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USING THE THEORY OF DESIGN OF EXPERIMENT AND THE SENSITIVITY ANALYSIS METHOD IN SIMULATION OF A GYROSCOPE RESONATOR

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Abstract. The paper proposes a little-distributed approach to a priori estimation of object characteristics. The mathematical model of the Coriolis vibratory gyroscope resonator which is being developed to determine the splitting of its eigenfrequency, has been chosen as an object. The frequency splitting is caused by the presence of imperfections of the resonator due to the manufacturing process. The theory of design of experiment and the sensitivity analysis method were used to estimate the model characteristics. The model is based on the determining relations of the theory of thin elastic tensile shells and on the Hamilton's variational principle. It was shown that the application of the taken methods allowed us to reveal the model parameters making the minimum contribution to the values of the output parameters and to simplify the model by their excluding.

Keywords: Coriolis vibratory gyroscope, sensitivity analysis, Sobol index, design of experiment

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ИСПОЛЬЗОВАНИЕ МЕТОДОВ ТЕОРИИ ПЛАНИРОВАНИЯ ЭКСПЕРИМЕНТА И АНАЛИЗА ЧУВСТВИТЕЛЬНОСТИ ПРИ РАБОТЕ С МОДЕЛЬЮ РЕЗОНАТОРА ГИРОСКОПА

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Аннотация. В статье предлагается малораспространенный подход к априорной оценке характеристик объекта. Математическая модель резонатора волнового твердотельного гироскопа, разрабатываемая для определения расщепления его собственной частоты, послужила объектом исследования. Расщепление частоты вызвано наличием несовершенств резонатора, обусловленных процессом производства. Для оценки характеристик модели использованы методы теории планирования эксперимента и анализа чувствительности. Модель построена на основе определяющих соотношений теории тонких упругих растяжимых оболочек, а также вариационного принципа

Гамильтона. Показано, что применение рассмотренных методов позволяет выявить параметры модели, вносящие минимальный вклад в значения выходных параметров, и упростить модель путем их исключения.

Ключевые слова: волновой твердотельный гироскоп, анализ чувствительности, индекс Соболя, теория планирования эксперимента

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Introduction

High-precision navigation systems based on solid-state wave gyroscopes (SSWG) are one of the most promising directions in modern gyroscopy [1–3]. The operating principles of the SSWG based on the Bryan effect are well known and described in the literature [4, 5]. However, the accuracy characteristics sufficient for constructing high-precision navigation systems [3] can only be achieved by involving a class of problems whose solutions require accurate mathematical modeling.

The following problems are related to mathematical modeling in the development of the SSWG:

prediction of operational natural frequencies and their splitting magnitudes;

identification of imperfection parameters in an imperfect resonator;

development of a balancing algorithm for the SSWG resonator;

development of a control algorithm for the SSWG.

It is important not only to be able to construct and verify the required mathematical model in the course of the simulation, but also to analyze the influence of its variable parameters on the output characteristics of the object to ultimately obtain the most compact configuration of the model describing the given process.

Methods of sensitivity analysis (SA) and, in particular, global sensitivity analysis (GSA) [6], as well as methods of design of experiment (DoE) [7, 8] are intended for solving such problems. Notably, these methods are widely used for processing the results of field tests carried out as part of the device optimization problem; they are not so common, however, for analysis of the analytical computational models constructed.

This paper provides an example of applying the DoE and GSA methods to a mathematical model of a hemispherical SSWG quartz resonator.

Description of the mathematical model

The mathematical model of a quartz hemispherical resonator is designed to determine the operating natural frequency, as well as its splitting due to the presence of imperfections appearing in the manufacturing process. We consider such imperfections as the variation of density, elasticity, and thickness. The model is based on the governing relations from the theory of thin elastic tensile shells [9], as well as Hamilton's variational principle [10]. The basis is the expression

$$\delta I = \delta \int_{t_0}^{t_1} L\left(q_1,\ldots,q_n,\dot{q}_1,\ldots,\dot{q}_n,t\right)dt = 0,$$

where δI is the variation of the required functional; $L = T - W$ (T, W are the kinetic energy of the considered unit volume of the shell and the potential strain energy, respectively); q_i are the generalized coordinates in $n$dimensional space; t is time.

According to the formulas presented in [9, 11], the expressions for kinetic and potential energy have the form:

$$T = \frac{1}{2}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \rho h V^2 A_1 A_2 d\theta d\varphi,$$

$$W = \frac{Eh}{2\left(1-\nu^2\right)}\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\left[\left(\varepsilon_1+\varepsilon_2\right)^2 - 2\left(1-\nu\right)\left(\varepsilon_1\varepsilon_2 - \left(\frac{\omega}{2}\right)^2\right)\right] A_1 A_2 d\theta d\varphi +$$

$$+\frac{Eh^3}{24\left(1-\nu^2\right)}\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\left[\left(\kappa_1+\kappa_2\right)^2 - 2\left(1-\nu\right)\left(\kappa_1\kappa_2-\tau^2\right)\right] A_1 A_2 d\theta d\varphi,$$

where ρ, kg/cm³, is the density of the material (quartz glass); **V**, m/s, is the absolute velocity vector of an arbitrary point of an elastic body; h, m, is the thickness of the hemispherical shell; A_1, A_2 are the Lamй parameters; ν is Poisson's ratio of the material; E, MPa, is its elastic modulus; ε_1, ε_2 are the parameters characterizing the elongation of the mid-surface; κ_1, κ_2 are the parameters characterizing the bending deformations of the mid-surface; ω, τ are the parameters characterizing the shear and torsion deformations of the shell, respectively; θ, φ, deg, are the zenith and azimuth angles, respectively.

These expressions take into account the imperfections of the parameters *E, h,* ρ by specifying their inhomogeneity with respect to the circumferential coordinate by the following law:

$$E = E_0 \cdot (1 + amp\_E \cdot \sin(k\_E \cdot \varphi + phase\_E \cdot \pi));$$
$$h = h_0 \cdot (1 + amp\_h \cdot \sin(s\_h \cdot \varphi + phase\_h \cdot \pi));$$
$$\rho = \rho_0 \cdot (1 + amp\_rho \cdot \sin(p\_rho \cdot \varphi + phase\_rho \cdot \pi)).$$

where E_0, h_0, ρ_0 *are* the base values of the elastic modulus, thickness and density; *amp_E*, *k_E*, *phase_E*, *amp_h*, *s_h*, *phase_h*, *p_rho, amp_rho*, *phase_ rho* are the parameters of harmonic perturbation (imperfections).

Using the Ritz method [12], the solution of the problem is reduced to solving the eigenvalue problem:

$$\left(A - \lambda^2 B\right)\mathbf{C} = 0,$$

where A, B are matrices associated with kinetic and potential energies, as well as the coordinate functions; **C** is a column vector of unknown coefficients; λ is a column vector of natural frequencies.

The values of natural frequencies corresponding to the second elliptical mode are determined as a result of the calculations.

For an ideal hemispherical resonator without imperfections, the difference between the paired values of the natural frequencies for the elliptical mode (f_1 and f_2) is zero. At the same time, in the case of a nonzero harmonic amplitude, the difference in the absolute value between f_1 and f_2 E, h, ρ is the required value of the splitting of the operational natural frequency. This is a property of any vibration mode with azimuthal angle variability for a hemispherical resonator.

We should note that in this paper we consider a mathematical model containing nine variable parameters associated with assigning the imperfections for the quantities E, h and ρ. These parameters are selected based on the computational resources available and the clarity of the results presented; we do not purport that this choice is sufficient for comprehensive analysis.

All calculations were performed using Matlab software combined with predictive modeling and robust optimization software pSeven (developed by DATADVANCE).

Global sensitivity analysis conducted for the mathematical model of the resonator

GSA methods are aimed at identifying the degree to which variable parameters of a function influence its output (analyzed) value. In practice, if the functional dependence between the input and output parameters is unknown, but their experimental values are available, then DoE methods are used to construct it. The latter imply the construction of an optimal experimental plan, i.e., the minimum number of experiments conducted, but still sufficient to construct a functional dependence between input variables (factors) and output parameters, as well as the assessment of the quality of the constructed function. A similar approach is used for conducting virtual experiments: a mathematical model is represented as a black box and serves only to obtain the values of the output parameters for the given input parameters.

As a result, a data array containing various combinations of factor values and the corresponding values of output parameters can be used to construct a simpler mathematical model describing the relationship between them, which generally reduces the computational time at the subsequent stages.

For example, a polynomial representation is often used as a simplified model:

$$y = a_0 + a_1 \cdot x_1 + a_2 \cdot x_2 + a_{11} \cdot x_1^2 + a_{22} \cdot x_2^2 + a_{12} \cdot x_1 \cdot x_2 ...,$$

where y is the value of the output parameter, a_{nm} are the required coefficients, x_n are the values of the factors.

The splitting S of the operational natural frequency is taken as the output parameter within the model considered, and the parameters of the imperfection harmonics at E, h and ρ (*k_E, amp_E, phase_E, s_h, amp_h, phase_h, p_rho, amp_rho, phase_rho*) are taken as factors.

A function of the form

$$S(k\_E,\ amp\_E,\ phase\_E,\ s\_h,\ amp\_h,\ phase\_h,\ p\_rho,\ amp\_rho,\ phase\_rho)$$

is a target or a response function. The geometric interpretation of this function is called the response surface.

An array of data is required to build a response surface; these data are the result of a real or virtual experiment (this follows from the above). The number of experiments should allow to construct a response surface describing the dependence of factors and output parameters in the considered region of the factor space with the required accuracy. It is preferable that the number of experiments should be minimal. There are numerous methods within the DoE for formulating an optimal experimental plan.

In this study, we used Latin hypercube sampling (LHS) [8], generating an almost random sample of factor values evenly filling the factor space.

Because there is no universal approximation method for constructing a continuous function based on known numerical values of parameters, various methods were used in this paper, chosen because they yield satisfactory results, and provide high quality indicators of the constructed functions. To determine the influence of factors on the output parameter S, the Sobol method was used as the GSA method [13].

It should be noted that the pSeven software uses a surrogate modeling approach that provides increased accuracy of constructing the response surface due to an additional iterative learning process (training, correction) of the primary response function [14] to the required accuracy parameters.

Sensitivity analysis of the mathematical model of the resonator: results and discussion

Three isolated cases with different initial data (ranges of factor variation) were considered for the model, which was done by reducing the dimension of the model by specifying some factors as constants (without the variation range). This feature was determined by the goals to minimize the computational time and to simplify the processing of the results obtained.

In all cases, the values setting the variation range of the physical and geometric characteristics of the material (density, elastic modulus, thickness) were selected based on the values of imperfection parameters determined by technological errors made during machining and manufacturing of the workpieces. The first 12 harmonics of imperfections are considered, the variation range of the phase angle of the harmonics corresponded to a variation in the phase angle from 0 to 2π.

The results were represented using data slices over the values of the full factor space obtained via the constructed target function. In other words, each slice is a data sample that allows to monitor the variation in splitting (as an output parameter) depending on the selected factor, while the rest are constant (based on the selected values).

Case I. Consideration of mathematical model to estimate the influence of factors amp_E, amp_h, amp_rho on the output parameter S. For this purpose, we used the initial data from Table 1, which contains the factors considered and their variation ranges.

Table 1

Factors used to construct response surfaces and their variation ranges for three cases

Factor	Variation range
Case I	
amp_E	0.0005–0.0500
phase_E	0.5000
k_E	4.000
amp_h	0.0005–0.0500
phase_h	0.5000
s_h	4.000
amp_rho	0.0005–0.0500
phase_rho	0.5000
p_rho	4.000
Case II	
amp_E	0.0005–0.0500
phase_E	0.000–2.000
k_E	4.000
amp_h	0.0005–0.0500
phase_h	0.000–2.000
s_h	4.000
amp_rho	0.0100
phase_rho	0.5000
p_rho	4.000
Case III	
amp_E	0.0100
phase_E	0.5000
k_E	1.000–12.000
amp_h	0.0100
phase_h	0.5000
s_h	1.000–12.000
amp_rho	0.0100
phase_rho	0.5000
p_rho	1.000–12.000

In accordance with the constructed experimental plan, the calculations carried out by the response surface model (RSM) [8, 14] yielded a response surface, with the Sobol indices calculated based on it [13]; they reflect the degree of influence of the studied parameters on the model. Fig. 1 shows data slices in the directions corresponding to the coordinate axes of Split (the value of frequency splitting) and each of the studied parameters. Evidently, the *amp_E* parameter has practically no effect on the output splitting value within the variation limits, while the other two parameters affect the splitting equally. In addition, the Sobol indices are shown graphically as columns in the lower right corners of the slices in Fig. 1 (their numerical values are given in Table 2). These indices actually express the variance magnitude of the model output relative to each of the factors. Importantly, the total value of the first-order Sobol indices is always equal to unity.

Analyzing the obtained results, we find that the calculations can be simplified in the case of a large-dimensional model by neglecting the variability of the elastic modulus, due to the small value of the Sobol index for the *amp_E* parameter.

Case II. Consideration of the mathematical model to minimize the splitting magnitude with a given (constant) imperfection in the form of parameters *amp_E, phase_E, amp_h, phase_h*. For this purpose, we used the initial data from Table 1, containing factors and their variation intervals (i.e., this is now a problem with four variables).

Fig. 2 shows data slices for the factors under consideration for the response surface constructed using the sparse Gaussian process (SGP), which is a modification of the standard Gaussian process (GP) [14, 15] for samples of large dimension.

Similarly to case I, we can see from Fig. 2 that the variation of the thickness parameters makes a decisive contribution to the change in splitting. The maximum splitting is observed when the phase component of the thickness variation coincides with the phase of the density variation (set as a constant). The phase value for which the splitting is reduced to zero is also observed in the slice, suggesting that the splitting caused by one defect can be balanced by introducing another defect with certain parameters. We should note that this result is possible because the factors considered in the mathematical model of the resonator are disconnected.

a) *b*) *c*)

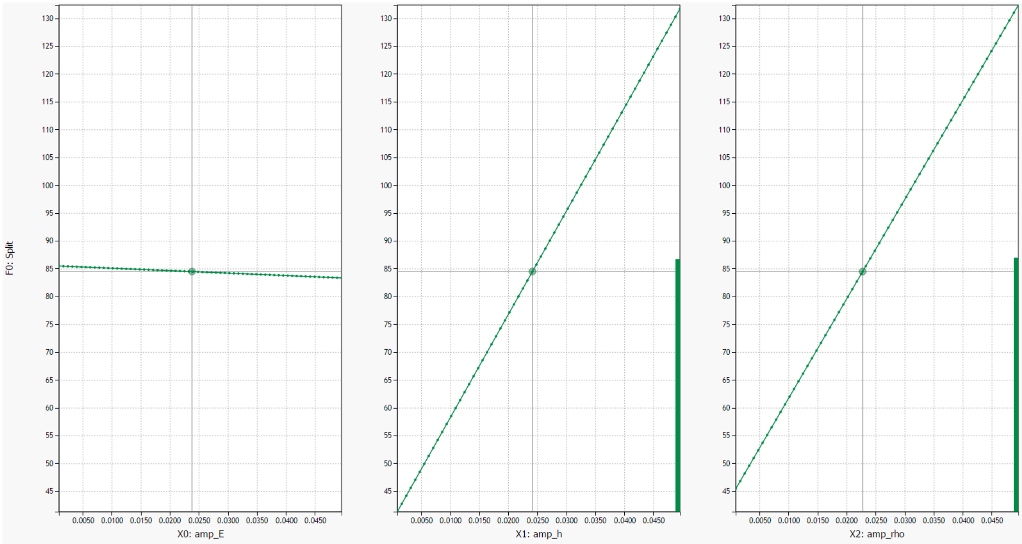

Fig. 1. Slices of data obtained for factors *amp_E*(*a*), *amp_h*(*b*) and *amp_rho*(*c*) (case I)

a) *b*) *c*) *d*)

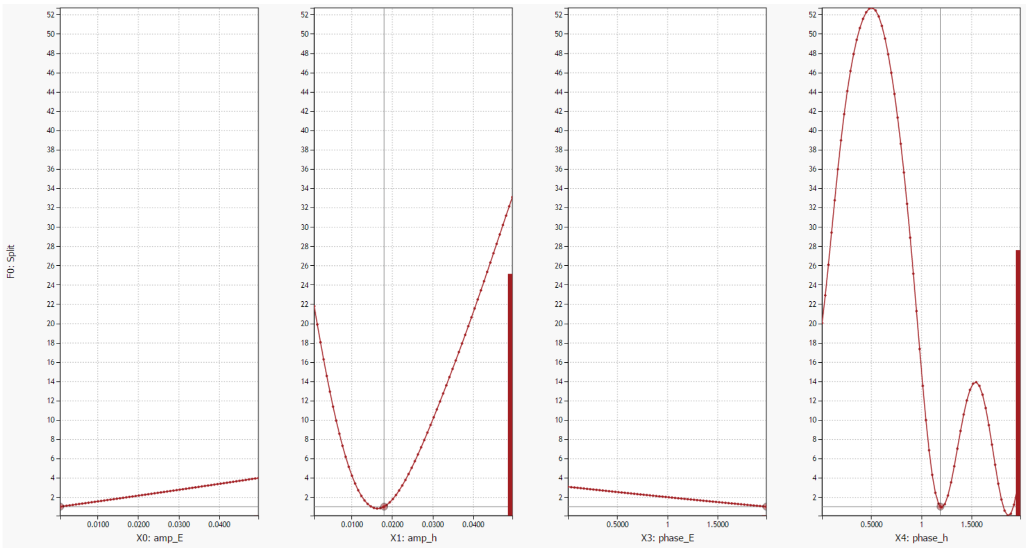

Fig. 2. Slices of data obtained for factors
amp_E (*a*), *amp_h* (*b*), *phase_E* (*c*) and *phase_h* (*d*) (Case II)

Table 2

Influence of parameters considered on the model for Case I (see Table 1 and Fig. 1)

Factor	*amp_E*	*amp_h*	*amp_rho*
Sobol index	0.00028	0.44800	0.45100

a) *b*) *c*) *d*)

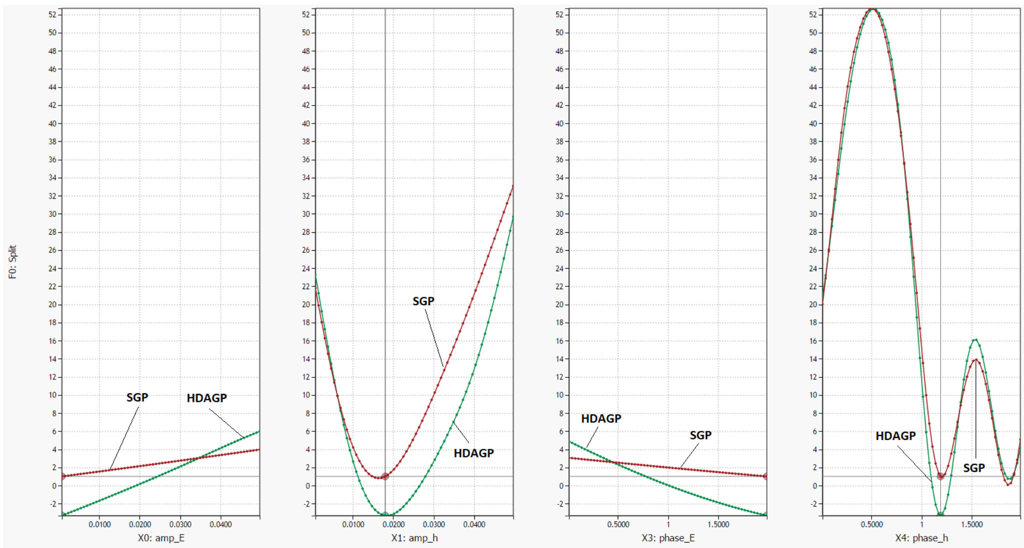

Fig. 3. Slices of data obtained for factors *amp_E*(*a*), *amp_h*(*b*), *phase_E*(*c*) and *phase_h* (*d*) (case II) for SGP and HDAGP surfaces (brown and green curves, respectively)

Fig. 3 shows a comparison of slices for these response surfaces constructed by SGP and high-dimensional approximation combined with the Gaussian process (HDAGP) [14, 15].

Considering the constructed response surfaces, we can observe their qualitative similarity, but the approximation performed by the HDAGP method gives negative splitting values. Such values are impossible, since the splitting values are calculated as the modulus of the difference between the natural frequencies f_1 and f_2 for an elliptical mode.

On the other hand, estimation of the approximation quality by the HDAGP method gives close yet better results compared to those obtained by the SGP method (Table 3). This result suggests that the constructed models must be tested for adequacy; furthermore, verification calculations using a mathematical model are necessary so that the approximation is performed with the accuracy sufficient for the task at hand at least in the area considered.

Case III. Consideration of the mathematical model to obtain a known result presented in [5] using DoE and GSA methods. The above study confirmed the decisive influence of the 4th imperfection harmonic on the frequency splitting of a hemispherical resonator with the 2nd elliptical operational mode. The initial data of Table 1 containing three factors and their variation intervals were used for this case. These are *k_E, s_h* and *p_rho.*

Table 3

Estimate of approximation quality of splitting values for case II

Estimation method	R^2	MSE	MAPE
		%	
HDAGP	0.983	3.169	7.368
SGP	0.979	3.497	7.583

Notations: R^2 is the determination coefficient, MSE, MAPE are the root-mean-square and maximum prediction errors, respectively.

Various approximation methods were tested for constructing the corresponding response surface, including the ones discussed above. However, the expected result was only obtained by the gradient boosted regression trees method (GBRT) [14, 15]. This method is applicable for approximating complex functions and working with large data sets; the output is a step function rather than a smooth one, which is characteristic for the influence of the 4th harmonic. In the case of the 2nd elliptical mode of vibrations, the 4th imperfection harmonic has a pronounced effect on splitting, increasing it by an order of magnitude, compared with the nearest harmonics (2nd and 3rd). Fig. 4 shows data slices for the factors under consideration for the surface constructed by the GBRT method.

a) *b*) *c*)

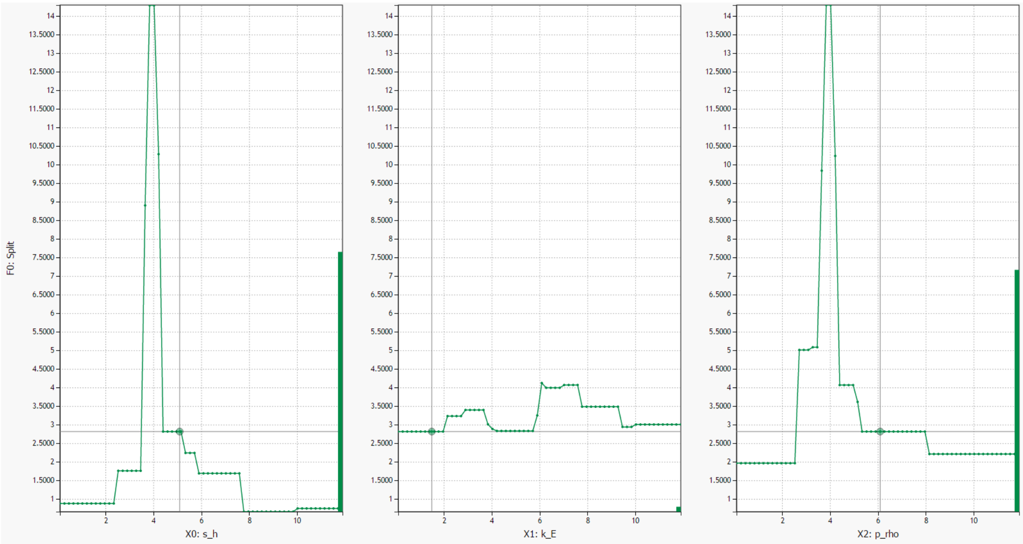

Fig. 4. Slices of data obtained for factors *s_h*(*a*), *k_E*(*b*) and *p_rho*(*c*) for surface constructed by GBRT method (case III)

The calculation results indicate that the method can be used to detect a local maximum in the region of the 4th harmonic of density variation and thickness variation. A maximum could not be detected in the region of the 4th harmonic for elasticity variation. This is probably caused by a significantly lower influence of the factor on frequency splitting (the Sobol index is 0.0098) and poor quality of the constructed model (the determination coefficient is 0.524).

Conclusion

This paper reports on the potential for application of the methods of design of experiment (DoE) and global sensitivity analysis (GSA) at a qualitative level to examine the behavior of the selected mathematical model and to assess the degree to which its parameters influence the output characteristics.

The application of the methods is illustrated by the example of a mathematical model of a hemispherical quartz resonator with the parameters describing the imperfection of its geometry and the physico-mechanical properties of the material from which it was manufactured.

We established that the parameters associated with the variation in thickness and density of the resonator have the greatest influence on the splitting of its natural frequencies. The influence of these parameters is comparable, which makes it possible to eliminate the effect of frequency splitting by compulsorily introducing one type of imperfection in the presence of another (for example, introducing thickness variation in the presence of density variation).

We established that the 4th harmonic of imperfections has the decisive influence on the frequency splitting described in the literature.

The DoE and GSA methods were confirmed to be effective for analysis of mathematical models. These methods allow to identify the model parameters that make a minimal contribution to the values of the output parameters, and simplify the model by excluding them. In addition, a polynomial model can then be constructed to describe the initial mathematical model with the required accuracy, reducing the time spent on the research with subsequent optimization carried out in accordance with the required parameters.

Notably, special attention should be paid to estimating the accuracy of constructing the response surface and estimating the quality of the corresponding response function.

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