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A GENERATIVE ADVERSARIAL NETWORK AS THE BASIS FOR A SEMI-INCLUSIVE DEEP INELASTIC LEPTON SCATTERING GENERATOR ON A POLARIZED PROTON

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Abstract. A neural network, that allows someone to obtain results for semi-inclusive deep inelastic scattering of charged leptons on polarized protons, with the production of pions or strange K mesons, has been developed in this study. The research covered both transverse and longitudinal polarizations of the proton. A range of initial energies of colliding particles was chosen from 20 to 100 GeV in a central mass system. The range is typical for electron-ion colliders currently being designed. It has been shown that it is possible to predict the physical characteristics of the final lepton and hadron with high accuracy as well as different variants of proton polarization using the proposed neural network.

Keywords: semi-inclusive deep inelastic scattering, asymmetries, machine learning, neural network, generative-adversarial network

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ГЕНЕРАТИВНО-СОСТЯЗАТЕЛЬНАЯ СЕТЬ КАК ОСНОВА ГЕНЕРАТОРА ПОЛУИНКЛЮЗИВНОГО ГЛУБОКОНЕУПРУГОГО РАССЕЯНИЯ ЛЕПТОНА НА ПОЛЯРИЗОВАННОМ ПРОТОНЕ

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Аннотация. В статье предложена разработанная нейронная сеть, позволяющая получать результаты полуинклюзивного глубоконеупругого рассеяния заряженных лептонов на поляризованных протонах с рождением пионов или странных K-мезонов. Рассмотрены состояния поляризации протона (поперечная и продольная). Выбран диапазон начальных энергий сталкивающихся частиц 20 – 100 ГэВ в системе центра масс, характерный для электрон-ионных коллайдеров, проектируемых в настоящее время. Показано, что с помощью предложенной разработки можно с высокой точностью предсказывать физические характеристики конечного лептона и адрона, а также различные варианты поляризации протона.

Ключевые слова: полуинклюзивное глубоконеупругое рассеяние, асимметрия, машинное обучение, нейронная сеть, генеративно-состязательная сеть

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Introduction

Deep inelastic scattering (DIS) of charged leptons by protons is one of the processes allowing to gain insight into the internal structure of the proton [1].

It is well-known that a large number of different particles are generated in the DIS process.

Experimental research and theoretical approaches to description of such processes are usually complex and require very sophisticated detector systems, involving various phenomenological models for the analysis of experimental results, related, for example, to hadronization [2]. For this reason, exclusive DIS studies have not yet been conducted.

However, as a rule, inclusive (with detection of only the scattered lepton) and semi-inclusive (with detection of the scattered lepton and one of the hadrons produced) DIS is considered.

Study of semi-exclusive DIS of leptons by protons becomes much more complicated if the lepton interacts with a polarized (longitudinally or transversely) proton [3].

At the same time, experiments with polarized particles are significantly more informative and allow to come close to solving problems related to the origins of proton spin.

Taking into account the polarization of the proton in the initial state of semi-inclusive GNR allows to measure various spin asymmetries that arise in the final state (after the process of semi-inclusive DIS) [3].

Transverse single-spin asymmetries occur during transverse polarization of the proton; these can be described within the framework of the Sivers [4] and Collins effects [3]. The Sivers asymmetry A_{Siv} can be used to generate the Sievers parton distribution functions, describing the correlations between the transverse momenta of quarks and the nucleon spin. The Collins asymmetries A_{Col} help measure the transverse functions of the parton distribution together with the Collins fragmentation functions [3]. All these functions are of great importance for describing the internal structure of the nucleon [3].

In the case of longitudinal polarization of the proton, a longitudinal single-spin asymmetry A_{ul} occurs. Its values allow to generate the T-odd correlation functions (parton distributions and fragmentation functions), which arise due to the exchange between quarks and longitudinally polarized gluons [5]. It was established relatively recently that non-zero T-odd parton distributions are compatible with the invariance of the strong interaction with respect to time reversal [5].

Large amounts of data obtained in each experiment are required to investigate any of the above-mentioned asymmetries and the associated mechanisms of their formation, which are determined by the parton distribution functions and fragmentation functions. In addition, it is necessary to carry out experimental studies with a large set of initial energies (reference points). Analysis of results of such experiments makes it possible to gain information about the parton distribution functions and fragmentation functions [6].

Due to limited experimental capabilities (particularly financial resources), it is impossible to obtain a sufficient number of reference points that can be used to approximate the distribution functions. As a result, it is necessary to develop programs that can interpolate or extrapolate experimental data by the selected parameters. The increase in data volumes (due to interpolation and extrapolation) should have a positive effect on the accuracy of the obtained distribution functions.

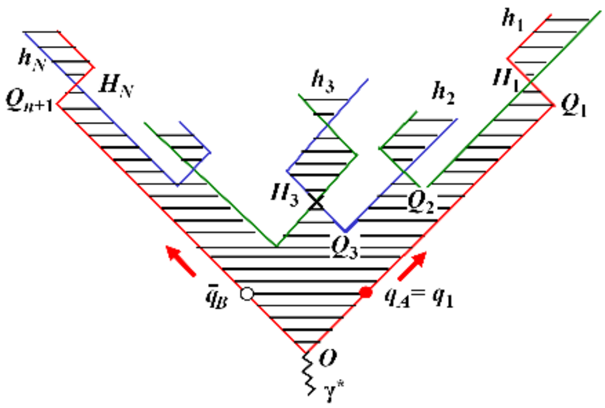

Fig. 1. Simplified scheme of hadronization process:
O is the interaction point; Q_1, Q_2, ..., Q_{n+1} are the string breaking points;
H_1, H_2, ..., H_N are the emission points of hadrons h_1, h_2, ..., h_N; q_A, $\bar{q}_B$ are the interacting quark A and the remnant B, respectively; γ^* is the virtual photon; arrows indicate the directions of motion of q_A and $\bar{q}_B$

Machine learning methods and, in particular, generative adversarial networks (GANs) can be used to solve problems related handling big data [7].

GANs allow to develop algorithms and write computer programs (called event generators) that can quickly obtain the necessary values from the original dataset, without specialized simulation of the interaction of particles and the detector. We should also note that such programs allow to avoid using large amounts of disk space, since they preserve the target distributions as small subsets of parameters [9].

Methodology

As noted in the introduction, the current state of experimental technology, the financial capabilities of the global scientific community and the presence of a large number of competing physical problems do not allow to experimentally obtain a sufficient number of data points that could be used for machine learning.

Due to this circumstance (lack of sufficient experimental data), the reference points were obtained in our study via modeling semi-inclusive deep-inelastic scattering of leptons by a polarized proton. The simulation was carried out using the PYTHIA8 program based on the Monte Carlo generator [10], expanded by the StringSpinner software package [11]. The latter includes the string+$^3P_0$ model [12], based on the Lund model [2], making it possible to account for the fragmentation of polarized quarks during hadronization.

The Lund model of hadronization can be illustrated by a simplified scheme (Fig. 1) [12].

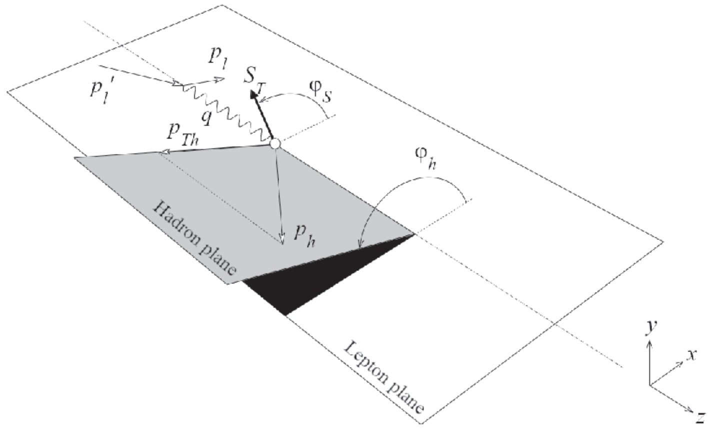

Fig. 2. Kinematics of semi-inclusive deep inelastic scattering [13]; planes of hadrons and leptons are shown (see the notations used in the text)

We assume that the proton interacting with the charged lepton consists of a quark A and a remnant B (q_A and $\bar{q}_B$, respectively, in Fig. 1) [12].

The virtual photon γ* emitted by the charged lepton is absorbed by one of the quarks of the unpolarized proton (for example, the quark q_A in Fig. 1). The photon γ* transfers its momentum to quark A, so that the separation of quark A and proton remnant B consequently begins. According to the Lund model, a relativistic string is stretched between objects A and B as a result of color interaction whose energy increases throughout the separation. The increase in tension continues until a quark-antiquark pair $q\bar{q}$ can be produced. The string then breaks with the production of a $q\bar{q}$ pair at the breaking points Q_1, Q_2, ..., Q_{n+1} [12]. This process can occur repeatedly, as long as the law of energy-momentum conservation allows it. In some cases, quarks and antiquarks can form a bound state, producing mesons [2]. This leads to semi-exclusive deep-inelastic scattering of charged leptons by unpolarized protons.

As noted above, the PYTHIA8+StringSpinner software package should be used for semi-exclusive deep-elastic scattering of a charged lepton by a polarized proton.

Using PYTHIA8+StringSpinner made it possible to simulate semi–inclusive deep inelastic lepton scattering by a polarized proton in the initial energy range $\sqrt{S_{lN}}$ = 20–100 GeV. The values of 20, 40, 60, 80 and 100 GeV were considered as reference initial energies.

100,000 events were generated for the considered charged leptons (e^+, e^-, μ^+, μ^-) and hadrons (π^0, π+, π^-, K^+, K^- at reference initial energies and at various polarizations of the proton (longitudinal, transverse, and without polarization). The four-momenta of the finite lepton p_l and hadron p_h were obtained from each event. These are referred to as real data. Using real data allows to obtain the Sivers and Collins asymmetries A_{Siv} and A_{Col} for transversely polarized proton and the asymmetry A_{ul} for longitudinally polarized proton.

The multiplicity distribution N_h for Collins asymmetry A_{Col} is proportional to the binomial in the case of transversely polarized proton [14]:

$$\frac{dN_h}{dx_{\text{Bj}}dzdp_{Th}d\varphi_{\text{Col}}} \propto 1 + D_{NN}S_T A_{\text{Col}} \sin\varphi_{\text{Col}}, \tag{1}$$

where D_{NN} is the depolarization factor, $D_{NN} = 2(1-y)/[1+(1-y)^2]$.

The quantity φ_{Col} in Eq. (1) is defined as

$$\varphi_{\text{Col}} = \varphi_h + \varphi_S + \pi,$$

where φ_S is the azimuthal angle between the transverse component of the spin vector **S** and the lepton scattering plane; φ_h is the azimuthal angle between the hadron emission plane and the lepton scattering plane (Fig. 2).

The hadron multiplicity distribution N_h for the Sivers asymmetry A_{Siv} is defined as [14]:

$$\frac{dN_h}{dx_{\text{Bj}}dzdp_{Th}d\varphi_{\text{Siv}}} \propto 1 + S_T A_{\text{Siv}} \sin\varphi_{\text{Siv}}, \quad (2)$$

where $\varphi_{\text{Siv}} = \varphi_h - \varphi_S$.

$\mathbf{S}_T$ in expressions (1), (2) is the nucleon spin vector perpendicular to both the virtual photon and the emitted hadron.

The values of asymmetry A_{ul} for longitudinallu polarized proton can be obtained from the hadron multiplicity distribution N_h, which is defined as [14]:

$$\frac{dN_h}{dx_{\text{Bj}}dzdp_{Th}d\varphi_h} \propto 1 + (1-y)A_{ul} \sin 2\varphi_h. \quad (3)$$

The following quantities were used in expressions (1)–(3):

$x_{\text{Bj}} = \dfrac{Q^2}{2Pq}$ is the Bjerken variable [13] describing the fraction of the proton momentum carried by the parton (P is the four-momentum of the proton, q is the four-momentum of the virtual photon, $Q^2 = -q^2$);

$z = \dfrac{P \cdot p_h}{P \cdot q}$ is the fraction of the four-momentum of the virtual photon transferred to the emitted hadron [13] (p_h is the four-momentum of the hadron produced);

p_{Th} is the projection of the hadron momentum, perpendicular to the virtual photon;

$y = \dfrac{P \cdot q}{P \cdot p_l'}$ is the fraction of the energy of the incident lepton transferred to the virtual photon [13] (p'_l is the four-momentum of the lepton before interaction).

The described method for calculating asymmetries was successfully tested by comparing it with experimental data obtained in the HERMES and COMPASS experiments [12, 13], yielding good agreement of the predictions with the experimental results. This success gives us reason to select and use a technique for calculating semi-inclusive scattering of leptons by a polarized proton. The range of initial energies typical for future electron-ion colliders is taken [16].

As noted above, specific calculations can be performed using the PYTHIA8+StringSpinner software package.

More details can be found in [17] (see the section "Methodology"), where PYTHIA8 is combined with a generative adversarial network (GAN) to build a generator for semi-exclusive deep-elastic scattering of charged leptons by polarized protons.

In this paper, the type of proton polarization was added as another input parameter of the generator (in addition to those used in [17]). Furthermore, the number of hidden layers of the GAN generator and discriminator was increased to 6

Results of neural network construction and discussion

Fig. 3 shows the values of the Collins asymmetry as a function of the Bjerken variable x_{Bj} for the hadrons π^- and π^+ with the electron e^- and muon μ^- scattered by transversely polarized protons at an initial energy of 40 GeV. Evidently, the asymmetries obtained based on GAN predictions coincide within the uncertainty range with the predictions obtained based on PYTHIA8+StringSpinner data.

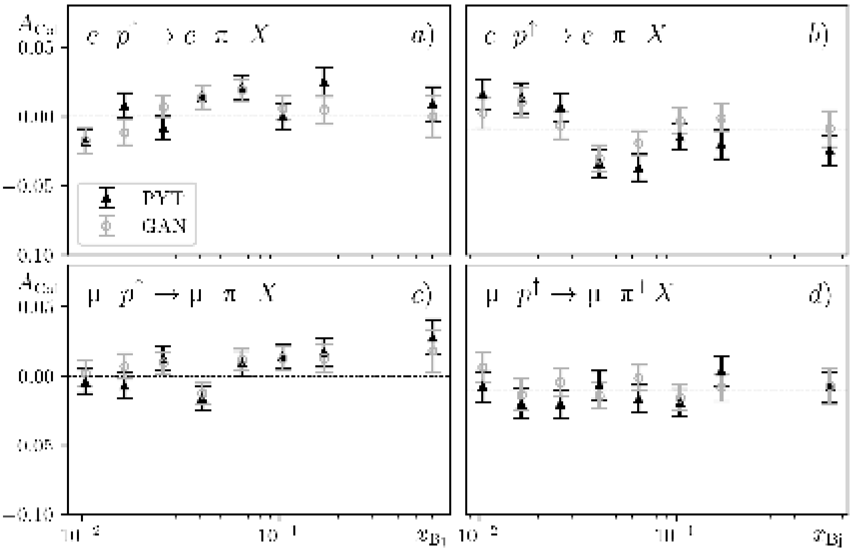

Fig. 3. Dependences of Collins asymmetry ACol on the Bjerken variable x_{Bj}
for scattering of electrons e^- (*a*, *b*) and muons μ^- (*c*, *d*) by transversely polarized protons
with the production of negative (π^-) (*a*, *c*) and positive (π^+) (*b*, *d*) pions.
The initial energy of the particles is 40 GeV.
The data were obtained using GAN (gray dots) and PYTHIA8+StringSpinner (black triangles)

Fig. 4 shows the values of the Sivers asymmetry as a function of the Bjerken variable x_{Bj} for kaons K^- and pions π^0 under scattering of positrons e^+ and antimuon μ^+ by transversely polarized protons at an interpolated initial energy of 70 GeV; the results were obtained based on GAN and PYTHIA8+StringSpinner. It follows from the data presented in Fig. 4 that the GAN-based generator retains the prediction accuracy with a different scattering configuration for Sivers asymmetries, including at energies that were not involved in the learning process.

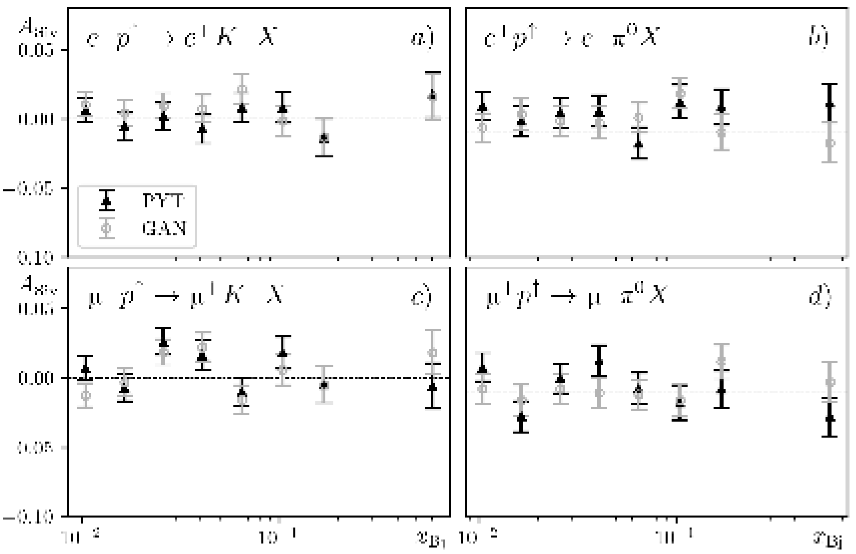

Fig. 4. Dependences of Sivers asymmetry A_{Siv} on the Bjerken variable x_{Bj}
for scattering of positrons e^+ (*a*, *b*) and antimuons μ^+ (*c*, *d*) by transversely polarized protons
with the production of negative kaons K^- (*a*, *c*) and neutral pions π^0 (*b*, *d*).
The data were obtained using GAN (gray dots) and PYTHIA8+StringSpinner (black triangles)

Fig. 5 shows the values of asymmetries A_{ul} depending on the values of the Bjerken variable x_{Bj} for kaons K^+ and pions π^- under scattering of electrons e^- and antimuons μ^+ by transversely polarized protons at an initial energy of 120 GeV, obtained based on GAN and PYTHIA8+StringSpinner. The analysis of these data shows that the GAN model can work with longitudinally polarized protons as well as at energies exceeding the energy range considered during training (extrapolated values).

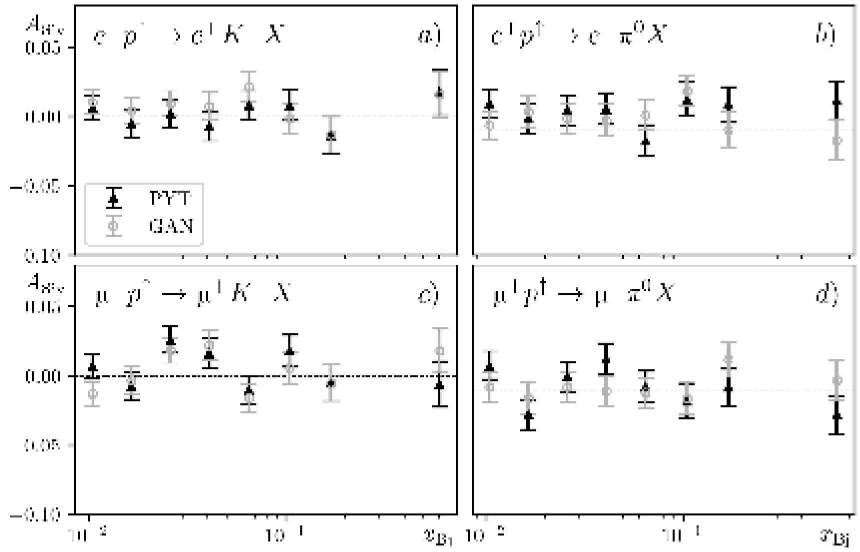

Fig. 5. Dependences of asymmetry A_{ul} on the Bjerken variable x_{Bj}
for scattering of electrons e^- (a, b) and antimuons μ^+ (c, d) by longitudinally polarized protons
with the production of positive kaons K^+ (a, c) and negative pions π^- (b, d).
The initial energy is 120 GeV.
The data were obtained using GAN (gray dots) and PYTHIA8+StringSpinner (black triangles)

Conclusion

In the presented study, a software package (event generator) was developed based on a generative-adversarial network model in order to predict the characteristics of the final state of a lepton and an additional hadron as a result of semi-exclusive deep-elastic scattering of a lepton on a polarized proton.

It is established that the constructed event generator can work accurately with various scattering configurations: incident leptons (e^+, e^-, μ^+, μ^-), hadrons (π^0, π^+, π^-, K^+, K^-), proton polarization states (longitudinal, transverse, without polarization) and initial energies (we considered the range of 20–100 GeV). Moreover, the generator works with the initial energies on which it was pre-trained (20, 40, 60, 80, 100 GeV), with the interpolated energies (between the reference values) and extrapolated ones (values above the considered range).

Studies indicate that the event generator can accurately (accounting for errors) predict various types of asymmetry (A_{Col}, A_{Siv}, A_{ul}) that occur in the presence of proton polarization. The prediction accuracy is preserved for various scattering configurations.

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