Original article
DOI: https://doi.org/10.18721/JPM.16313

NUMERICAL STUDY OF THE EFFICIENCY OF MODAL FILTER METHOD AND OBSERVER METHOD FOR IMPLEMENTATION OF MODAL CONTROL OF VIBRATIONS OF ELASTIC SYSTEMS

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Abstract. The article compares the efficiency of two methods for implementing modal control for active vibration suppression of distributed elastic systems. The former is the modal filter method, which implies a linear transformation of measured and control signals; the latter is the method of modal observers, which uses the object model to reconstruct the state vector from the measurement signals. For this purpose, the problem of suppression of forced bending vibrations of a thin metal beam at several lower resonance frequencies has been solved numerically for two different objects. The simulation results showed an undeniable advantage of the observer method over the modal filter one. The inherent effects of signal transmission in the control loop, occurring in real systems but usually neglected in numerical studies were analyzed. It was established that these phenomena had a significant impact on the efficiency of the synthesized control systems. Therefore, they must be taken into account in numerical simulations.

Keywords: active vibration suppression, modal control, modal filter, distributed elastic systems

Funding. The research was supported by the Ministry of Science and Higher Education of the Russian Federation (Agreement No. 075-15-2021-573).

Citation: Fedotov A. V., Belyaev A. K., Polyanskiy V. A., Numerical study of the efficiency of modal filter method and observer method for implementation of modal control of vibrations of elastic systems, St. Petersburg State Polytechnical University Journal. Physics and Mathematics. 16 (3) (2023) 160–173. DOI: https://doi.org/10.18721/JPM.16313

Научная статья
УДК 531.391+681.5
DOI: https://doi.org/10.18721/JPM.16313

ЧИСЛЕННОЕ ИССЛЕДОВАНИЕ ЭФФЕКТИВНОСТИ МЕТОДА МОДАЛЬНЫХ ФИЛЬТРОВ И МЕТОДА НАБЛЮДАТЕЛЕЙ ПРИ РЕАЛИЗАЦИИ МОДАЛЬНОГО УПРАВЛЕНИЯ КОЛЕБАНИЯМИ УПРУГИХ СИСТЕМ

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Аннотация. Статья посвящена сравнению эффективности двух методов реализации модального управления для активного гашения колебаний распределенных упругих систем: метода модальных фильтров, предполагающего линейное преобразование измеренных и управляющих сигналов, и метода наблюдателей, использующих модель объекта для восстановления вектора состояния по результатам измерений. Для этого численно решается задача (в двух постановках) гашения вынужденных изгибных колебаний тонкой металлической балки на нескольких низших резонансах с помощью пьезоэлектрических сенсоров и актуаторов. Полученные результаты показали бесспорное преимущество метода наблюдателей перед методом модальных фильтров. Проанализированы эффекты передачи сигнала в контуре управления, возникающие в реальных системах, но не принимаемые, как правило, во внимание в численных исследованиях. Установлено, что эти эффекты существенно влияют на эффективность синтезируемых систем управления, поэтому их необходимо учитывать при моделировании.

Ключевые слова: активное гашение колебаний, модальное управление, модальный фильтр, распределенные упругие системы

Финансирование: Работа выполнена при поддержке Министерства науки и высшего образования Российской Федерации (соглашение № 573-2021-15-075).

Ссылка для цитирования: Федотов А. В., Беляев А. К., Полянский В. А. Численное исследование эффективности метода модальных фильтров и метода наблюдателей при реализации модального управления колебаниями упругих систем // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2023. Т. 3 № .16. С. 160–173. DOI: https://doi.org/10.18721/JPM.16313

Introduction

Active control of vibrations in distributed elastic systems has received much attention in recent decades. Similar problems are found in many fields, such as construction, robotics, mechanical engineering, automotive, aerospace, etc. The control problem is often formulated as suppressing the forced vibrations of an object, since such vibrations can worsen the operational characteristics of a structure, causing unwanted noise or even damage and failure. Our paper addresses this specific problem.

Active feedback control of mechanical vibrations in elastic systems is carried out by sensors in the control system (CS), feeding input signal to actuators (drives) applying control action to the object, which is the output of the CS. Piezoelectric elements capable of performing the functions of sensors and actuators due to direct and reverse piezoelectric effects have become widespread for these purposes. Such elements are simple and convenient to use, are easily molded into the required shape and can operate in a wide frequency range, which makes them very attractive for active oscillation vibration control. Here we consider sensors and actuators that are piezoelectric plates coated with electrodes and glued to the control object (metal beam).

Our previous studies presented experimental [1] and numerical [2, 3] comparison of various methods for active control for the problem of damping forced bending vibrations of a thin metal beam in a frequency range including two lower resonance frequencies. We considered a local approach (each control loop contains one sensor and one actuator located on both sides of the beam in the same region), a modal approach (each control loop corresponds to a specific vibration mode and uses all available sensors and actuators), as well as a modal control method (only one control loop is used, configured to compensate for a known form of external perturbation). Recent research indicates that the modal approach has the greatest efficiency among the methods for damping forced vibrations in objects at several resonance frequencies.

The modal approach in these studies is based on mode separation in the CS by the method of modal filters, i.e., the matrices giving the linear transformation of measured and control signals. This procedure allows to 'filter out' the unnecessary modes to ensure that each control loop corresponds to one specific mode of the object's vibrations. This method is easy to use, but its efficiency is limited, especially given a small number of sensors and actuators. There is another, more advanced method for mode separation in control loops: the observer method. It uses an object model to reconstruct the state vector based on the measurement results, therefore providing more efficient mode separation. However, this approach also does not completely solve the main problem of modal control that is the spillover effect, i.e., the transfer of energy to higher uncontrolled modes [4], which can destabilize a closed system.

The main goal of our study is to compare the two described methods of modal control, the modal filter method and the observer method, under the same conditions.

Our focus was for the problem formulated in the numerical study to match the experimental conditions as closely as possible, namely, to take into account the effects of signal transmission in the control loop that are integral in real systems. Such effects as phase shift and amplitude variation of the control signal, occurring due to delay and additional elements in the control loop, are also considered.

As a rule, these phenomena are not accounted for in numerical simulation [5, 6], however, this study presents clear proof that they significantly affect the result of CS synthesis and its efficiency, and therefore must be taken into account in numerical studies if their ultimate goal is the experimental implementation of the given systems.

Theoretical fundamentals of the approaches under consideration

The modal approach has originated in the 1960s, when its basic principles were first formulated in [7]. The proposed method was subsequently developed in [8, 9]. The modal approach has become the classical one, as it is well-studied and used in various fields of technology [10–12]. This approach can integrate both the modal filter method (for distributed sensors and actuators [13, 14] or their discrete systems [15, 16]) and the observer method [5, 17, 18].

This section provides a general overview of these methods, explaining their application within the framework of the study.

Modal filter method. Consider the problem of controlling bending vibrations of beams using piezoelectric sensors and actuators. Let us imagine the transverse displacement of the beam points $w(x,t)$ as an expansion in terms of eigenmodes:

$$w(x,t)=\sum_{i=1}^{n}X_i(x)q_i(t), \tag{1}$$

where n is the number of eigenmodes of the beam vibrations taken into account in the model; $X_i(x)$ are the vibrational eigenmodes; $q_i(t)$ are the generalized coordinates.

Let us write the equations for the beam vibrations in matrix form for the eigenmodes:

$$\ddot{q}+2\xi\Omega\dot{q}+\Omega^2 q=Q^c+Q^d, \tag{2}$$

where $q_{n\times 1}(t)$ is the vector of generalized coordinates with the length n; $\Omega_{n\times n}$ is the diagonal matrix of the beam's vibrational eigenmodes; ξ is the scalar damping coefficient (taken to be the same for all modes for simplicity); $Q^c_{n\times1}(t)$, $Q^d_{n\times1}(t)$ are the vectors of generalized forces corresponding to control and external perturbation.

For simplicity, let us assume that the number of sensors and actuators is the same and equal to m ($m \le n$). Their operation is described by the following equations:

$$y_{m\times1} = \Theta^s_{m\times n} q_{n\times1}, \tag{3}$$

$$Q^c_{n\times1} = \Theta^a_{n\times m} u_{m\times1}, \tag{4}$$

where $y_{m\times1}(t)$ is the vector of sensor signals; $u_{m\times1}(t)$ is the vector of control signals applied to the actuators; $\Theta^s_{m\times n}$, $\Theta^a_{n\times m}$ are the influence matrices for sensors and for actuators (characterizing how intensely each sensor reacts to each mode, and each actuator excites each mode).

Sensors and actuators are arranged consistently in the problems considered in the paper as sensor-actuator pairs on both sides of the beam. This means that the influence matrices are also related:

$$\Theta^a_{n\times m} = k^{as}\left(\Theta^s_{m\times n}\right)^T = \Theta, \tag{5}$$

where k^{as} is a scalar coefficient depending on the physical and geometric parameters of sensors and actuators.

The numerical values of the elements of the influence matrices depend on how the actuators and sensors are arranged on the control object. There are criteria that allow optimizing the locations of these elements, formulated specifically for the matrix Θ. For example, it is proposed in [6, 17] to maximize the minimum singular number of the matrix Θ, or the minimum eigenvalue of the matrix $\Theta^T\Theta$, which is essentially the same.

The piezoelectric elements in this study are arranged on the beam so as to maximize their influence on the first and second eigenmodes, since the control problem consists in dampening the vibrations specifically at the two lower frequencies.

Finally, let us give definition to modal filters. Modal filters are matrices giving the linear transformations of measured and control signals and ensuring that each control loop corresponds to a certain mode of the object's vibrations.

We assume that control is carried out for k modes. The matrix T, which is the mode analyzer, is responsible for processing the measured signals:

$$\hat{q}_{k\times1} = T_{k\times m} y_{m\times1}, \tag{6}$$

where $\hat{q}_{k\times1}(t)$ is the estimate vector of k lower generalized coordinates $q_i(t)$.

The control actions are set in accordance with the matrix F, which the mode synthesizer:

$$u_{m\times1} = F_{m\times k} \hat{Q}_{k\times1}, \tag{7}$$

where $\hat{Q}_{k\times1}(t)$ is the vector of required control actions on k lower eigenmodes.

Since we consider modal control, the vector of required actions depends on the vector of estimates of generalized coordinates as follows:

$$\hat{Q}_{k\times1} = -R_{k\times k}\hat{q}_{k\times1}, \tag{8}$$

where $R_{k\times k}$ is a diagonal matrix of gain coefficients.

The elements of this matrix generally define the control laws in each loop, so they can be written as functions of a complex variable s:

$$R_{ii} = R_i(s). \tag{9}$$

Evidently, the matrices T and F should be defined as follows in the simplest case $k = m = n$:

$$T = \left(\Theta^s\right)^{-1}, F = \left(\Theta^a\right)^{-1}. \tag{10}$$

This would mean satisfying the equalities

$$\hat{q} = q, Q^c = \hat{Q}.$$

However, the number of modes n to be taken into account in control of distributed systems generally exceeds the number of sensors and actuators m, while the number of modes k controlled may also differ from these numbers.

In this case , it is possible to represent the influence matrices in the following form:

$$\Theta^s_{m\times n} = \begin{bmatrix} \bar{\Theta}^s_{m\times k} & \tilde{\Theta}^s_{m\times(n-k)} \end{bmatrix}, \ \Theta^a_{n\times m} = \begin{bmatrix} \bar{\Theta}^a_{k\times m} \\ \tilde{\Theta}^a_{(n-k)\times m} \end{bmatrix}, \tag{11}$$

the analyzer and synthesizer of the modes are defined as pseudo-inverse to the corresponding components of the influence matrices:

$$\Theta^s_{m\times n} = \begin{bmatrix} \bar{\Theta}^s_{m\times k} & \tilde{\Theta}^s_{m\times(n-k)} \end{bmatrix}, \ \Theta^a_{n\times m} = \begin{bmatrix} \bar{\Theta}^a_{k\times m} \\ \tilde{\Theta}^a_{(n-k)\times m} \end{bmatrix}, \tag{12}$$

This is the method for determining modal filters used in this study. For example, in the case of control by two lower modes using two sensor-actuator pairs ($k = m = 2$), the described method ensures that the first mode is not excited and does not affect the first control loop, and the second mode does not affect the second loop.

However, higher modes are both excited and affect both control loops. This phenomenon is called the spillover effect (the transfer of energy to higher modes) and is the main obstacle limiting the efficiency of modal control.

We earlier proposed an algorithm called the experimental identification method [19]: it allows experimentally implementing the described method for determining modal filters.

Meanwhile, there are other approaches to setting the matrices T and F. The first of them [20] is close to the one already considered:

$$\left(\Theta^s_{m\times n}\right)^+ = \begin{bmatrix} T_{k\times m} \\ \tilde{T}_{(n-k)\times m} \end{bmatrix}, \ \left(\Theta^a_{n\times m}\right)^+ = \begin{bmatrix} F_{m\times k} & \tilde{F}_{m\times(n-k)} \end{bmatrix}. \tag{13}$$

It is also possible to determine the mode analyzer T by approximating the displacement function of the beam points $w(x,t)$, using the displacement values measured by sensors [20] (if piezoelectric plates measuring and acting on the curvature of the beam are used, it is more correct to approximate the curvature function $w''(x,t)$); the mode synthesizer F can be determined similarly.

The proposed method can be described approximately by the following formulas:

$$T_{k\times m} = M^s_{k\times k}\left(\bar{\Theta}^s_{m\times k}\right)^T, \ F_{m\times k} = \left(\bar{\Theta}^a_{k\times m}\right)^T M^a_{k\times k}, \tag{14}$$

where $M^s_{k\times k}$, $M^a_{k\times k}$ are some diagonal matrices whose elements give the degree of excitation and response of the control system to individual modes.

If the number of sensors and actuators is increased (so that they more or less cover the entire surface of the control object), the methods considered produce the same results, since the eigenmodes of an elastic body are orthogonal.

An advantage of the modal filter method is that it is simple to implement; additionally, if a sufficiently large number of sensors and actuators are used, it allows to avoid destabilization of the object's higher modes [20]. However, this effect can still be dangerous in such a system if there is a small number of sensors and actuators, instability in the higher modes.

Observer method We describe the method by representing system (2) in the state space:

$$\dot{q}^n = Aq^n + Bu + Dd, \tag{15}$$

where q^n is the state vector of the system associated as follows with the vector of generalized coordinates from (2):

$$q^n = \begin{pmatrix} q_1 & \dots & q_n & \dot{q}_1 & \dots & \dot{q}_n \end{pmatrix}^T; \tag{16}$$

d is the vector of external influences with the length $\bar{m}$, and the matrix D has the form

$$D = \begin{bmatrix} 0_{n\times\bar{m}} \\ \tilde{D}_{n\times\bar{m}} \end{bmatrix}, \tag{17}$$

so that the following equality holds true:

$$\tilde{D}_{n\times\bar{m}} d_{\bar{m}\times 1} = Q^d_{n\times 1}. \tag{18}$$

The remaining matrices used in Eq. (15) can also be expressed in terms of matrices from Eqs. (2) and (4):

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -\Omega^2_{n\times n} & -2\xi\Omega_{n\times n} \end{bmatrix},\ B = \begin{bmatrix} 0_{n\times m} \\ \Theta^a_{n\times m} \end{bmatrix}, \tag{19}$$

where $0_{n\times n}$ and $0_{n\times m}$ are matrices consisting of zeros; $I_{n\times n}$ is a unit matrix.

The equation for sensor signals is written similarly:

$$y = Cq^n = \begin{bmatrix} \Theta^s_{m\times n} & 0_{m\times n} \end{bmatrix} q^n. \tag{20}$$

To formulate the laws for observers, we rewrite Eqs. (15) and (20), dividing the system into two parts. We assume here that observation is carried out for the same k lower modes for which control is carried out ($k < n$):

$$\dot{q}^k = A^{(1)} q^k + B^{(1)} u + D^{(1)} d, \tag{21}$$

$$\dot{q}^{n-k} = A^{(2)} q^{n-k} + B^{(2)} u + D^{(2)} d, \tag{22}$$

$$y = C^{(1)} q^k + C^{(2)} q^{n-k}. \tag{23}$$

Here, two state vectors appear instead of one:

$$q^k = \begin{pmatrix} q_1 & \dots & q_k & \dot{q}_1 & \dots & \dot{q}_k \end{pmatrix}^T, \tag{24}$$

$$q^{n-k} = \begin{pmatrix} q_{k+1} & \dots & q_n & \dot{q}_{k+1} & \dots & \dot{q}_n \end{pmatrix}^T, \tag{25}$$

and the matrices from Eqs. (15) and (20) are divided into two parts accordingly. Next, we formulate the law of observers assuming that the matrices $A^{(1)}$, $B^{(1)}$, $C^{(1)}$ are known:

$$\dot{\hat{q}} = A^{(1)}\hat{q} + B^{(1)} u + L\left(y - C^{(1)}\hat{q}\right). \tag{26}$$

Here $\hat{q}_{2k\times 1}$ is the estimate of the state vector q^k, or the vector of estimates k of the first generalized coordinates and velocities.

Let us formulate the control law similar to Eqs. (7), (8) with a constant control matrix R:

$$u_{m\times 1} = F_{m\times k}\hat{Q}_{k\times 1} = -F_{m\times k} R_{k\times 2k} \hat{q}_{2k\times 1}. \tag{27}$$

Notice that unlike Eq. (8), the matrix R here is not square but rectangular (with the size $k \times 2k$), since the vector $\hat{q}$ contains estimates not only of generalized coordinates, but also of velocities.

To write a general equation for the entire closed system, we introduce the observation error vector $e_{2k\times 1}$ and rewrite Eqs. (21), (22), (26) and (27):

$$e = \hat{q} - q^k, \tag{28}$$

$$u = -FR\left(q^k + e\right), \tag{29}$$

$$\dot{q}^k = \left(A^{(1)} - B^{(1)}FR\right)q^k - B^{(1)}FRe + D^{(1)}d, \tag{30}$$

$$\dot{q}^{n-k} = A^{(2)}q^{n-k} - B^{(2)}FR\left(q^k + e\right) + D^{(2)}d, \tag{31}$$

$$\dot{e} = \left(A^{(1)} - LC^{(1)}\right)e + LC^{(2)}q^{n-k} - D^{(1)}d. \tag{32}$$

Thus, the equation of the whole system can be written as follows:

$$\dot{\bar{q}} = \bar{A}\bar{q} + \bar{D}d, \tag{33}$$

introducing the following notations:

$$\bar{q} = \begin{pmatrix} q^k & q^{n-k} & e \end{pmatrix}^T, \tag{34}$$

$$\bar{A} = \begin{bmatrix} A^{(1)} - B^{(1)}FR & 0_{2k\times 2(n-k)} & -B^{(1)}FR \\ -B^{(2)}FR & A^{(2)} & -B^{(2)}FR \\ 0_{2k\times 2k} & LC^{(2)} & A^{(1)} - LC^{(1)} \end{bmatrix}, \tag{35}$$

$$\bar{D} = \begin{bmatrix} D^{(1)} \\ D^{(2)} \\ -D^{(1)} \end{bmatrix}. \tag{36}$$

Evidently, if $k = n$, the second row and the second column of the matrix of the closed system $\bar{A}$ disappear, and the poles of the system are given by two matrices:

$$\bar{A}_{11} = A^{(1)} - B^{(1)}FR \text{ and } \bar{A}_{33} = A^{(1)} - LC^{(1)}.$$

Thus, the observation and control problems are separated, and we can independently synthesize the matrices L and R, based on the required properties of a closed system. The presence of higher uncontrolled modes (at $k < n$) immediately complicates the problem: the previously mentioned spillover effect appears. The effect consists of two distinct components: the first one is observation spillover, occurring due to activation of higher modes, for which the matrix component $\bar{A}_{32} = LC^{(2)}$ is responsible; the second one is the control spillover, i.e., excitation of higher modes by the control system, expressed by the components of the matrix

$$\bar{A}_{21} = \bar{A}_{23} = -B^{(2)}FR.$$

The spillover effect interferes with the operation of the control system and limits its efficiency; in particular it can lead to destabilization of a closed system.

Apparently, if the dynamics of a system with control is described by Eq. (33) with matrix (35), then this description allows to find an explicit solution in the time or frequency domain as well as analyze the stability of the system by the eigenvalues of matrix $\bar{A}$. However, this method is suitable only if the control matrix R is constant, i.e., its components do not depend on the complex variable s. Unlike theoretical models, there is always a delay in the control loop in real-life conditions, which can be described just by setting the dependence $R(s)$. The control loop may also include additional elements, such as low-pass filters and amplifiers that have their own amplitude and phase frequency characteristics, which also affects the transfer functions (TF) in the control loops.

In this case, the dynamic response of the system using observers is calculated via the following formulas for the Laplace transforms of variables introduced earlier:

$$\hat{q}(s) = \left(sI_{2k} - A^{(1)} + LC^{(1)} + B^{(1)}FR(s)\right)^{-1} Ly(s), \tag{37}$$

$$u(s) = -FR(s)\hat{q}(s) = -K(s)y(s), \tag{38}$$

$$K(s) = FR(s)\left(sI_{2k} - A^{(1)} + LC^{(1)} + B^{(1)}FR(s)\right)^{-1} L, \tag{39}$$

$$q(s) = \left(sI_{2n} - A + BK(s)C\right)^{-1} Dd(s). \tag{40}$$

The matrix R is considered constant in the object models used in the observers in this paper. In particular, this assumption is necessary to analyze the stability of observation for the A_{obs} matrix:

$$A_{obs} = A^{(1)} - LC^{(1)} - B^{(1)}FR. \tag{41}$$

Thus, the observer's operation does not account for the above-mentioned additional effects on signal transmission in control loops, while the general model of the system does in fact account for these effects. One of the goals of the study is precisely to analyze how these phenomena influence the efficiency of control with observers.

Let us focus more closely on the structure of the observation matrix L and the control matrix R. Since we use a modal approach, these matrices should provide separate control of different modes of the object's vibrations. For this purpose, we give the matrices in the following manner:

$$L_{2k\times k} = \begin{bmatrix} K^{L}_{k\times k} \\ K^{Ld}_{k\times k} \end{bmatrix} T_{k\times m},\; R_{k\times 2k} = \begin{bmatrix} K^{R}_{k\times k} & K^{Rd}_{k\times k} \end{bmatrix}, \tag{42}$$

where $T_{k\times m}$ is the mode analyzer; $K^{L}_{k\times k}$, $K^{Ld}_{k\times k}$, $K^{R}_{k\times k}$, $K^{Rd}_{k\times k}$ are diagonal matrices with elements K^{L}_{i}, K^{Ld}_{i}, K^{R}_{i} and K^{Rd}_{i}, $i = 1,..., k$.

The elements K^{R}_{i} and K^{Rd}_{i}, characterize how the control effect on the ith mode depends on the estimates of the ith generalized coordinate and velocity; the elements K^{L}_{i} and K^{Ld}_{i} reflect the influence of the estimate error of the ith generalized coordinate on the dynamics of estimates of the ith generalized coordinate and velocity. The specified error is obtained from the errors in determining the measured signals $\hat{y} - y$ using a linear transformation given by the matrix T.

Notably, the matrices introduced earlier for the modal filter method participate in observation and control. This is the mode analyzer and syntesizer matrices T and F (see Eqs. (27) and (42)). They perform the same function, separating the object's vibrational modes in the control system; they can be determined by the methods described in the section 'Modal filter method'.

To summarize, we should note that the observer method is more difficult to apply than the modal filter method, and it can be expected to provide a more accurate estimate of generalized coordinates, since the observer method involves a control object model to solve this problem. Moreover, the observer method can be further sophisticated to improve the efficiency by expanding the vector of estimated variables or using distributed observers as well as nonlinear control methods [21–24].

Hinged-support beam

The first part of the study considers modal CS for damping vibrations in a hinged-support beam. For the system considered in this section, the problem of synthesizing a CS using modal filters was solved in [25] by one of the authors of this study.

Thus, we consider a hinged-support aluminum beam with a cross section of 3 × 35 mm and a length of 1 m, making bending vibrations in the *xz* plane (Fig. 1).

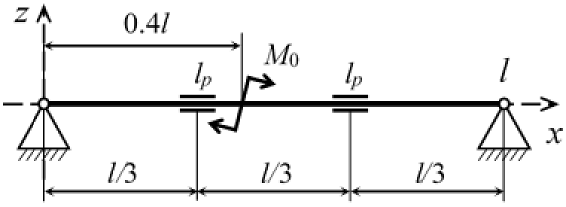

Fig. 1. Schematic representation of hinged-support beam with sensors and actuators; l is the beam length, M_0 is the external bending moment

An external harmonic perturbation is applied to the beam in the cross section with the coordinate x_0 = 0.4 m, which is the bending moment with the magnitude M_0 = 0.1 N·m. The control system includes two sensors and two actuators, which are thin rectangular piezoelectric plates measuring 50 × 30 mm, coated with electrodes and glued to the beam by sensor-actuator pairs in two segments.

The problem of the control system is to dampen the forced bending vibrations of the beam in a frequency range that includes two lower resonance frequencies. The numerical model of the beam takes into account n = 33 lower modes.

The coordinates of sensors and actuators on the beam are selected in accordance with the goal of control: they are arranged symmetrically relative to the middle of the beam so as to efficiently respond and act on the first and second modes of bending vibrations. For this purpose, we decided to place them evenly, at distances $l/3$ = 0.333 m from the ends of the beam.

Let us consider the synthesis stages of a modal CS by the modal filter method. First we need to set these modal filters, i.e., matrices T and F (mode analyzer and synthesizer). They are given by inversion of the components of the influence matrices, in accordance with Eq. (12) for the simplest case $k = m = 2$; the result is discussed in [25]. The next step is to set the control laws for each loop. At this stage, the TF coefficients in the control loops were varied via the optimization procedure [3, 26] and the most efficient of them were determined. The stability condition of a closed system was monitored using the Nyquist criterion for the case of two control loops [2].

Synthesis of the modal control system with observers consisted of similar stages. First, the matrices T and F had to be set in exactly the same way. Secondly, it was necessary to set the observation and control matrices L and R: their structure was chosen in accordance with the Eq. (42), so it remained to determine only the diagonal elements of the matrices $K^{L}_{2\times2}$, $K^{Ld}_{2\times2}$, $K^{R}_{2\times2}$, $K^{Rd}_{2\times2}$; the above optimization procedure was applied for this purpose.

Notably, the experimental specifics, i.e., phase delay and a decrease in the signal amplitude at higher frequencies were taken into account in the control loops for the first problem that was CS synthesis by the modal filter method. These phenomena are modeled by the following component of the TF:

$$R^{del}\left(s\right)=\frac{1}{1+0.0005s}. \tag{43}$$

This component was included in the general TF of the control loops obtained by the optimization method.

The second problem, CS synthesis using observers, is solved by two approaches: with and without taking into account the delay (43). If the delay is not taken into account, the closed system can be described by Eq. (33) with matrix (35). Describing the system becomes more complex if the delay is taken into account; Eqs. (37)–(40) are used for this purpose. As noted earlier, the matrix R in the system model used in the observer is considered constant.

TF in control loops for the modal filter method are given in [25]. The following are the optimal parameters of the CS for the observer method. The parameters for the case when the delay is not taken into account take the form

$$\begin{cases} K^L = diag\{0.75 \quad 3.6\}, K^{Ld} = diag\{-155 \quad -2650\}, \\ K^R = 0_{2\times2}, K^{Rd} = diag\{195 \quad 640\}. \end{cases} \tag{44}$$

If the delay is taken into account, then

$$\begin{cases} K^L = diag\{0.015 \quad 2\}, K^{Ld} = diag\{-163 \quad -2600\}, \\ K^R = 0_{2\times2}, K^{Rd} = diag\{3600 \quad 1120\}. \end{cases} \tag{45}$$

Now let us compare the results obtained for different control systems. Fig. 2 shows the frequency response of the closed system with different types of control, in the frequency range from 0 to 40 Hz.

The intensity of beam vibrations E in stationary mode is used as a controlled value, determined as follows:

$$E = \sqrt{\frac{1}{l}\int_0^l \left|w(x,t)\right|^2 dx}. \tag{46}$$

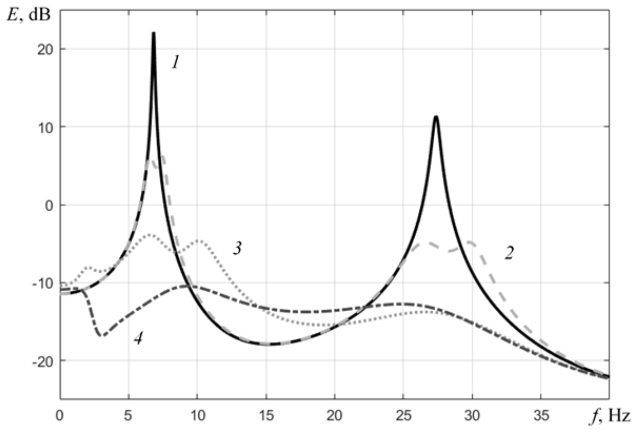

Fig. 2. Frequency response of hinged-support beam without control (curve *1*), with control by modal filter method (*2*)and by observer method without (*3*) and with (*4*) accounting for the delay in the control loop

The results of damping forced vibrations in the first (I) and second (II) resonances for different control systems are given in Table 1.

Analyzing the data in Fig. 2 and in Table 1, we can conclude that the observer method (curves *3*, *4*) is much more efficient than the modal filter method (curve *2*). At the same time, taking into account the delay in the control loop has a considerable influence on the result, small near the second resonance, but leading to a dramatic change in the vibration amplitude near the first one, increasing the control efficiency.

Elastically restrained beam

The second part of the study focuses on creating modal control systems for vibrations in another elastic system, a beam elastically restrained in the midsection. This numerical study logically continues the experiment we conducted earlier [1].

Table 1

Decrease in resonance amplitudes of vibrations in hinged-support beams for different control systems

Control system	Amplitude decrease, dB, for resonance	
	I (ΔE_1)	II (ΔE_2)
With modal filters	–15.9	–16.2
With observers:		
without accounting for the delay	–26.0	–25.1
accounting for the delay	–32.6	–24.1

The problem statement repeats the one used in the experiment. The experimental setup is shown schematically in Fig 3. Aluminum beam *1* with a cross section of 3 × 35 mm and a length of 70 cm is arranged vertically and fixed at one point at a distance of 10 cm from the lower end. The fastener connecting the beam to fixed base *3* includes piezoelectric stack actuator *2*, whose longitudinal vibrations induce vibrations in the support, acting as an external excitation.

The control system consists of two sensor-actuator pairs (sensors *4*, actuators *5*), the same as in the problem discussed in the section "Hinged-support beam".

The purpose of the control system, as before, is to dampen the bending vibrations of the beam at the first (I) and second (II) resonances. The location of sensors and actuators on the beam is selected in accordance with the intended purpose [1].

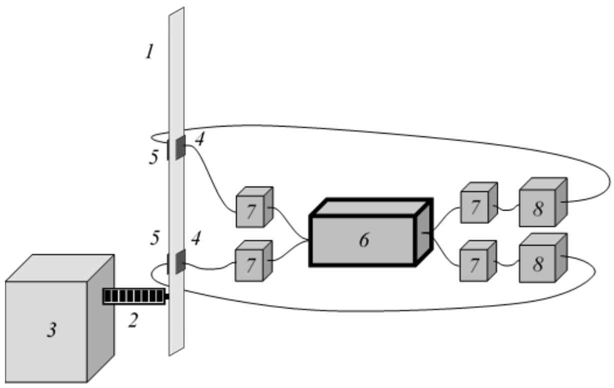

Fig. 3. Schematic of experimental setup:
aluminum beam *1*; piezoelectric stack actuator *2*; fixed base *3*; sensors *4*; actuators *5*; digital controller *6*; low-pass filters *7*; amplifiers *8*

In addition to digital controller *6* converting the measured signals into control signals, the control loop includes additional elements, low-pass filters *7* and amplifiers *8*. Filters smooth out the high-frequency components of the signal arising from discretization in the controller, and generally increase the stability of the closed system; amplifiers increase the amplitude of the control signal 25 times before it is fed to the actuators. In addition, it is also necessary to account for the signal transformations in the control loop that occur due to charge relaxation of piezoelectric sensors when voltage is measured in them, appearing at low frequencies. This effect was discovered experimentally and then taken into account in the simulation.

Fig. 4 shows the logarithmic amplitude and phase frequency characteristics of filters, amplifiers, and piezoelectric sensors, since they all affect signal transmission in the control loop, which means they must be taken into account in the system model.

Numerical simulation of the closed system is carried out in several stages.

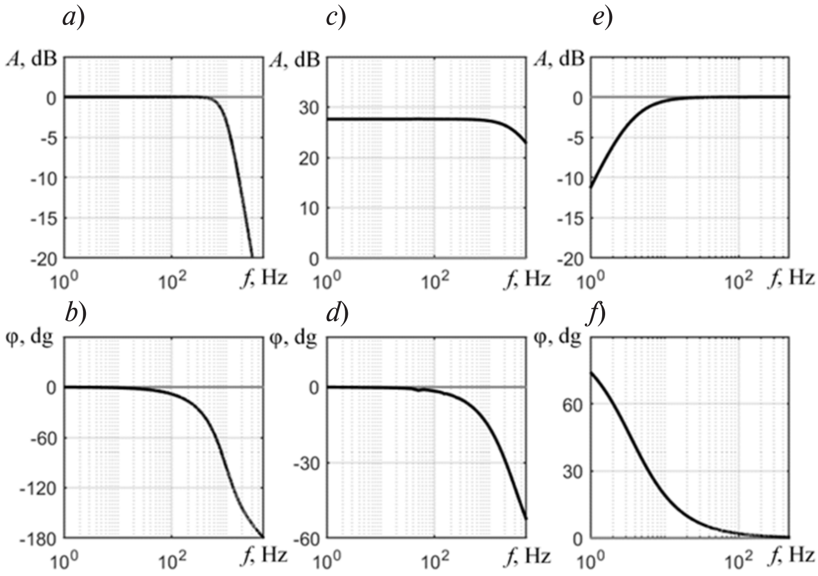

Fig. 4. Frequency (*a*, *c*, *e*) and phase (*b*, *d*, *f*) response of control loop elements affecting signal transmission: low-pass filters (*a*, *b*), amplifiers (*c*, *d*), piezoelectric sensors (*e*, *f*)

Firstly, we constructed a three-dimensional model of the beam with piezoelectric elements and fastener in the ANSYS package, conducting harmonic analysis to obtain various frequency and phase responses of the system. In this study, a finite element (FE) model containing 3,534 elements and 21,088 nodes is used; the damping coefficient in such a model is assumed to be the same for all modes: $\xi = 0.002$.

The FE model is described in more detail in [2, 26]; there is also a comparison of the results of FE calculations and the experiment, establishing high accuracy for the model used.

At the second stage of simulation, the parameters are set for each type of control system tested, with the frequency and phase responses determined, after which the frequency characteristics of the closed system are calculated by combining the obtained characteristics [2]. The control efficiency is determined by analyzing the amplitude of vibrations in a point at the upper end of the beam.

The results of the CS synthesized earlier using the optimization procedure and based on the modal filter method are given in [3, 26]. When we constructed the CS by the observer method, we decided to control three rather than two lower modes ($k = 3$), while the number of sensors and actuators and their locations on the beam remained the same as for the modal filter method ($m = 2$). This can be considered an indisputable advantage of the observer method: the system model in the observation loop can be arbitrarily complex, even with a small number of sensors and actuators used. On the other hand, this number imposes a limit on the number of eigenmodes controlled independently in the modal filter method: $k \le m$.

The matrices T and F for the CS with observers were calculated by Eq. (12):

$$T = \begin{bmatrix} 0.38 & 0.28 \\ -1.32 & 2.84 \\ 2.29 & 2.21 \end{bmatrix} \cdot 10^{-6}, \; F = \begin{bmatrix} 8.16 & -28.48 & 49.17 \\ 6.06 & 61.12 & 47.54 \end{bmatrix}. \tag{47}$$

The observation matrices L and control matrices R obtained by the optimization procedure have the following structure (taking into account the form of both matrices (42)):

$$\begin{cases} K^L = diag\{90 \quad 5.08 \quad -2\}, \; K^{Ld} = diag\{-4280 \quad -4920 \quad -4660\}, \\ K^R = 0_{3\times3}, \; K^{Rd} = diag\{9800 \quad 610 \quad 3600\}. \end{cases} \tag{48}$$

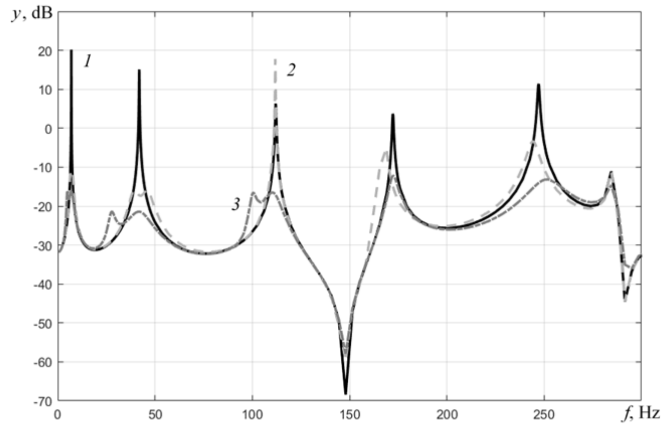

Fig. 5. Frequency response of elastically restrained beam without control system (curve *1*), with control based on modal filter (*2*) and observer (*3*) methods

Fig. 5 shows the results obtained by damping the beam vibrations by different control systems, namely, the frequency response of the closed system, where the vibrational amplitude of the point at the upper end of the beam, in the frequency range from 0 to 300 Hz, acts as the monitored value.

Table 2 shows the results for damping of forced beam vibrations by different control systems at resonances from the first (I) to the fifth (V). Notably, the first CS was intended for controlling only two lower modes, and the second CS for controlling three, however, as it turned out, these systems can operate at higher resonances.

It can be seen from the data in Fig. 5 and in Table 2 that the observer method, as in the case of the hinged-support beam, is more efficient than the modal filter method: it produces a greater decrease in the amplitude of forced beam vibrations not only at the first (I) and second (II), but also at several subsequent resonances.

Table 2

Decrease in resonance amplitudes of vibrations in elastically restrained beam for different control systems

Control system	Amplitude decrease, dB, for resonance				
	I (Δy_1)	II (Δy_2)	III (Δy_3)	IV (Δy_4)	V (Δy_5)
With modal filters	–31.96	–31.06	+11.42	–8.73	–14.72
With observers	–36.50	–36.52	–22.85	–15.94	–24.51

Conclusion

We carried out a numerical comparison of two methods for modal control of vibrations in distributed systems: the modal filter method and the observer method. We established that the second method allows to construct more efficient control systems than the first, producing a greater decrease in the amplitudes of the object's forced vibrations with a greater number of resonance frequencies. An important advantage of the observer method is the simplicity of control synthesis: it is sufficient to optimize the elements of several diagonal matrices instead of setting cumbersome transfer functions.

In addition, we studied the influence of the delay in the control loop occurring in real systems, in particular due to the presence of additional elements (such as filters and amplifiers), on the result of synthesis of the modal control system with observers. It was found that such influence is significant, so it should be taken into account in numerical models.

Several directions are interesting for further numerical research: search for the optimal way to set the matrices for the mode synthesizer and analyzer, modification of the observer by expanding the observation vector as well as using nonlinear observation and control laws. Another promising direction is experimentally confirming the comparative efficiency of various methods for modal control obtained in the paper by theoretical methods.

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Received 05.05.2023. Approved after reviewing 29.06.2023. Accepted 29.06.2023.
Статья поступила в редакцию 05.05.2023. Одобрена после рецензирования 29.06.2023. Принята 29.06.2023.