MECHANICS

Original article
DOI: https://doi.org/10.18721/JPM.16111

METHOD OF SPECIFYING THE CENTRIFUGAL LOADS OF SHAFTS IN CALCULATION OF THE ROTOR UNBALANCE BEHAVIOR

A. R. Usmanov✉, *V. S. Modestov*

Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia

✉ ysman2d@gmail.com

Abstract. In the paper, a calculation method for specifying the centrifugal loads (SCLM) of free turbine rotors (turbine shaft unbalance method) has been put forward. The traditional method does not take this effect into account whereas the unbalance can lead to significant rotor deflections when approaching the critical frequencies. The SCLM in the solution of the problem within the elasticity theory in beam approximation using the finite element method (FEM) and the eigenfunction expansion method was exemplified by a three-shaft system. The loads caused by thickness variation, radial runout of outside surfaces and the one of mounting surfaces were considered. An experiment consisting of vibration tests of a gas turbine engine was carried out. The test task showed that taking the shaft unbalance into account gave results being closer to the experimental ones.

Keywords: unbalance, shaft deflection, centrifugal load, thickness variation, radial runout, tolerance

Citation: Usmanov A. R., Modestov V. S., Method of specifying the centrifugal loads of shafts in calculation of the rotor unbalance behavior, St. Petersburg State Polytechnical University Journal. Physics and Mathematics. 16 (1) (2023) 132–139. DOI: https://doi.org/10.18721/JPM.16111

Научная статья
УДК 629.7.036
DOI: https://doi.org/10.18721/JPM.16111

МЕТОД ЗАДАНИЯ ЦЕНТРОБЕЖНЫХ НАГРУЗОК ВАЛОВ ПРИ РАСЧЕТЕ ДИСБАЛАНСНОГО ПОВЕДЕНИЯ РОТОРОВ

А. Р. Усманов✉, *В. С. Модестов*

Санкт-Петербургский политехнический университет Петра Великого,

Санкт-Петербург, Россия

✉ ysman2d@gmail.com

Аннотация. В статье предложен расчетный метод задания центробежных нагрузок роторов свободной турбины, учитывающий дисбаланс ее валов. Традиционный метод этот эффект не учитывает, тогда как при приближении к критическим частотам дисбаланс может привести к значительным прогибам ротора. На примере системы из трех валов описан метод задания центробежных нагрузок при решении задачи теории упругости в балочной аппроксимации с применением метода конечных элементов и метода разложения по формам собственных колебаний. Рассмотрены нагрузки от разнотолщинности, биения наружной поверхности и биения посадочных поверхностей. Проведен эксперимент, который состоял в вибрационных испытаниях газотурбинного двигателя. На тестовой задаче показано, что учет дисбаланса валов приводит к результатам, более близким к экспериментальным.

Ключевые слова: дисбаланс, прогиб вала, центробежная нагрузка, разнотолщинность, разностенность, радиальное биение, допуск

Ссылка для цитирования: Усманов А. Р., Модестов В. С. Метод задания центробежных нагрузок валов при расчете дисбалансного поведения роторов // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2023. Т. 1 № .16. С. 132–139. DOI: https://doi.org/10.18721/JPM.16111

Introduction

The most common reason for rotor deflections is unbalance in the centers of gravity of its sections due to manufacturing inaccuracies.

The main method for reducing the operational deflection of the rotor is balancing by adding loads that compensate for the unbalance in the parts. It was established in [1] that balancing a flexible rotor as a rigid one can lead to an increase in the loads on the supports during operation at critical rotational speeds. Another approach to reducing the operational deflection of flexible rotors is to optimize their design by mathematical modeling.

The initial loads required to calculate the operational deflection of flexible rotors are set by specifying the unbalance of the impellers and the residual unbalance in the rotor balancing planes [2–5]. This does not take into account the unbalance of the shaft, which can lead to significant deflections of the rotor approaching critical frequencies.

The goal of this paper is to develop a method for setting centrifugal loads that would take into account the unbalance of the shafts, comparing its capabilities with the traditional approach.

Experimental methods

The problem of elasticity theory in the beam approximation using the finite element method and method of eigenmode expansion is reduced to a system of equations taking the following form [6]:

$$\bar{M}\ddot{e} + \sum_{i=1}^{n_L} \bar{C}_d(i)\dot{e} + \bar{K}e = \bar{Q}(t), \tag{1}$$

where e is a column vector of modal coordinates; $\bar{C}_d(i)$ is the damping matrix of the ith damper; n_L is the number of dampers; $\bar{M} = q^T Mq$; $\bar{K} = q^T Kq$; $\bar{Q} = q^T Qq$; (M is the inertia matrix, K is the stiffness matrix, Q is the column vector of generalized external forces in the nodes of the system, q is the eigenmode matrix); t is time.

If we take a rotating coordinate system for rotating sections and neglect balanced centrifugal loads, we can prove that the system of inertial forces in these sections is reduced to the action of unbalanced centrifugal loads and gyroscopic moments [7].

In view of this statement, system (1) takes the form

$$\bar{M}\ddot{e} + \left[\bar{C}_g + \sum_{i=1}^{n_L} \bar{C}_d(i)\right]\dot{e} + \bar{K}e = \bar{Q}(t), \tag{2}$$

where $\bar{C}_g$ is the total gyroscopic matrix.

In this case, unbalanced loads are given as a component of the external load Q.

The matrix elements $\bar{C}_d(i)$ are determined based on the analytical solution of the Reynolds equations in the oil film region in the gaps between the rotor and the stator [8].

The example of a system of simple cylindrical shafts can be used to confirm (see report [9]) that unbalanced centrifugal loads on shafts are induced by the following factors:

thickness variation,

radial runout of the outer surface,

radial runout of the mounting surfaces.

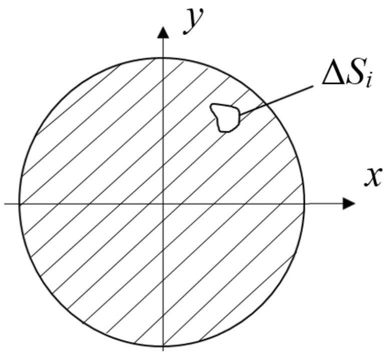

Fig. 1. Shaft section showing the unit segment of the partition ΔS_i

Let us prove that the unbalance of a certain section of the shaft is determined by the formula

$$\bar{D} = m\mathbf{r}, \tag{3}$$

where **r** is the radius vector of the section's gravity center, m is the mass of the section.

We divide the shaft section into elementary segments ΔS_i, as shown in Fig. 1.

Then the unbalance of the section is determined by the formula

$$\bar{D} \approx \sum_{i=1}^{N} m_i \mathrm{r}_i = \sum_{i=1}^{N} \rho \Delta S_i h \mathrm{r}_i, \tag{4}$$

where h is the thickness of the section, ρ is the density of its material.

The projection D_y of the section unbalance is expressed as

$$D_y \approx \sum_{i=1}^{N} \rho \Delta S_i h y_i. \tag{5}$$

Passing to the limit, we obtain the exact formula for determining the projection D_y:

$$D_y = \lim_{\max_i \Delta S_i \to 0} \sum_{i=1}^{N} \rho \Delta S_i h y_i = \rho h \int_S y dS = \rho h S \frac{\int_S y dS}{S} = \rho h S y_c = m y_c, \tag{6}$$

where S is the cross-sectional area of the shaft, y_c is the projection of the vector **r** on the axis OY.

Similarly, we can prove that

$$D_x = m x_c. \tag{7}$$

As a result, it is proved that the unbalance of a certain section of the shaft is determined by Eq. (3).

The unbalance modulus in the case of thickness variation, taking into account Eq. (3), is determined by the expression:

$$D = m y_c = \frac{m d_o^{\,2} s}{2\left(d^2 - d_o^{\,2}\right)}, \tag{8}$$

where m is the mass of the shaft; d, d_0 are the outer and inner diameters of the shaft section, respectively; s is the value of the thickness variation, understood as the difference between the maximum and minimum thickness of the shaft section.

In the case of runout of the outer surface, the unbalance modulus is determined by the formula

$$D = ma, \tag{9}$$

where a is the runout of the outer surface of the shaft relative to its supporting surfaces.

In the case of runout of the mounting surface of the shaft, the unbalance modulus per unit length of the shaft is determined by the formula

$$dD(x) = \left(b_I - \frac{x}{x_{II}} b_I\right) \rho_l dx, \tag{10}$$

where ρ_I is the density of the shaft per unit length, b_I is the runout of the first mounting surface, x_{II} is the coordinate of the second mounting surface.

In the case of unbalance of the impeller d_{run} and residual unbalance in the balancing planes d_{res}, the unbalance modulus is determined by the formulas:

$$D = d_{run}, \tag{11}$$

$$D = d_{res}. \tag{12}$$

In the general case when a spatial system consisting of n forces is acting on the system, the formulas for projections of displacements at arbitrarily selected points in the amount of m take the form

$$\delta_i^x = D_0\delta_{i0}\cos\varphi_0 + D_1\delta_{i1}\cos\varphi_1 + ... + D_n\delta_{in}\cos\varphi_n, \tag{13}$$

$$\delta_i^y = D_0\delta_{i0}\sin\varphi_0 + D_1\delta_{i1}\sin\varphi_1 + ... + D_n\delta_{in}\sin\varphi_n, \tag{14}$$

where δ_i^x, δ_i^y are projections of displacement at the ith point ($i = 1, 2, ..., m$); δ_{ij} is the displacement at the ith point from the unit jth unbalance D_j; φ_j is the angle of the jth unbalance.

Now we assume that the random variables s, b, a, φ_i, d_{run}, d_{res} are uniformly distributed and independent, with each random variable φ_i distributed in the interval $[0, 2\pi]$, and the maximum values of all six random variables determined by the tolerance values.

If we assume each factor to be independent, then we can model random variables by δ_i^x, δ_i^y generating s, b, a, φ_i, d_{run}, d_{res} and performing transformations by Eqs. (8)–(14).

The compensating unbalance caused by balancing is non-random and is calculated based on the rotor equilibrium condition for each implementation of random variables s, b, a, φ_i, d_{run}, excluding d_{res}.

Given the experimental values δ_i^E, the null hypothesis is formulated as follows:

the simulated and experimental samples δ_i, δ_i^E belong to the same statistical population.

Problem statement

Consider the rotor of a free turbine consisting of three shafts. Their cross-sections are shown in Fig. 2,*a*; the solution domain of Eqs. (2) with boundary conditions is shown in Fig. 2,*b*. It is required to calculate the vertical velocity at the measuring point.

Fig. 2,*b* shows the following boundary conditions:

$$2,\ F_i^{x,y}/U_i^{x,y} = K;\ 3,\ F_i^{x,y}/U_i^{x,y} = K_{x,y};$$

$$4,\ U_i^{x,y} = U_j^{x,y},\ \theta_i^{x,y} = \theta_j^{x,y};$$

$$5,\ (F_i^{x,y} - F_j^{x,y})/(U_i^{x,y} - U_j^{x,y}) = K_{x,y},$$

where F, U and θ are the nodal force, displacement and angle of rotation, respectively; K is the stiffness coefficient.

The calculations were carried out in six stages.

1. The amplitude of the vertical velocity δ_{ij} was determined at the measuring point MP from the action of unit unbalances.
2. The sample of s, b, a, φ_i, d_{run} was generated.
3. For each implementation of s, b, a, φ_i, d_{run}, we calculated the compensating unbalances acting in the rotor balancing planes, serving to stabilize the system of forces due to these factors.
4. A sample of d_{res}, φ, modulus and phase of residual unbalances in the balancing planes was generated.
5. The vertical velocity sample was calculated at the measuring point MP by Eqs. (8)–(14).
6. The hypothesis that the computational and experimental samples belonged to the same statistical population in accordance with the Wilcoxon criterion was verified.

Two simulation scenarios were generated, differing by the action of loads.

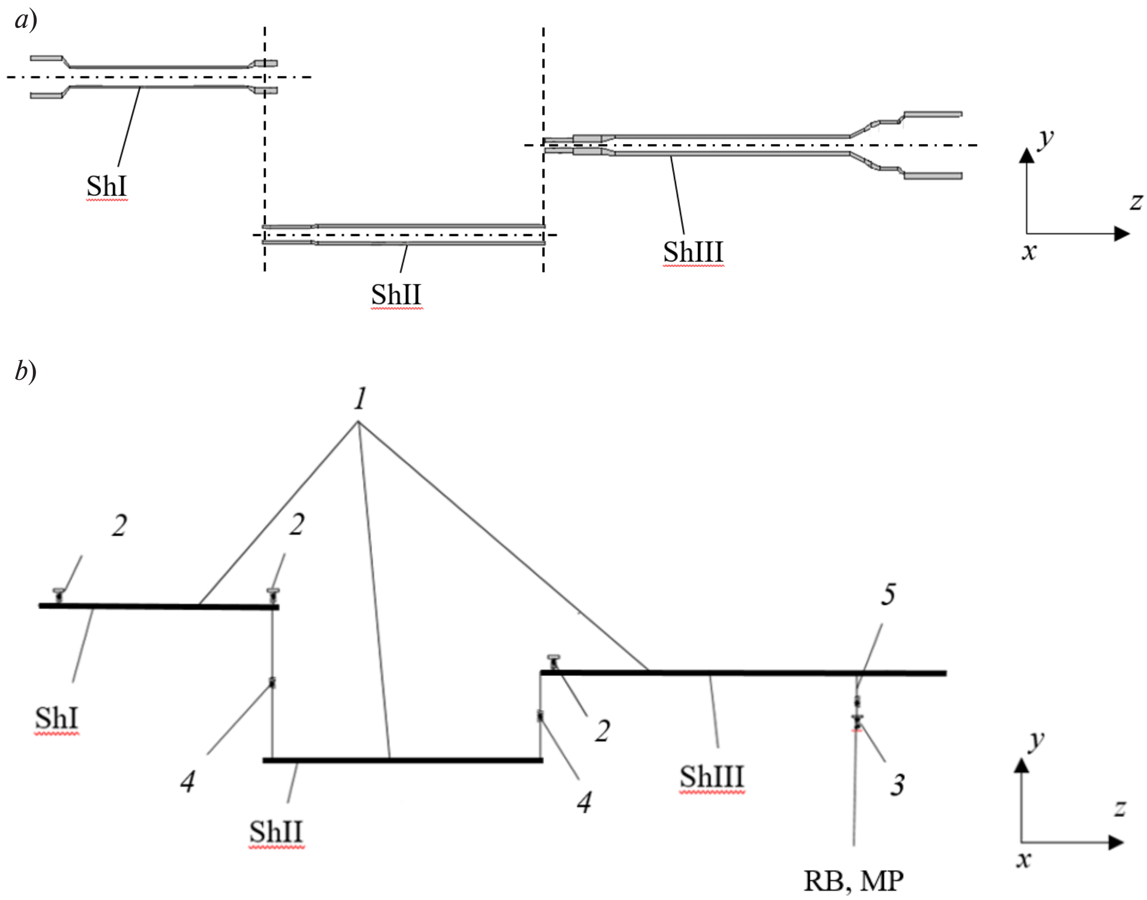

Fig. 2. Schematics for problem statement of free turbine rotor consisting of three shafts (ShI, ShII, ShIII): *a* are the shaft cross-sections; *b* is the solution domain of Eqs. (2) with boundary conditions (*2*–*5*), *1* is the solution domain; RB is the rigid beam; MP is the measuring point.

Because the boundary conditions are rather cumbersome, they are given in the text.

Scenario 1. The loads are induced by thickness variation of the shafts ShI, ShII, ShIII (see Fig. 2), the runout of the outer surface of these shafts, the runout of the mounting surfaces of the shaft ShII, the unbalance of the impeller, as well as compensating and residual unbalances in the rotor balancing planes.

Scenario 2. The loads are induced by unbalance of the impeller, compensating and residual unbalances in the rotor balancing planes.

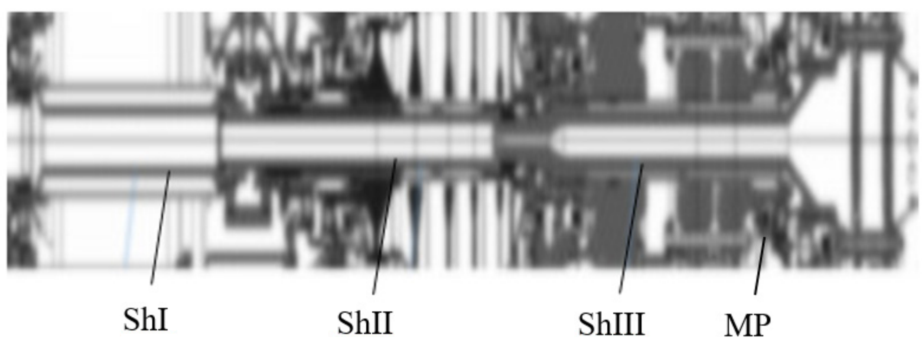

Fig. 3. Photograph of free turbine fragment: ShI, ShII, ShIII are rotor shafts; MP is the velocity measuring point on a rigid beam RB (see Fig. 2,*b*)

The calculation was carried out for the critical rotor velocity closest to the maximum frequency of the operating rotation range.

The vertical velocity from single unbalances was found using Dynamics R4 software, and the vertical velocity sample was calculated using an original program we developed, written in Python.

Experimental

The experiment consisted of vibrational tests of a free turbine as part of a gas turbine engine (Fig. 3).

The vertical vibration velocities were recorded at a given point MP of the engine (see Fig. 3), at each instant, at a constant angular rotation frequency of the shafts ShI–ShIII. The harmonic component of the vibration velocity with a frequency corresponding to the angular frequency of rotation of the three shafts was isolated using the discrete Fourier transform algorithm. Measurements were carried out for 13 engines where the values of the parameters s, b, a, φ_i, d_{run}, d_{res} for the rotor of the free turbine lie within the tolerances. Variation of the parameters is inevitable due to the inaccuracies introduced by manufacturing of the parts.

Experimental testing of a gas turbine engine allowed to generate a sample from the amplitudes of vertical vibration velocities.

Analysis of results

Fig. 4 shows a comparison of experimental and calculated histograms of the vertical vibration velocity distribution.

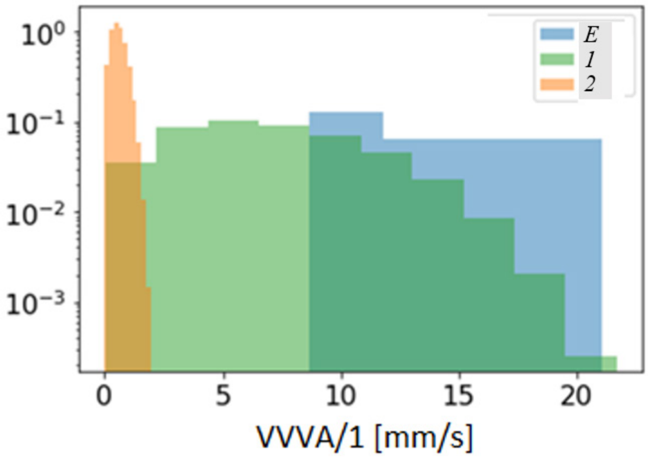

Fig. 4. Comparison of experimental (*E*) and two computational histograms (cases *1* and *2*) of the amplitude distribution of dimensionless vertical vibration velocity (VVVA/1[mm/s]) of the measuring point MP (see Figs. 2,*b* and 3).
Dimensionless values of probability density are plotted along the vertical axis

Table

Hypothesis testing results using the Wilcoxon criterion

Simulation scenario	Computational value	
	statistic	significance level
1	23.0	0.1159
2	0.0	0.0015

Note. The calculated statistical values and significance levels were found using the computational and experimental samples.

The results obtained by testing the hypothesis that the computational and experimental samples belong to the same statistical population are given in the table. Analyzing the results, we conclude that taking into account the unbalance of the shafts yields a much higher significance level, which means that the Scenario *1* is in better agreement with the experimental results.

Conclusion

We proposed a method for setting loads to determine the operational deflection of the rotor, taking into account the unbalance of the shafts.

A test problem was formulated to illustrate that taking into account the unbalance of the shafts yields results closer to experimental data.

Importantly, the study assumed the parameters s and a to be constant along the shaft length, which limits the scope of the method to the value of the shaft length; the method is only applicable for unloaded shafts.

In the future, we plan to obtain more reliable results (with *a* higher significance level) by conducting in-depth studies taking into account the influence of such factors as the non-linearity of bearing stiffness, transient processes in the rotor (for example, torque variability), etc., as well as testing the gas turbine engine for an extended range of experimental samples.

REFERENCES

1. **Laskin A. S., Sebelev A. A., Zabelin N. A.,** Energeticheskie mashiny. Vibroprochnost i ustoychivost elementov turbomashin [Energy machines. Vibration resistance and stability of turbomachine elements], POLITEKH-PRESS, St. Petersburg, 2018 (in Russian).

2. **Hajmovich A. I., Bolotov M. A., Pechenina E. E.,** Model of virtual balancing of rigid rotors, Vestnik of Samara University. Aerospace and Mechanical Engineering. 21 (1) (2022) 99–109 (in Russian).

3. **Fegade R., Patel V., Nehete R. S., Bhandarkar B. M.,** Unbalanced response of rotor using ANSYS parametric design for different bearings, Int. J. Eng. Sci. Technol. 7 (1) (2014) 506–515.

4. **Ahobal N., Ajit prasad S. L.,** Study of vibration characteristics of unbalanced overhanging rotor, IOP Conf. Ser.: Mater. Sci. Eng. 577 (2019) 012140.

5. **Longxi Zh., Shengxi J., Jingjing H.,** Numerical and experimental study on the multiobjective optimization of a two-disk flexible rotor system, Int. J. Rot. Machinery. 2017 (January) (2017) 1–10.

6. **Ivanov A. V., Leont'ev M. K.,** Modal analysis of dynamic rotor systems, Russian Aeronautics. 48 (3) (2005) 45–53.

7. **Birger I. A., Shorr B. F., Iosilevich G. B.,** Raschet na prochnost' detaley mashin [The calculation of machine parts strength], Mashinostroenie [Mechanical Engineering], Moscow, 1993 (in Russian).

8. **Leont'ev M. K.,** Konstruktsiya i raschet dempfernykh opor rotorov GTD [Design and calculation of GTE rotors damper supports], Moscow Aviation Institute Publ., Moscow, 1998 (in Russian).

9. **Usmanov A. R., Modestov V. S.,** Determination of centrifugal loads caused by shaft thickness variation, Proceeding of the All-Russian Scientific Conference "Week of Science at FizMekh", 4 – 9 Apr. 2022, POLITEKH-PRESS, St. Petersburg, 2022. 248–250 (in Russian).

10. **Bryukhanov S. A., Krisanov A. E., Zharikov V. V., Filyankin S. V.,** The effect of thermal unbalance on the rotor of pump-compressor and turbine equipment, Eurasian Union of Scientists. (12-5 (21)) (2015) 35–37 (in Russian).

11. **Murillo-Marrodán A., Garcна E., Barco J., Cortйs F.,** Analysis of wall thickness eccentricity in the rotary tube piercing process using a strain correlated FE model, Metals. 21 (8) (2020) 1045.

СПИСОК ЛИТЕРАТУРЫ

1. **Ласкин А. С., Себелев А. А., Забелин Н. А.** Энергетические машины. Вибропрочность и устойчивость элементов турбомашин. Санкт-Петербург: ПОЛИТЕХ-ПРЕСС, 2018. 128 с.

2. **Хаймович А. И., Болотов М. А., Печенина Е. Ю.** Модель виртуального уравновешивания жестких роторов // Вестник Самарского университета. Аэрокосмическая техника, технологии и машиностроение. 2022. Т. 21. № 1. С. 99–109.

3. **Fegade R., Patel V., Nehete R. S., Bhandarkar B. M.** Unbalanced response of rotor using ANSYS parametric design for different bearings // International Journal of Engineering Sciences & Emerging Technologies. 2014. Vol. 7. No. 1. Pp. 506–515.

4. **Ahobal N., Ajit prasad S. L.** Study of vibration characteristics of unbalanced overhanging rotor // IOP Conference Series: Materials Science and Engineering. 2019. Vol. 577. P. 012140.

5. **Longxi Zh., Shengxi J., Jingjing H.** Numerical and experimental study on the multiobjective optimization of a two-disk flexible rotor system // International Journal of Rotating Machinery. 2017. Vol. 2017. January. Pp. 1 –10.

6. **Леонтьев М. К., Иванов А. В.** Модальный анализ динамических систем роторов // Известия высших учебных заведений. Авиационная техника. 2005. № 3. С. 31–35.

7. **Биргер И. А., Шорр Б. Ф., Иосилевич Г. Б.** Расчет на прочность деталей машин. М.: Машиностроение, 1993. 640 с.

8. **Леонтьев М. К.** Конструкция и расчет демпферных опор роторов ГТД. М.: Изд-во МАИ, 1998. 44 с.

9. **Усманов А. Р., Модестов В. С.** Определение центробежных нагрузок, вызванных разнотолщинностью валов // Неделя науки ФизМех: сборник статей Всероссийской научной конференции. 4 – 9 апреля 2022., г. Санкт-Петербург: ПОЛИТЕХ-ПРЕСС, 2022. С. 248–250.

10. **Брюханов С. А., Крисанов А. Е., Жариков В. В., Филянкин С. В.** Влияние теплового дисбаланса на ротор насосно-компрессорного и турбинного оборудования // Евразийский союз ученых. 2015. № 12-5 (21). С. 35–37.

11. **Murillo-Marrodán A., Garcнa E., Barco J., Cortйs F.** Analysis of wall thickness eccentricity in the rotary tube piercing process using a strain correlated FE model // Metals. 2020. Vol. 21. No. 8. P. 1045.

THE AUTHORS

USMANOV Anton R.
Peter the Great St. Petersburg Polytechnic University
29 Politechnicheskaya St., St. Petersburg, 195251, Russia
ysman2d@gmail.com
ORCID: 0000-0003-1574-4638

MODESTOV Victor S.
Peter the Great St. Petersburg Polytechnic University
29 Politechnicheskaya St., St. Petersburg, 195251, Russia
vmodestov@spbstu.ru
ORCID: 0000-0003-0845-638X

СВЕДЕНИЯ ОБ АВТОРАХ

УСМАНОВ Антон Равильевич – *аспирант Высшей школы механики и процессов управления Санкт-Петербургского политехнического университета Петра Великого.*
195251, Россия, г. Санкт-Петербург, Политехническая ул., 29
ysman2d@gmail.com
ORCID: 0000-0003-1574-4638

МОДЕСТОВ Виктор Сергеевич – *ведущий программист Высшей школы механики и процессов управления Санкт-Петербургского политехнического университета Петра Великого.*
195251, Россия, г. Санкт-Петербург, Политехническая ул., 29
vmodestov@spbstu.ru
ORCID: 0000-0003-0845-638X

Received 17.06.2022. Approved after reviewing 11.12.2022. Accepted 11.12.2022.
Статья поступила в редакцию 17.06.2022. Одобрена после рецензирования 11.12.2022. Принята 11.12.2022.