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THE INTERACTION OF MIXED FORCED, PARAMETRIC AND SELF-EXCITED OSCILLATIONS AT LIMITED EXCITATION AND DELAYS

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Abstract. On the basis of a dynamic model of a frictional self-oscillating system, the influence of delays in elasticity and friction causing self-oscillations on mixed forced, parametric and self-oscillations during the interaction of an oscillating system with an energy source has been considered. The solution of nonlinear differential equations of motion of an oscillatory system and an energy source was constructed using the method of direct linearization. The latter differs from the known methods for the analysis of nonlinear systems by many advantages, including ease of use. Based on the Routh – Hurwitz criteria, the stability conditions for the analysis of stationary motions were obtained. Calculations were carried out to obtain information on the influence of delays on the oscillation modes. This influence was established to be very significant. The stability of stationary oscillations depends both on the characteristics of the energy source and on the magnitude of the delay; a weak or very weak stability appears.

Keywords: limited excitation, oscillations, delay, Routh–Hurwitz criteria, direct linearization

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ВЗАИМОДЕЙСТВИЕ ВЫНУЖДЕННЫХ, ПАРАМЕТРИЧЕСКИХ И АВТОКОЛЕБАНИЙ ПРИ ОГРАНИЧЕННОМ ВОЗБУЖДЕНИИ И ЗАПАЗДЫВАНИЯХ

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Аннотация. На основе динамической модели фрикционной автоколебательной системы рассмотрено влияние запаздываний в упругости и вызывающей автоколебания трения на смешанные вынужденные, параметрические и автоколебания при взаимодействии колебательной системы с источником энергии. Решение нелинейных дифференциальных уравнений движения колебательной системы и источника энергии построено с использованием метода прямой линеаризации. Последний отличается от известных методов анализа нелинейных систем множеством преимуществ, в том числе простотой применения. На основе критериев Рауса – Гурвица получены условия устойчивости для анализа стационарных движений. Чтобы получить информацию о влиянии запаздываний на режимы колебаний, проведены расчеты. Установлено, что это влияние очень существенное. Устойчивость стационарных колебаний зависит как от характеристики источника энергии, так и от величины запаздывания, появляется слабая или очень слабая устойчивость.

Ключевые слова: ограниченное возбуждение, колебания, запаздывание, критерии Рауса – Гурвица, прямая линеаризация

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Introduction

As global environmental problems (climate change, acid rain, air and water pollution, accumulation of nuclear waste and its effects on the environment) intensify, they have become a great concern to the general public around the world. Numerous studies on the subject see energy conservation as a rational solution. While the energy industry is the backbone for the basic industries, growing at a rate are higher than those of other industries, it is also one of the major contributors to environmental and social challenges. Uncontrolled energy consumption, high anthropogenic load on the environment and their climatic impacts can lead to a global environmental crisis. For this reason, no problem attracts such close attention from researchers as the problem of climate change. Reducing energy consumption, switching to sustainable energy sources and adopting conservation practices are the main issues facing humanity.

The theory of the interaction between an energy source and an oscillatory system can offer some solutions to the related problems. Even though the effect was discovered by Sommerfeld in 1902, a consistent theoretical framework was developed by Kononenko only in the 1950s. Kononenko's research is summarized in the world-famous monograph [1, 2], laying the foundations for a new direction in the theory of oscillations. This theory has been further developed by researchers around the world, for example, in [3–5] and many others. The relationship of environmental problems with the level of energy consumption, metrology, accuracy of models for calculating systems and processing parts was established in [6].

Various issues related to the phenomenon of hysteresis (delay) are considered in [7–23], etc. Accounting for delays is necessary for many problems, for example, in various branches of technology (transportation, electronics, automatic regulation, radio engineering, non-ferrous metallurgy, mass transfer processes, paper and glass production, etc.), in optics, management, biology (blood, brain research, etc.).

Dynamic hysteresis is observed under cyclic stresses with a maximum amplitude and significantly below the elastic limit, due to inelasticity or viscoelasticity. In mechanical systems, the delay is caused by internal friction in materials, imperfection of their elastic properties, etc. This considerably affects the tuning process and the stability of the system, while its presence can be both harmful and useful. The delay can lead to oscillations in tracking systems, belt conveyors, controllers, rolling mills, etc.

The goal of this study is to analyze the effect of delays in elasticity and friction inducing self-oscillations on mixed forced, parametric and self-excited oscillations during the interaction of an oscillatory system with an energy source.

Model and equations

Let us consider a model of a frictional self-oscillating system (Fig. 1). It adequately describes the self-oscillations that often occur in machinery due to friction (brakes, textile equipment, metal-cutting machines, etc.), is widely used for their analysis (see, for example, [24–26]). A body with mass m lies on a belt driven by an engine with the torque characteristic $M(\varphi)$, where φ is the rotation speed of the engine rotor.

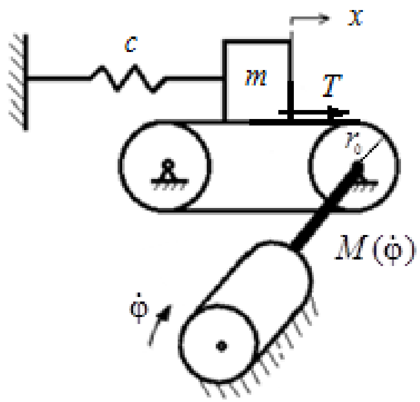

Fig. 1. Model of self-oscillating system: body with mass *m* lies on a belt driven by an engine with the torque $M(\varphi)$; φ is the rotational speed of the motor rotor, *c* is the spring constant, *T* is the friction force and r_0 is the radius of its application point

The friction force $T(U)$ arising between the body and the belt depends on the relative velocity $U = V - x$, where $V = r_0\varphi$ ($r_0 = const$ is the radius of the point at which the friction force is applied) and can induce its self-oscillations. In real conditions, the friction $T(U)$ (also observed in space experiments [27]) is widely used as

$$T(U) = R(\operatorname{sgn} U - \alpha_1 U + \alpha_3 U^3),$$

where *R* is the normal reaction force; $\alpha_1 = const$, $\alpha_3 = const$ are positive values; $\operatorname{sgn} U = 1$ at $U > 1$, $\operatorname{sgn} U = -1$ at $U < 1$; $-R \le T(0) \le R$ at rest ($U = 0$).

The nonlinear function $T(U)$ based on the method of direct linearization (see [28 –31]) can be replaced by the function

$$T_* = R(\operatorname{sgn} U + B_T + k_T \dot{x}), \tag{1}$$

where the linearization coefficients B_T, k_T are determined by the expressions

$$B_T = -\alpha_1 u + \alpha_3 u^3 + 3\alpha_3 N_2 u a^2 p^2,$$
$$k_T = \alpha_1 - 3\alpha_3 u^2 - \alpha_3 \bar{N}_3 \upsilon^2,$$

$$N_2 = (2r+1)/(2r+3),$$
$$\bar{N}_3 = (2r+3)/(2r+5),$$
$$a = \max|x|,$$
$$\upsilon = \max|\dot{x}|.$$

The interval within which the linearization accuracy parameter *r* is selected is not limited, but a choice within 0–2 is sufficient.

Taking into account the action of the external driving force $\lambda\sin\nu_1 t$ on the body, the parametric excitation $(c+b\cos\nu t)x$, the delays $\tau = const$ and $\Delta = const$, the mathematical model of the system is comprised of nonlinear differential equations

$$\begin{aligned} m\ddot{x} + k_0\dot{x} + c_0 x + c_\tau x_\tau &= T(U_\Delta) - \lambda\sin\nu_1 t - bx\cos\nu t, \\ I\ddot{\varphi} &= M(\dot{\varphi}) - r_0 T(U), \end{aligned} \tag{2}$$

where $k_0 = const$, $c_0 = const$ are, –respectively, the damping and stiffness coefficients; *I*– is the total inertia moment of the rotating parts;

$$c_\tau = const,\ U_\Delta = r_0\dot{\varphi} - \dot{x}_\Delta,\ \dot{x}_\Delta = \dot{x}(t-\Delta),\ x_\tau = x(t-\tau).$$

Eqs. (2) accounting for the form of function (1) take the form

$$\begin{aligned} m\ddot{x} + k_0\dot{x} + c_0 x + c_\tau x_\tau &= T(U_\Delta) - \lambda\sin\nu_1 t - bx\cos\nu t, \\ I\ddot{\varphi} &= M(\dot{\varphi}) - r_0 T(U), \end{aligned} \tag{3}$$

Solution of the differential equations

The following general nonlinear equation with nonlinear functions $\bar{F}(\dot{x})$ and $\bar{f}(x)$, linearized by the direct linearization method, is considered in [28]:

$$\ddot{x} + \bar{F}(\dot{x}) + \bar{f}(x) = H(t, x). \tag{4}$$

Using the substitution of variables $x = \upsilon p^{-1}\cos\psi$, $\dot{x} = -\upsilon\sin\psi$, $\psi = pt + \xi$, $\upsilon = \max|\dot{x}|$, we obtained standard equations for determining nonstationary values of υ and ξ:

$$\frac{d\upsilon}{dt} = -\frac{k\upsilon}{2} - H_s(\upsilon, \xi), \frac{d\xi}{dt} = \frac{\omega^2 - p^2}{2p} - \frac{1}{\upsilon} H_c(\upsilon, \xi), \tag{5}$$

where

$$H_s(\upsilon, \xi) = \frac{1}{2\pi}\int_0^{2\pi} H(\cdots)\sin\psi\, d\psi, \; H_c(\upsilon, \xi) = \frac{1}{2\pi}\int_0^{2\pi} H(\cdots)\cos\psi\, d\psi.$$

Using Eqs. (5), we obtain from Eqs. (3), in view of the expressions

$$\upsilon = ap, \; p = \nu/2, \; \dot{x}_\Delta = -\upsilon\sin(\psi - p\Delta), \; x_\tau = \upsilon\nu^{-1}\cos(\psi - \nu\tau),$$

the following equations (for two cases) to determine the non-stationary values of a, ξ, u:

a) $u \geq ap$

$$\begin{aligned} \frac{da}{dt} &= -\frac{1}{4pm}(2aA + 2\lambda\cos\xi - ba\sin 2\xi), \\ \frac{d\xi}{dt} &= \frac{1}{4pma}(2aE + 2\lambda\sin\xi + ab\cos 2\xi), \\ \frac{du}{dt} &= \frac{r_0}{I}\left[M\left(\frac{u}{r}\right) - r_0 R(1 + B_T)\right]; \end{aligned} \tag{6a}$$

b) $u < ap$

$$\begin{aligned} \frac{da}{dt} &= -\frac{1}{4pm}\left[2aA - \frac{8R}{\pi ap}\sqrt{a^2p^2 - u^2} + 2\lambda\cos\xi - ba\sin 2\xi\right], \\ \frac{d\xi}{dt} &= \frac{1}{4pma}(2aE + 2\lambda\sin\xi + ab\cos 2\xi), \\ \frac{du}{dt} &= \frac{r_0}{I}\left[M\left(\frac{u}{r}\right) - r_0 R(1 + B_T) - \frac{r_0 R}{\pi}(3\pi - 2\psi_*)\right], \end{aligned} \tag{6b}$$

where $A = p(k_0 - qk_T\cos p\Delta) - c_\tau\sin p\tau$, $E = m(\omega_0^2 - p^2) + c_\tau\cos p\tau - pqk_T\sin p\Delta$,

$$\omega_0^2 = c_0/m, \; \psi_* = 2\pi - \arcsin(u/ap).$$

A technique described in [3] is used to formulate Eqs. (6b); $V = r_0\dot{\varphi}$ is replaced by $u = r_0\Omega$ during averaging.

The conditions $\dot{a} = 0$, $\dot{\xi} = 0$, $\dot{u} = 0$ in Eqs. (6a) and (6b) give equations for stationary motions.

In the case when $u \geq ap$ (a), we have the following relations for amplitude and phase:

$$\begin{gathered} (a^2D - 2\lambda^2)^2 - 4\lambda^2(\lambda^2 + 2bGa^2) = 0, \; tg\,\xi = L(baL - \lambda)/aA, \\ G = b + 2E, D = 4A^2 + G^2 - 2bG, \; L = (\lambda \pm \sqrt{\lambda^2 + 2ba^2(b + 2E)})/2ab, \end{gathered} \tag{7}$$

and in the case when $u < ap$ (b), the amplitude is determined by the approximate formula $ap \approx u$.

The condition $\dot{u} = 0$ allows to determine the stationary values of the velocity u from the equation

$$M(u/r_0) - S(u) = 0,$$

where the expression $S(u)$ represents the load on the energy source and has the form:

a) for $u \geq ap,\ S(u) = r_0 R(1 + B_T),$

b) for $u < ap,\ S(u) = r_0 R\left[(1 - B_T) + \pi^{-1}(3\pi - 2\psi_*)\right].$

Taking into account the approximate equality $ap \approx u$, the expression $S(u)$ is simplified in the case when $u < ap$(b).

Conditions of stability for stationary oscillations

Composing the equations in variations for the cases (6a), (6b) and using the Routh–Hurwitz criteria, we obtain the following stability conditions:

$$D_1 > 0,\ D_3 > 0,\ D_1 D_2 - D_3 > 0, \tag{7}$$

where $D_1 = -(b_{11} + b_{22} + b_{33}),\ D_2 = b_{11}b_{33} + b_{11}b_{22} + b_{22}b_{33} - b_{23}b_{32} - b_{12}b_{21} - b_{13}b_{31},$

$$D_3 = b_{11}b_{23}b_{32} + b_{12}b_{21}b_{33} - b_{11}b_{22}b_{33} - b_{12}b_{23}b_{31} - b_{13}b_{21}b_{32}.$$

The following coefficients appear in the case of velocities $u \geq ap$ (a):

$$b_{11} = \frac{r_0}{I}\left[Q + r_0 R(\alpha_1 - 3\alpha_3 u^2 - 3\alpha_3 N_2 a^2 p^2)\right],\ b_{12} = -6N_2\alpha_3 Ruap^2 r_0^2 I^{-1},\ b_{13} = 0,$$

$$b_{21} = -\frac{3}{m}\alpha_3 Rua\cos p\Delta,\ b_{22} = \frac{1}{4pm}\left[2(c_\tau \sin p\tau + pRk_T \cos p\Delta) + b\sin 2\xi\right],$$

$$b_{23} = \frac{1}{2pm}(\lambda\sin\xi + ba\cos 2\xi),\ b_{31} = \frac{3}{m}\alpha_3 Ru\sin p\Delta,$$

$$b_{32} = \frac{1}{2pma^2}\left(2\bar{N}_3\alpha_3 Ra^3 p^3 \sin p\Delta - \lambda\sin\xi\right),\ b_{33} = \frac{1}{2pma}(\lambda\cos\xi - ab\sin 2\xi),$$

where $Q = \frac{d}{du}M(\frac{u}{r})$.

In the case of velocities $u < ap$ (b), only the coefficients

$$b_{11} = \frac{r_0}{I}\left[Q + r_0 R(\alpha_1 - 3\alpha_3 u^2 - 3\alpha_3 N_2 a^2 p^2) - \frac{2r_0 R}{\pi\sqrt{a^2p^2 - u^2}}\right],$$

$$b_{12} = -\frac{2Rr_0^2}{I}\left[3N_2\alpha_3 uap^2 + \frac{u}{\pi a\sqrt{a^2p^2 - u^2}}\right],$$

$$b_{21} = -\frac{auR}{m}\left[3\alpha_3\cos p\Delta - \frac{2}{\pi a^2 p^2\sqrt{a^2p^2 - u^2}}\right],$$

$$b_{22} = \frac{1}{4pm}\left[2(c_\tau \sin p\tau + pRk_T \cos p\Delta) + b\sin 2\xi\right] - \frac{2Ru^2}{\pi ma^2p^2\sqrt{a^2p^2 - u^2}},$$

change, and the others are the same as for velocities $u \geq ap$(a).

Computational results

The information about the effect of delays on the dynamics of the system was obtained by performing computations for the following parameter values: $\omega = 1$ s^{-1}, $m = 1$ kgf·s^2·cm^{-1}, $b = 0.07$ kgf·cm^{-1}, $k = 0.02$ kgf·s·cm^{-1}, $c_\tau = 0.05$ kgf·cm^{-1}, $R = 0.5$ kgf, $\alpha_1 = 0.84$ s·cm^{-1}, $\alpha_3 = 0.18$ s^3·cm^{-3}, $r_0 = 1$ cm, $I = 1$ kgf·s·cm^2.

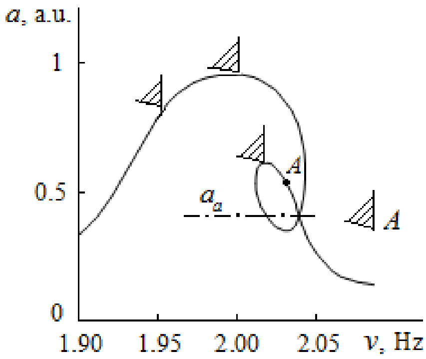

Fig. 2. The dependence of the amplitude on the frequency in the absence of delays ($\Delta = 0$, $\tau = 0$). We use the value of linearization accuracy $r = 1.5$

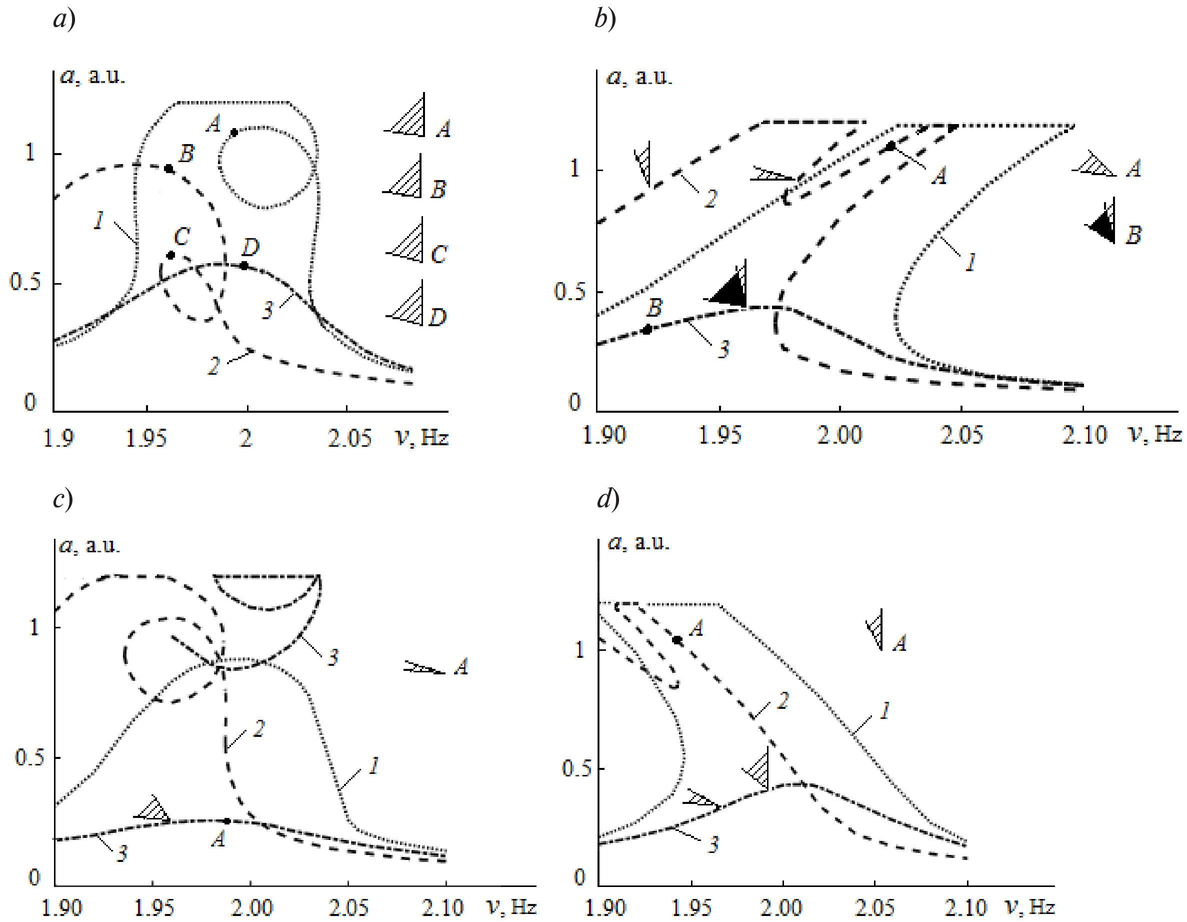

Fig. 3. The dependence of the amplitude on the frequency at different values of the delay parameter Δ: 0 (*a*), π/2 (*b*), π (*c*) and 3π/2 (*d*). The value of the parameter $p\tau$ varies in the graphs: π/2 (curves *1*), π (curves *2*), 3π/2 (curves *3*)

The value u = 1.2 cm·s^{-1} was used for the velocity, and the values of delays $p\tau$ and $p\Delta$ were selected from the interval (0,3π/2).

The amplitude-frequency curves $a(p)$, shown in Figs. 2 and 3, are constructed using the accuracy parameter r = 1.5 ($\bar{N}_3 = 3/4$ for k_T). We can conclude from the analysis that the data completely coincide with the results that can be obtained by the Bogolyubov–Mitropolsky asymptotic averaging method [32]. The results shown in Fig. 2, where a_a corresponds to the amplitude of self-oscillations, represent the case with the absence of delays (Δ = 0, τ = 0) and are given for comparison.

Oscillations with corresponding amplitudes are stable within the shaded sectors for the slope of the energy source characteristic $Q = dM(u/r_0)/du$. These sectors should be given in the graph for the load $S(u)$ on the energy source, but are shown in the amplitude curves for brevity. A weak stability appears within the sectors shaded with black, i.e., the stability condition (or conditions) (8) are satisfied in the form $0.000X > 0$, where X ≤ 9. We should also note that stability is even weaker in some segments of the curves in the form $0.0000Y > 0$, where $Y \leq 9$. Such weak or very weak stability is observed for all values of the delays, although it is shown only for $p\Delta = \pi/2$.

Conclusion

In this paper, we consider the dynamics of a frictional self-oscillating system with an energy source of limited power in the presence of a driving force and parametric excitation, delays in elasticity and friction inducing self-oscillations. Calculations based on the equations formulated for stationary motion we carried out to obtain information about the effect of these delays on the dynamic characteristics of mixed forced, parametric and self-oscillations. The graphical results are clear evidence for the substantial influence of the delays on the dynamics of oscillations. It is also associated with the load, accordingly, the dynamics and energy consumption of the source. This influence leads to a number of important conclusions:

a shift of the amplitude curves occurs in the amplitude-frequency plane;

the amplitude curves considerably change shape, assuming the appearance characteristic for nonlinear elasticity (soft, hard);

the stability of stationary oscillations depends both on the characteristics of the energy source and on the magnitude of the delay, weak or very weak stability of the oscillations appears.

Our findings make it possible to plan further studies. The approach used in the study is applicable to analysis of various oscillatory processes. The data obtained in this direction can serve as a tool for practical simulations of real objects for optimal selection of their parameters and characteristics of energy sources, minimizing energy consumption and harmful environmental effects.

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