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Structure of acoustic Lauegram on the Ewald circle of reflection for the Rayleigh wave scattering

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Abstract. The acoustic Lauegram of the Rayleigh wave the Laue–Bragg–Wulff high-frequency scattering on a rectangle rough band of an isotropic solid, having periodic lattice of an arbitrary number of the roughness discontinuities, is theoretically investigated in details in dependence on the angle of scattering φ_s at a fixed ratio of the lattice unit cell size to the wavelength and parameters of the lattice. The Ewald conception of the circle of reflection is used. The problem of an arbitrary number, defined beforehand, of the resonances of scattering, i.e. nodes of the reciprocal lattice, for any φ_s, defined beforehand, lying on the Ewald circle of reflection, is first solved analytically in the present work in the classical case, i.e. without influence of the amplitude form-factor of the lattice. It is found, that increasing of the number of resonances for any φ_s is necessarily accompanied by the increasing of the Ewald circle of reflection radius, i.e. of the Rayleigh wave frequency, at fixed sizes of a discontinuities lattice. It is obtained first, that amplitude form-factor of the discontinuities lattice strongly influences the structure of the acoustic Lauegram: arbitrary number of the resonances of scattering for any φ_s can be placed on the Ewald circle of reflection without variation of its radius by using of the appropriate amplitude form-factor of a discontinuities lattice of a solid roughness.

Keywords: Rayleigh wave, Laue scattering, Bragg–Wulff reflection

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Структура акустической лауэграммы на окружности отражения Эвальда для рассеяния волны Рэлея

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Аннотация. Получены новые закономерности рассеяния Лауэ-Брэгга-Вульфа для поверхностной акустической волны Рэлея в рамках концепции окружности отражения Эвальда.

Ключевые слова: волна Рэлея, рассеяние Лауэ, отражение Брэгга-Вульфа

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Introduction

It is known, that the Bragg-Wulff mirror reflection and the more general Laue scattering of the waves on the periodically arranged inhomogeneities are broadly used in the science and technologies [1—18]. Both these types of the physical phenomena are described by the Bragg-Wulff law of reflection and the Laue conditions of scattering respectively [1—7]. These laws of reflection and scattering are conditions of the phase synchronism of the reflected and scattered waves respectively [7—9]. The great interest is aroused by the phenomena of the wave scattering on the non-perfectly periodic lattices of inhomogeneities, which take place in the real nature [9—13]. Theoretically these physical phenomena are investigated in a frame of the kinematic and dynamic theories of scattering [6, 11]. The first one deals with the Born approximation of the perturbation theory, the second one solves the problems of the multiple scattering of waves on inhomogeneities. The new physical topological [14, 15] laws of scattering were obtained first in [16—18] in a frame of the Born approximation of the perturbation theory in inhomogeneity amplitude. These laws are topological ones [14, 15], since they are the same for the sets of inhomogeneities, having definite configuration properties. They include both the Rayleigh and the Laue—Bragg—Wulff scatterings. It was earlier well-known [5] that the high-frequency Laue—Bragg—Wulff scattering takes place mostly on the medium discontinuities because continuous inhomogeneities are physically smooth for the wave, having the wavelength more less than the character size of inhomogeneity. The periodic space arrangement of the medium discontinuities has the great importance for the conditions of the phase synchronism of the scattered waves according to the Laue—Bragg—Wulff laws [1—7]. But the amplitude form-factor of discontinuities lattice, i.e. dependence of the roughness left and right limit values difference in a point of discontinuity on a number of this discontinuity in a lattice [16—18], was not taken into account [1—13]. As for the Laue—Bragg—Wulff scattering, the new topological laws of scattering reveal the important role of this amplitude form-factor of the discontinuities lattice in a wave scattering, up to violation [16—18] of the Laue—Bragg—Wulff laws [1] of scattering.

It is interesting to investigate the fundamental properties of the acoustic Lauegram of scattering, i.e. dependence of the indicatrix of scattering on the angle of scattering, in the frame of a conception of the Ewald sphere of reflection [3] without influence of the lattice amplitude form-factor on a scattering, for example, when all discontinuities have equal amplitudes, as it is described by the Laue—Bragg—Wulff laws, and with account of this form-factor. The Ewald sphere of reflection conception states that necessary and enough condition that definite angle of scattering corresponds to the resonance of scattering is location of the corresponding node of the reciprocal lattice on the Ewald sphere of reflection. But the conception of the Ewald sphere of reflection does not consider and solve the next problem: what is the radius of the Ewald sphere of reflection, i.e. the frequency of the incident wave, containing arbitrary number, defined beforehand, of resonances for an arbitrary angles of scattering, defined beforehand? This problem is solved first in the present work for the scattering of the surface acoustic Rayleigh wave on a rectangle rough band of an isotropic solid, having periodic lattice of an arbitrary number of the roughness discontinuities [18]. In this case two transverse with respect to direction of the incident

Rayleigh wave propagation edges of the rough band violate the Laue–Bragg–Wulff law, but all the discontinuities amplitudes of the longitudinal lattice are the same, and so they do not influence the scattering. The scattered Rayleigh wave is cylindrical, so the Ewald sphere of reflection becomes the Ewald circle of reflection. The great influence of the amplitude form-factor of the transverse and longitudinal discontinuities lattices, i.e. of the new topological laws of scattering [16—18], on the solution of this problem is obtained and investigated.

The statement of the problem and method of a solution

Let us consider the theoretical problem of the surface acoustic Rayleigh wave scattering on a rough finite size part of an isotropic solid in the Laue–Bragg–Wulff case and in a more general case of the new topological laws of scattering [16–18]. The results of this completely concrete problem, obtained from the first principles of the dynamical theory of elasticity [5], give the possibility to understand the general laws of a wave scattering physical phenomena [1–13].

The problem on the number of the reciprocal lattice nodes, lying on the Ewald circle of reflection

Let the plane surface acoustic Rayleigh wave, propagating along the x_1-axis of a free surface of an isotropic homogeneous solid, occupying half-space $x_3 \geq 0$ of the Cartesian coordinates system (x_1,x_2,x_3), is incident on the surface rough region, having the form of a rectangle with the finite sizes L_1 and L_2 along the x_1- and x_2-axes respectively. That is the roughness occupies a rectangle region $-L_{1,2}/2 < x_{1,2} < L_{1,2}/2$. It is described by the next function

$$\begin{gathered} x_3 = f^{(2)}\left(x_1,x_2\right) = \delta_0 f_0\left(x_1,x_2\right) = \\ = \delta_0 f_\theta\left(x_1; -L_1/2; L_1/2\right) f_\theta\left(x_2; -L_2/2; L_2/2\right) f_1\left(x_1\right) \end{gathered} \tag{1}$$

where δ_0 is the roughness amplitude, having dimension of a length; the step function $f_\theta(x,a,b) = 1$ for $a < x < b$ and 0 otherwise; $f_1(x_1)$ is arbitrary dimensionless deterministic (not statistical) function. The problem of the plane Rayleigh wave scattering on the roughness (1) into the cylindrical Rayleigh wave in the Laue–Bragg–Wulff short-wavelength limit $\lambda \ll L_{1,2}$ is solved in [18] in the Rayleigh–Born approximation of the perturbation theory in a roughness amplitude (1). Conditions of the Rayleigh wave scattering resonances are expressed through the wave-vector $\vec{q}$ ((12) in [18]), transmitted from the incident to the scattered Rayleigh wave. This vector is defined uniquely by the absolute value of the wave-vector of incident or scattered wave k_R and the angle of scattering φ_s. So, two patterns of scattering can be considered. The first one is the frequency spectrum of the indicatrix of scattering, i.e. its dependence on the Laue–Bragg–Wulff parameter p_{N_l} ((10) in [18]) at arbitrary fixed value of the angle of scattering φ_s. The new topological laws of scattering [16–18] reveal that the frequency spectrum of scattering is always periodic independently on the periodicity or aperiodicity of the discontinuities lattice and on amplitude form-factor of a lattice [17]. The second pattern of scattering, mentioned above, is the structure of the indicatrix of scattering in dependence on the angle of scattering φ_s at a constant value of the Laue–Bragg–Wulff parameter p_{N_l}. This dependence is known to name the acoustic Lauegram of the surface roughness on the analogy of the X-Rays Lauegrams of a solid surface [1,10]. Let's investigate the fundamental properties of the acoustic Lauegram. The results and designations of [18] are used for this investigation of the present work. The frequency spectrum of scattering contains the resonances of scattering in a general case, but there is not such general case for the Lauegram of scattering. The question about the presence of resonances in the Lauegram is considered by the conception of the Ewald sphere of reflection [3]. The Lauegram contains resonances of scattering if and only if it contains the nodes of the reciprocal lattice in the space of the transmitted wave-vectors $\vec{q}$, corresponding to the scattering lattice and to the conditions of resonances of scattering ((18–21) in [18]). But this conception and result [3] does not consider and solve the analytical problem about the number of such resonances and angles of scattering, corresponding to them. For the cylindrical scattered Rayleigh wave the sphere of reflection becomes the Ewald circle of reflection (Fig. 1). Let us consider and solve the next analytical problem: what is the radius k_R of the Ewald circle of reflection, i.e. the frequency of the incident wave, containing arbitrary number, defined beforehand, of resonances for arbitrary angles of scattering φ_s, defined beforehand?

Let's investigate this problem in the case without influence of the longitudinal lattice amplitude form-factor, accounting only the presence of the two transverse edges of the lattice (1), [18], violating the classical Laue–Bragg–Wulff law of scattering [18]; and consider the role of the amplitude form-factors of the longitudinal and transverse lattices in the formation of the acoustic Lauegram of scattering.

Solution of the problem

Let us consider analytical solution of the stated problem.

The main scattering phase equations on the Ewald circle of reflection

The physical elastic process of the Rayleigh wave scattering corresponds to the circle of the radius k_R in the space of the wave-vectors. Different points on this circle gives the angles of scattering φ_s. The $\varphi_s = 0$ is the direction of propagation of the incident wave. This circle, constructed in the two-dimensional space of the reciprocal lattice ((18) in [18]), is the Ewald circle of reflection (Fig. 1), [3]. The condition, that the node of the reciprocal lattice $\vec{q}^{(r)}(n_1,n_2)$ ((18) in [18]) lies on the Ewald circle of reflection (Fig. 1), having the radius $k_R = p_{Nl}/(L_1/N_l)$, gives the next main scattering phase equations, connecting the Laue–Bragg–Wulff parameter p_{N_l} of the given physical process of scattering and the numbers (n_1,n_2), defining the node of the reciprocal lattice, lying on the Ewald circle of reflection of this process, and consequently the phase of the angular spectrum of scattering, i.e. of the acoustic Lauegram of scattering,

$$\begin{cases} n_1\left(\dfrac{p_{N_l}}{\pi}-n_1\right)=\left(2n_2+1\right)^2 m^{(L)}, \\ \tan\left(\dfrac{\varphi_s}{2}\right)=\pm\dfrac{m_l\left(n_1,n_2\right)}{\sqrt{m^{(L)}}}, \end{cases} \quad (2)$$

where $n_1 = 1, 2, \ldots;\ n_2 = -1/2, 0, 1, 2, \ldots;\ m^{(L)} = \left(L_1/N_l\right)^2/\left(4L_2^2\right)$, m_l is given by (22) in [18]. The second equation of (2) gives the angle of scattering for the resonance (n_1,n_2) ((18–21) in [18]). Since the system (2) connects integer numbers n_1 and $2n_2 + 1$, let us consider the next approximation of the real values p_N/π and the form parameter $m^{(L)}$ by means of the rational numbers, which are the ratio of any two natural, i.e., positive integer, numbers

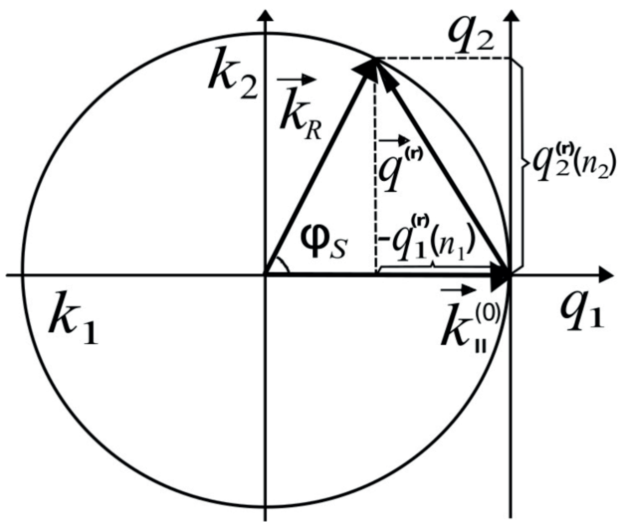

Fig. 1. Ewald circle of reflection. $q_1^{(r)}$, $q_2^{(r)}$ ((18–21) in [18]) are the coordinates of the nodes of the reciprocal lattice in the space (q_1,q_2). $n^{(E)}$ nodes of the reciprocal lattice lie on the Ewald circle, if the angle of scattering $0 < \varphi_s < \pi$. The same amount of the resonance reciprocal lattice nodes, lying on the Ewald circle for the $\pi < \varphi_s < 2\pi$, have positions symmetrical with respect to the q_1-axis

$$\frac{p_{N_l}}{\pi} = \frac{m_5}{m_6} C; \; m^{(L)} = \frac{m_7^2}{m_8^2}; \; n_1 / m_5 \equiv n_3, \tag{3}$$

where $C, n_3, m_5, m_6, m_7, m_8$ are natural numbers; $p_{N_l} = k_R \left(L_1 / N_l \right)$ is a positive real number. Representation (3) gives the next final system of equations, connecting sought natural numbers $C,\ n_3,\ 2n_2+1$ with each other and with the angle of scattering φ_s, instead of the system (2)

$$\begin{cases} m_8^2 m_5^2 n_3 \left(C - m_6 n_3 \right) = \left(2n_2 + 1 \right)^2 m_7^2 m_6 ; \\ \tan \dfrac{\varphi_s}{2} = \pm \dfrac{m_l \left(n_1, n_2 \right)}{\sqrt{m^{(L)}}} = \pm \dfrac{m_5 n_3}{\left(2n_2 + 1 \right)} \dfrac{m_8}{m_7} , \end{cases} \tag{4}$$

where C, n_3 are natural numbers, $(2n_2 + 1)$ is non-negative integer.

One possible schema of the solution

One possible schema of the (4) solution can have the next form

$$\begin{cases} V_1^{(1)^2} \chi_1 m_7^2 m_6^2 + V_1^{(2)^2} \chi_1 = C ; \quad V_2^{(1)^2} \chi_2 m_7^2 m_6^2 + V_2^{(2)^2} \chi_2 = C ; \quad \ldots \\ \chi_m \left(V_m^{(1)^2} m_7^2 m_6^2 + V_m^{(2)^2} \right) = C ; \; \ldots \; V_{n^{(E)}}^{(1)\,2} \chi_{n^{(E)}} m_7^2 m_6^2 + V_{n^{(E)}}^{(2)\,2} \chi_{n^{(E)}} = C , \end{cases} \tag{5}$$

where $V_m^{(1)},\ V_m^{(2)}$ are arbitrary natural numbers, $m = 1, 2, \ldots, n^{(E)}$;

$$V_m^{(1)2} V_m^{(2)2} \chi_m^2 m_8^2 m_5^2 = \left(2n_2 + 1 \right)^2 , \quad \text{where } m = 1, 2, \ldots, n^{(E)}. \tag{6}$$

It follows from (6), that $V_m^{(1)},\ V_m^{(2)}$, where $m = 1, 2, \ldots, n^{(E)}$, are arbitrary odd natural numbers. Arbitrary natural number $n^{(E)}$ shows how many nodes of reciprocal lattice lie on the Ewald circle of reflection.

Results and Discussion

The Ewald circle of reflection in a frame of the Laue–Bragg–Wulff laws of scattering

It follows from the system of equations (5), that sought natural number C is the least common multiple **(lcm)** of the next natural numbers

$$C = \operatorname{lcm} \left\{ V_m^{(1)^2} m_7^2 m_6^2 + V_m^{(2)^2} \right\}, \quad m = 1, 2, \ldots, n^{(E)}. \tag{7}$$

The next relations follow from the (4–6)

$$\begin{gathered} \chi_m = C / \left(V_m^{(1)^2} m_7^2 m_6^2 + V_m^{(2)^2} \right), \; n_3^{(m)} = \chi_m V_m^{(1)^2} m_7^2 m_6, \\ n_1^{(m)} = m_5 n_3^{(m)} = \chi_m V_m^{(1)2} m_7^2 m_6 m_5, \\ \left(2n_2^{(m)} + 1 \right) = V_m^{(1)} V_m^{(2)} \chi_m m_8 m_5, \\ m_l^{(m)} = \frac{V_m^{(1)}}{V_m^{(2)}} \frac{m_6}{m_8} m_7^2, \; m = 1, 2, \ldots, n^{(E)}. \end{gathered} \tag{8}$$

It follows from (4), (8) that the angles of scattering $\varphi_s^{(m)}$, where $m = 1, 2,..., n^{(E)}$, corresponding to the two nodes of the reciprocal lattice under number m, have the next form

$$\tan\frac{\varphi_s^{(m)}}{2} = \pm\frac{V_m^{(1)}}{V_m^{(2)}} m_6 m_7, \quad m = 1, 2, \ldots, n^{(E)} \tag{9}$$

It follows from the solution (9), that predefining of the pairs of arbitrary odd natural numbers $V_m^{(1)}$, $V_m^{(2)}$ means predefining of the arbitrary angles of scattering $\varphi_s^{(m)}$, where $m = 1, 2,..., n^{(E)}$, corresponding to the full resonances of scattering ((18–21) in [18]), lying on the Ewald circle of reflection (Fig. 1), having the next sought radius $k_R = (m_5 / m_6)\pi C / (L_1 / N_l)$.

The new topological laws of scattering and the Ewald circle of reflection

The new topological laws of scattering [16—18] include first the amplitude form-factor of the discontinuities lattices ((11) in [18]) into the theory of scattering on the basis of the first principles of the dynamical theory of elasticity. Let us consider the influence of the lattice amplitude form-factor on the presence of the resonances of scattering, i.e. nodes of the reciprocal lattice, on the Ewald circle of reflection (Fig. 1), [3]. For the Rayleigh wave scattering on the rectangle periodic lattice of discontinuities (1), ((1) in [18]) with $f_1(x_1)f_2(x_2)$ instead of $f_1(x_1)$, having $N_l^{(1)}$ and $N_l^{(2)}$ unit cells along the x_1- and x_2- axes respectively, bounded by the discontinuities, located at the next points $l_{m_{1,2}}^{(1,2)} = -L_{1,2}/2 + \tilde{l}_{m_{1,2}}^{(1,2)}$, where $\tilde{l}_{m_{1,2}}^{(1,2)} = (m_{1,2} - 1) L_{1,2} / N_l^{(1,2)}$, $m_{1,2} = 1, 2, \ldots, N_l^{(1,2)}$, the indicatrix of scattering has the next form analogically to ((10) in [18])

$$I_{\parallel,3}^{(R)} = \beta^2 \frac{c_R^4}{c_t^4} \frac{p_{N_l}^5}{8\pi R_2^2} \frac{(L_2/L_1)^2 A_{1,2}^2(x_3)}{N_l^{(1)2\left(n_d^{(1)}-3/2\right)} N_l^{(2)2\left(n_d^{(2)}+1\right)}} \times$$
$$\times \frac{\left|\tilde{J}^{(1)}(q_1)\right|^2 \left|\tilde{J}^{(2)}(q_2)\right|^2 (1-\cos\varphi_s)^2 (\gamma + \cos\varphi_s)^2}{\left(q_1 L_1 / N_l^{(1)}\right)^{2\left(n_d^{(1)}+1\right)} \left(q_2 L_2 / N_l^{(2)}\right)^{2\left(n_d^{(2)}+1\right)}}, \tag{11}$$

where

$$\left|\tilde{J}^{(1,2)}(q_{1,2})\right|^2 = \tilde{F}_0^{(1,2)} + 2\sum_{n=1}^{N_l^{(1,2)}} \tilde{F}_n^{(1,2)} \cos\left(q_{1,2}\frac{L_{1,2}}{N_l^{(1,2)}} n\right),$$
$$\tilde{F}_n^{(1,2)} = \sum_{m=1}^{N_l^{(1,2)}+1-n} F_m^{(1,2)} F_{m+n}^{(1,2)}, \tag{12}$$
$$F_m^{(1,2)} = L_{1,2}^{n_d^{(1,2)}} \left(\frac{d^{n_d^{(1,2)}}}{d\, x_{1,2}^{n_d^{(1,2)}}} f_{1,2}(x_{1,2}) \right) \Bigg|_{l_m^{(1,2)}+0}^{l_m^{(1,2)}-0}.$$

$F_m^{(1,2)}$ are dimensionless amplitude form-factors of the longitudinal and transverse lattices, i.e. lattices, arranged along the $x_{1,2}$-axes respectively.

Let us the amplitude form-factors (12) of the lattice have the next form

$$F_{m_{1,2}}^{(1,2)} = 1 + \sum_{i=1}^{n_{1,2}^{(a)}} \cos\left(a_i^{(1,2)} m_{1,2}\right),\ m_{1,2} = 1, 2, \ldots, N_l^{(1,2)}. \tag{13}$$

Then the conditions of the full resonances of scattering have the next form [16, 17]

$$q_1 L_1 / N_l^{(1)} \pm a_{i_1}^{(1)} = -2\pi n_1,\ q_2 L_2 / N_l^{(2)} \pm a_{i_2}^{(2)} = \pm 2\pi n_2,$$
$$n_{1,2} = 1, 2, \ldots;\ i_{1,2} = 1, 2, \ldots, n_{1,2}^{(a)}. \tag{14}$$

It follows from the (14), that the reciprocal resonance lattice with account of the lattice amplitude form-factor (12), (13), i.e. in the frame of the new topological laws (11), [16]-[18], has the next form

$$\vec{q}^{(r)} = -\vec{a}_1^{\left(N_l^{(1)}\right)}\left(2\pi n_1 \pm a_{i_1}^{(1)}\right)\Big/\left(a_1^{\left(N_l^{(1)}\right)}\right)^2 \pm \vec{a}_2^{\left(N_l^{(2)}\right)}\left(2\pi n_2 \mp a_{i_2}^{(2)}\right)\Big/\left(a_2^{\left(N_l^{(2)}\right)}\right)^2, \quad (15)$$

where $n_{1,2} = 1, 2, \ldots;\ 0 \le a_{i_{1,2}}^{(1,2)} \le 2\pi,\ i_{1,2} = 1, 2, \ldots, n_{1,2}^{(a)}$; all the "±" are independent in the (15), but $q_1^{(r)} < 0$ (Fig. 1), ((18), (19) in [18]). The conditions (14) are the system of the linear algebraic equations with respect to the unknowns $a_{i_{1,2}}^{(1,2)}$ (13), defining the amplitude form-factors of the lattice (12), (13).

The radius of the Ewald circle of reflection k_R and the resonance angles of scattering φ_s are the parameters of this system. It means that arbitrary number of the resonances of scattering (14), (15) can be placed on the Ewald circle of reflection, i.e. on the acoustic Lauegram of scattering, without increasing of its radius, i.e. the frequency of the Rayleigh wave at fixed sizes of the lattice, contrary to the case (7–10), when the amplitude form-factor of the longitudinal lattice does not influence the scattering, and one of the transverse lattice violates ((18), (21) in [18]) the Laue–Bragg–Wulff law of scattering [1, 2].

Conclusion

The fundamental properties of the acoustic Lauegram of scattering are obtained in the frame of the new topological laws of scattering [16–18]. These results can be used in the physical research: from the solid state physics up to investigations on the X-Ray and acoustic microscopy materials imaging, and in acoustoelectronic technologies and physics of the acoustic metamaterials [10–13].

REFERENCES

1. **Bijvoet J.M., Burgers W.G., Hagg G.** (Eds.), Early Papers on Diffraction of X-Rays by Crystals, Vol. I, Springer, 1969; Vol. II, Springer, 1972.
2. **Laue M., Friedrich W., Knipping P.**, in **Bacon G. E.** (Ed.), X-Ray and Neutron Diffraction (in English), Pergamon Press, 1966.
3. **Ewald P.P.,** Acta Cryst. A25 (1969) 103.
4. **Jenkins F.A., White H.E.,** Fundamentals of Optics, McGraw-Hill, 2001.
5. **Biryukov S.V., Gulyaev Yu.V., Krylov V.V., Plesskii V.P.,** Surface Acoustic Waves in Inhomogeneous Media, Springer-Verlag, 1995.
6. **Khapachev Yu.P., Dyshekov A.A.**, Theory of the Dynamic Roentgen Diffraction in Superlattices, Nalchik, Russia, 2002.
7. **Prokhorov A.M.** (Ed.), The Physics Encyclopedia, Moscow, 1988.
8. **Gorodnichev E.E., Dudarev S.I., Rogozkin D.B., Ryazanov M.I.,** Sov. Phys. JETP. 69 (1989) 17.
9. **Savchenkov E.N., Schandarov S.M., et al.,** JETP Letters. 110 (2019) 165.
10. **Miao J., Ishikawa T., Robinson I. K., Murnane M.M.**, Science. 348 (2015) 530.
11. **Lee M.**, X-Ray Diffraction for Materials Research. From Fundamentals to Applications, CRC Press, 2016.
12. **Maev R.G.,** Acoustic Microscopy. Fundamentals and Applications, Wiley-VCH, Weinheim, 2008.
13. **Deymier P.A.** (Ed.), Acoustic Metamaterials and Phononic Crystals, Springer, 2013.
14. **Alexandryan R.A., Mirzahanyan E.A.,** General Topology, Moscow, 1979.
15. **Shapiro I.S., Ol'shanetsky M.A.,** Lectures on Topology for Physicists, Moscow, 2001. ISBN: 5-7029-0311-0.
16. **Chukov V.N.**, Topological Laws of the Rayleigh Wave Scattering, LAP, 2018, ISBN: 978-613-9-84837-9.
17. **Chukov V.N.,** 2020, ISBN: 978-5-7422-7029-4, p. 87, https://acoust-conference.iapras.ru.
18. **Chukov V.N.**, J. Phys.: Conf. Ser. 2103 (2021) 012157.

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