damping ratios (logarithmic decrements) were set to be the same and equal to 0.01.

The inertial and stiffness parameters were set in the adopted model. The damping ratios were assumed to be the same and equal to 0.01. Then, a proportional damping matrix was calculated at each natural frequency by the specified damping parameters. Forces with a white noise spectrum were applied to masses $M_1 - M_8$. The values of the force amplitude and other parameters of the system are given in Table 1. The loading modeled was non-uniformly distributed over the degrees of freedom: the amplitude of the force at degrees of freedom 7 and 8 was increased by 10 times.

Next, vibrational responses at all degrees of freedom were determined as time series with a given frequency from the exact solution of the dynamic problem.

The measurement data obtained by this approach were used to test the FDD technique and to subsequently identify the damping parameters by the proposed method. The results were compared with the parameters set for the model. The frequency function of the first singular value is shown in Fig. 4. Evidently, the six peaks corresponding to the natural frequencies (1–5 and 8) can be regarded as, i.e., the problem of determining the damping ratios corresponds to the case of a single natural frequency. The damping ratios corresponding to frequencies 1–5 and 8 were calculated by Eq. (19) based on to the algorithm described above (Table 2, upper lines).

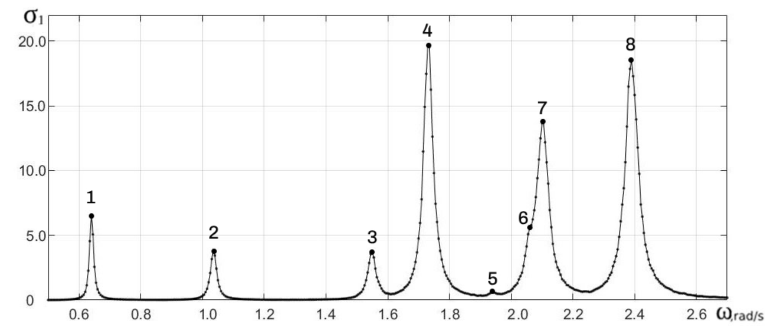

Fig. 4. Frequency spectrum for function of first singular value in CSDM for system with 8 degrees of freedom (see Fig. 3)

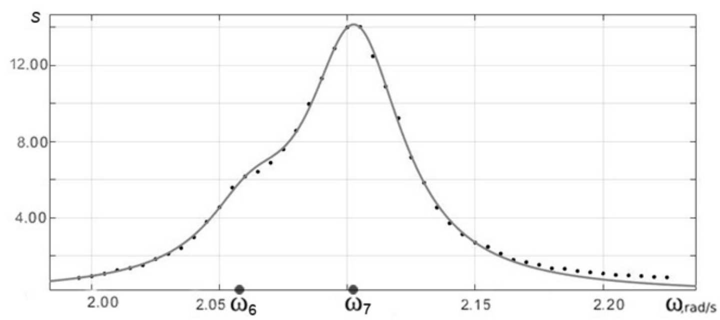

Fig. 5. Calculated sum of first two singular values (points) and obtained dependence approximated by Eq. (32) (solid line

Table 2

Calculated damping ratios and corresponding coefficients of determination

Peak	Natural frequency	Damping ratio	Coefficient of determination
Case of 'single' natural frequency			
1	0.64	0.0107	0.9985
2	1.03	0.0109	0.9995
3	1.54	0.0104	0.9957
4	1.73	0.0104	0.9975
5	1.93	0.0140	0.9357
8	2.39	0.0106	0.9983
Case of two 'converged' natural frequencies			
6	2.06	0.01014	0.9977
7	2.10	0.01070	0.9981

Note. The least squares method was used to find the values of the coefficient of determination.

The graph in Fig. 5 for the case of converged frequencies (these are frequencies 6 and 7 in the example) shows the points corresponding to the sum of the first and second singular values (obtained by the simulation model) as well as a function approximating them by Eq. (32).

The values of the identified damping ratios and the corresponding coefficients of determination found by the above method are also given in Table 2.)

Conclusion

The paper proposes a simple method for determining the damping ratios after identifying the natural frequencies of a structure based on experimental data using the FDD technique. Analytical expressions are obtained in the vicinity of natural frequencies for the first singular value as well as for the sum of the first two singular values as frequency functions. The method for determining the damping ratios is based on approximating the values obtained from processing the experimental data by analytical expressions with unknown parameters. The least squares method allows to determine the damping ratios. The first singular value is approximated in the case of a single natural frequency, the sum of the first two singular values is approximated in the case of converged natural frequencies.

The damping ratios were identified for a model problem. The proposed method has an advantage over the well-known EFDD method presented in [4], due to its lower complexity; furthermore, unlike the EFDD method, it allows determining the damping characteristics in the case of natural frequencies that are close in value.

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