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Stability of steady states of plasma diodes with counter-streaming electron and positron flows

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Abstract. Stability of steady states of a planar geometry diode with counter flows of electrons and positrons is studied. The study is related to the elucidation of pulsar RF radiation nature. The equation for the electric field perturbation is derived. Its exact solution is obtained for the case of a homogeneous steady-state field. The study of the dispersion equation obtained has shown that there is a threshold for the inter-electrode gap value, above which steady-state solutions are unstable. The instability threshold turned out to be √2 times higher than the known Pierce threshold.

Keywords: plasma diode, electron and positron flows, plasma instability

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Материалы конференции
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Устойчивость стационарных состояний плазменного диода со встречными пучками электронов и позитронов

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Аннотация. Исследована устойчивость стационарных состояний плоского вакуумного диода со встречными пучками электронов и позитронов. Исследование связано с выяснением природы радиоизлучения пульсаров. Получено уравнение, описывающее эволюцию малого возмущения электрического поля. В случае однородного стационарного электрического поля найдено его аналитическое решение. Исследование полученного дисперсионного уравнения показало, что существует порог по длине вакуумного зазора, выше которого стационарные состояния неустойчивы. Порог устойчивости оказался в √2 раз выше известного порога Пирса.

Ключевые слова: плазменный диод, потоки электронов и позитронов, плазменная неустойчивость

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Introduction

Since the discovery of pulsars in 1967, the understanding of the physical processes responsible for their RF radiation has not advanced much compared to the classical works [1, 2]. Neither the mechanism of this radiation nor the reason for switching between modes have been understood so far. It is only in recent years that astrophysicists have come to understanding that the radiation is associated with collective processes in plasma formed by electrons and positrons in a pulsar diode [3]. In this case, it can be argued that the RF radiation of pulsars is caused by electric field oscillations that occur during the development of instability in the plasma. It is like the Bursian-Pierce instability that is characteristic of diodes with collisionless plasma [4].

Steady-state solutions for a planar geometry diode with counter flows of charged particles of opposite signs moving in plasma without collisions were studied in detail in [5]. It was shown that there are two modes: 1) all charged particles move in the inter-electrode gap without reflection and reach the opposite electrodes, 2) the potential distribution has extrema reflecting the portion of the particles. The paper examines the stability of steady-state solutions of the first type. The equation for the electric field perturbation is derived. Its analytical solution is found for the case of a homogeneous steady-state field. The study of dispersion relation has shown that there is a threshold for the gap value, when exceeded, an aperiodic instability develops in the diode plasma.

Derivation of equations

We consider a diode of planar geometry. We assume that electrons enter the plasma from the left electrode with a non-relativistic velocity $v_{e,0}$ and density $n_{e,0}$, and positrons enter from the right electrode with a non-relativistic velocity $-v_{p,0}$ and density $n_{p,0}$. Charged particles move without collisions, and when reaching the electrode they are absorbed. We assume that electrons and positrons enter into inter-electrode gap with the same kinetic energies, i.e., $W_0 \equiv m_e v_{e,0}^2/2 = m_p v_{p,0}^2/2$ (here m_e and m_p are the masses of the electron and positron).

For the convenience, we turn to dimensionless quantities, choosing the electron energy at the left boundary W_0 and the Debye-Hückel length $\lambda_D = [2\varepsilon_0 W_0/(e^2 n_{e,0})]^{1/2}$ as units of energy and length (here e is the electron charge, and ε_0 is the vacuum permeability).

For the density of electrons entering from the left electrode and moving without reflection in a time-dependent field, the following expression is obtained in Ref. [6]

$$n_e(\eta,\tau) = \left[\left(1+2\eta+2G_e(\eta)\right)^{-1/2} - Q_e(\eta)\right]^{-1}. \tag{1}$$

Here

$$G_e(\zeta,\tau;u_{e,0},\tau_{e,0}) = -\int_{\tau_0}^{\tau} dt \frac{\partial}{\partial t}\eta(\zeta,t),$$

$$Q_e(\zeta,\tau;u_{e,0},\tau_{e,0}) = \tag{2}$$

$$= -\int_0^{\tau-\tau_{e,0}} dt(\tau-\tau_{e,0}-t)\frac{d}{d\tau_{e,0}}\varepsilon\left[\zeta(t+\tau_{e,0});u_{e,0},\tau_{e,0}\right]_{u_{e,0}=const}.$$

For the density of positively charged particles entering from the right electrode and moving without reflection in a time-dependent field, an expression similar to Eq. (1) is obtained in Ref. [7]

$$n_p(\eta,\tau) = \left\{\left|-\left[1-2(\eta-V)+2G_p(\eta)\right]^{1/2} - Q_p(\eta)\right|\right\}^{-1}. \tag{3}$$

Here

$$\begin{gathered} G_p(\zeta,\tau;u_{p,0},\tau_{p,0}) = \int_{\tau_0}^{\tau} dt \frac{\partial}{\partial t}\eta(\zeta,t), \\ Q_p(\zeta,\tau;u_{p,0},\tau_{p,0}) = \\ = \int_0^{\tau-\tau_{p,0}} dt(\tau-\tau_{p,0}-t)\frac{d}{d\tau_{p,0}}\varepsilon\left[\zeta(t+\tau_{p,0});u_{p,0},\tau_{p,0}\right]_{u_{p,0}=const}. \end{gathered} \tag{4}$$

In Eqs. (1–4), τ, ζ, η and ε are dimensionless time, coordinate, potential and electric field strength, while $\tau_{e,0}$, $u_{e,0}$, $\tau_{p,0}$ and $u_{p,0}$ are the time moments and velocities of the departure of electrons and positrons from the corresponding electrodes. Note that in Eqs. (1) and (3), the value of G is equal to the amount of energy that a charged particle acquires ($G > 0$) or loses ($G < 0$) when moving in a time-dependent field, and the function Q characterizes the additional (compared to motion in a stationary field) compression (or extension) experienced by a group of particles that have left the boundary with velocities within a narrow range.

The potential distribution (PD) at each moment τ is found as a solution of the second order equation, which is obtained after substituting expressions for the densities of electrons (1) and positrons (3) into Poisson's equation

$$\begin{gathered} \frac{d^2\eta}{d\zeta^2} = \left[\left(1+2\eta+2G_e(\eta)\right)^{1/2} - Q_e(\eta)\right]^{-1} - \\ -\left\{\left[1-2\left(\eta-V\right)+2G_p(\eta)\right]^{1/2} + Q_p(\eta)\right\}^{-1}. \end{gathered} \tag{5}$$

Studying the stability features of solutions, we consider small perturbations of PD in the form

$$\begin{gathered} \eta(\zeta,\tau) = \eta_0(\zeta) + \tilde{\eta}(\zeta)\exp(-i\Omega\tau), \\ |\tilde{\eta}(\zeta)| << \eta_0(\zeta). \end{gathered} \tag{6}$$

In this case, we can assume that $G_{e,i}(\zeta,\tau) = \tilde{G}_{e,i}(\zeta)\exp(-i\Omega\tau)$, $Q_{e,i}(\zeta,\tau) = \tilde{Q}_{e,i}(\zeta)\exp(-i\Omega\tau)$, and both of these functions are quantities of the order of $\tilde{\eta}$ [6], [7]. Expanding both parts of Eq. (5) in a power series in small potential perturbation amplitude and taking into account the linear terms only, we get

$$\begin{gathered} \tilde{\eta}''(\zeta) = -u_{e,0}^{-3}(\zeta)\left[\tilde{\eta}(\zeta)+\tilde{G}_e(\zeta)\right] + u_{e,0}^{-2}(\zeta)\tilde{Q}_e(\zeta) - \\ -u_{p,0}^{-3}(\zeta)\left[\tilde{\eta}(\zeta)-\tilde{G}_p(\zeta)\right] + u_{p,0}^{-2}(\zeta)\tilde{Q}_p(\zeta). \end{gathered} \tag{7}$$

In Eq. (7),

$$\begin{gathered} u_{e,0}(\zeta) = \left[1+2\eta_0(\zeta)\right]^{1/2}, \\ u_{p,0}(\zeta) = \left[1-2\eta_0(\zeta)+2V\right]^{1/2} \end{gathered} \tag{8}$$

are the undisturbed velocities of electrons and positrons corresponding to the monoenergetic particle velocity distributions. The following expressions are obtained for the functions $\tilde{G}_e$, $\tilde{Q}_e$, $\tilde{G}_p$ and $\tilde{Q}_p$ in Refs. [8] and [7]:

$$\begin{gathered}
\tilde{G}_e(\zeta) = -\tilde{\eta}(\zeta) + \int_0^{\zeta} dx \tilde{\eta}'(x) \exp\{i\Omega[\sigma_e(\zeta) - \sigma_e(x)]\}, \\
\tilde{Q}_e(\eta) = -i\Omega u_{e,0}(\zeta) \int_0^{\zeta} dx [u_{e,0}(x)]^{-3} \int_0^{x} dy \tilde{\eta}'(y) \exp\{i\Omega[\sigma_e(\zeta) - \sigma_e(y)]\}, \\
\tilde{G}_p(\zeta) = \tilde{\eta}(\zeta) + \int_{\zeta}^{\delta} dx \tilde{\eta}'(x) \exp\left\{i\Omega[\sigma_p(\zeta) - \sigma_p(x)]\right\}, \\
\tilde{Q}_p(\zeta) = i\Omega u_{p,0}(\zeta) \int_{\zeta}^{\delta} dx [u_{p,0}(x)]^{-3} \int_{x}^{\delta} dy \tilde{\eta}'(x) \exp\left\{i\Omega[\sigma_p(\zeta) - \sigma_p(x)]\right\}.
\end{gathered} \tag{9}$$

Here

$$\sigma_e(\zeta) = \int_0^{\zeta} dx [u_{e,0}(x)]^{-1},\ \sigma_p(\zeta) = \int_{\zeta}^{\delta} dx [u_{p,0}(x)]^{-1} \tag{10}$$

are times of electron and positron flight from the corresponding electrode to the point ζ in an undisturbed field.

After substituting (9) into (7), we obtain an integral-differential equation for an amplitude of the potential perturbation, which, after a single integration using the boundary condition for the potential perturbation at the right boundary $\tilde{\eta}(\delta) = 0$ takes the following form:

$$\begin{gathered}
\tilde{\eta}'(\zeta) + \int_0^{\zeta} dx [u_{e,0}(x)]^{-3} \int_0^{x} dy \tilde{\eta}'(y) \exp\{i\Omega[\sigma_e(\zeta) - \sigma_e(y)]\} + \\
+ \int_{\zeta}^{\delta} dx [u_{p,0}(x)]^{-3} \int_{x}^{\delta} dy \exp\left\{i\Omega[\sigma_p(\zeta) - \sigma_p(y)]\right\} \tilde{\eta}'(y) = A.
\end{gathered} \tag{11}$$

Here

$$A = \tilde{\eta}'(\delta) + \int_0^{\delta} dx [u_{e,0}(x)]^{-3} \int_0^{x} dy \exp\{i\Omega[\sigma_e(\delta) - \sigma_e(y)]\} \tilde{\eta}'(y), \tag{12}$$

and the value of the derivative $\tilde{\eta}'(\delta)$ is an arbitrary parameter.

Thus, we have obtained for the first time an equation for the amplitude of the electric field perturbation in a diode with counter flows of electrons and positrons.

Study of homogeneous steady-state solutions stability

Consider the special case of $\eta_0(\zeta) \equiv 0$. Here $u_{e,0}(\zeta) = 1$, $u_{p,0}(\zeta) = 1$ (the latter is the velocity absolute value), $\sigma_e(\zeta) = \zeta$, $\sigma_p(\zeta) = \delta - \zeta$. After making simple calculations and getting rid of 2-fold integrals, for the Eq. (11) we get

$$\tilde{\eta}'(\zeta) + \int_0^{\zeta} dx K_+(\zeta - x) \tilde{\eta}'(x) - \int_{\zeta}^{\delta} dx K_-(\zeta - x) \tilde{\eta}'(x) = A. \tag{13}$$

Here

$$A = \tilde{\eta}'(\delta) + \int_0^{\delta} dx K_+(\delta - x) \tilde{\eta}'(x),\ K_{\pm}(t) = t \exp(\pm i\Omega t). \tag{14}$$

Eq. 13 is an integral equation of the convolution type. It is solved using the Laplace transform. For the image $f(p)$ of the function $\tilde{\eta}'(\zeta)$ we get the following expression:

$$f(p)=\frac{\left(p^2+\Omega^2\right)^2}{P_4(p)}\left[\frac{g_1}{(p+i\Omega)^2}-\frac{g_2}{p+i\Omega}+\frac{A}{p}\right]=$$
$$=\frac{g_1}{(p+i\Omega)^2}-\frac{g_2}{p+i\Omega}+\frac{A}{p}-\frac{2g_1(p^2-\Omega^2)}{(p+i\Omega)^2P_4(p)}+\frac{2g_2(p^2-\Omega^2)}{(p+i\Omega)P_4(p)}-\frac{2A(p^2-\Omega^2)}{pP_4(p)}. \quad (15)$$

Here

$$P_4(p)=\left(p^2+\Omega^2\right)^2+2\left(p^2-\Omega^2\right),$$
$$g_1=\int_0^\delta dx\exp(i\Omega x)\tilde{\eta}'(x),\quad g_2=\int_0^\delta dx x\exp(i\Omega x)\tilde{\eta}'(x), \quad (16)$$

and it is taken into account that the image of $K_\pm$ kernels look like $k_\pm(p)=(p\mp i\Omega)^{-2}$. The polynomial $P_4(p)$ has 4 roots:

$$\alpha_i=\pm\left[-\left(\Omega^2+1\right)\pm\sqrt{4\Omega^2+1}\right]^{1/2},\ i=1,\ldots,4. \quad (17)$$

The function $\tilde{\eta}'(\zeta)$ is found by the inverse Laplace transform using (15–17):

$$\tilde{\eta}'(\zeta)=\left[\sum_{i=1}^{4}\frac{\left(\alpha_i^2+\Omega^2\right)^2\exp(\alpha_i\zeta)}{\alpha_i\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}+\frac{\Omega^4}{\prod\limits_{i\neq j}(-\alpha_i)}\right]A+\left[\sum_{i=1}^{4}\frac{\left(\alpha_i-i\Omega\right)^2\exp(\alpha_i\zeta)}{\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}\right]g_1-$$
$$-\left[\sum_{i=1}^{4}\frac{\left(\alpha_i-i\Omega\right)^2(\alpha_i+i\Omega)\exp(\alpha_i\zeta)}{\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}\right]g_2. \quad (18)$$

It can be seen from Eq. 18 that the function $\tilde{\eta}'(\zeta)$ depends on three values: A, g_1 and g_2. They, in turn, depend on this function and are the solution of the system of linear equations obtained after substitution of Eq. (18) to Eqs. (14) and (16).

$$\begin{aligned}(1-B_{1,1})\cdot A-B_{1,2}\cdot g_1-B_{1,3}\cdot g_2&=\tilde{\eta}'(\delta),\\ B_{2,1}\cdot A+(B_{2,2}-1)\cdot g_1+B_{2,3}\cdot g_2&=0,\\ B_{3,1}\cdot A+B_{3,2}\cdot g_1+(B_{3,3}-1)\cdot g_2&=0.\end{aligned} \quad (19)$$

Here the coefficients $B_{k,l}$ depend on the roots α_i, and the values Ω and δ. We do not write them out explicitly because of their cumbersomeness. The solution of the system (19) has the form:

$$A=\frac{\Delta_A}{\Delta}\tilde{\eta}'(\delta),\quad g_1=\frac{\Delta_1}{\Delta}\tilde{\eta}'(\delta),\quad g_2=\frac{\Delta_2}{\Delta}\tilde{\eta}'(\delta). \quad (20)$$

Here

$$\Delta_A=(B_{2,2}-1)(B_{3,3}-1)-B_{2,3}B_{3,2},$$
$$\Delta_1=-[B_{2,1}(B_{3,3}-1)-B_{2,3}B_{3,1}],\ \Delta_2=B_{2,1}B_{3,2}-B_{3,1}(B_{2,2}-1),$$
$$\Delta=\begin{vmatrix}1-B_{1,1} & -B_{1,2} & -B_{1,3}\\ B_{2,1} & B_{2,2}-1 & B_{2,3}\\ B_{3,1} & B_{3,2} & B_{3,3}-1\end{vmatrix}. \quad (21)$$

Substituting (20) into (18), we find the function $\tilde{\eta}'(\zeta)$. After integrating the resulting expression over ζ from 0 to δ and taking into account $\tilde{\eta}(0)=0$, we obtain the dispersion equation

$$\frac{1}{\Delta}\left\{\left[\sum_{i=1}^{4}\frac{\left(\alpha_i^2+\Omega^2\right)^2[\exp(\alpha_i\delta)-1]}{\alpha_i^2\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}+\frac{\Omega^4\delta}{\prod\limits_{i\neq j}(-\alpha_i)}\right]\Delta_A+\left[\sum_{i=1}^{4}\frac{\left(\alpha_i-i\Omega\right)^2[\exp(\alpha_i\delta)-1}{\alpha_i\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}\right]\Delta_1-\right.$$
$$\left.-\left[\sum_{i=1}^{4}\frac{\left(\alpha_i-i\Omega\right)^2(\alpha_i+i\Omega)[\exp(\alpha_i\delta)-1]}{\alpha_i\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}\right]\Delta_2\right\}=0. \tag{22}$$

The solutions of Eq. 22 are the dependences of the growth rate Γ and the frequency ω of the eigen mode on the magnitude of the inter-electrode gap δ (the so-called dispersion branches). We have calculated and constructed some dispersion branches. In particular, several aperiodic branches, i.e. dependencies of $\Gamma(\delta)$ at $\omega = 0$, are shown in Fig. 1. It can be seen that they intersect the axis of $\Gamma = 0$, i.e. solutions corresponding to the values of δ with $\Gamma > 0$, are unstable. Calculations show that the growth rates of all oscillatory branches lie below the $\Gamma = 0$ axis, i.e., all oscillatory perturbations are stable.

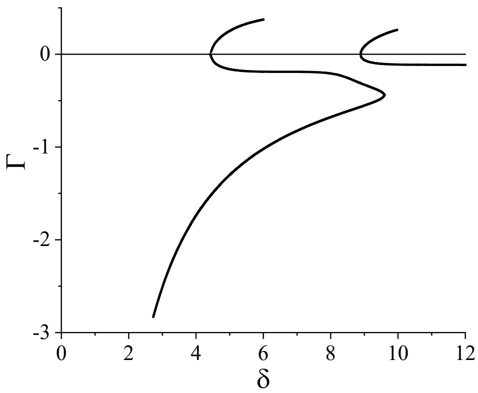

Fig. 1. Dependence of the growth rate on the inter-electrode distance for the first two aperiodic branches

We found the exact δ values relevant to the points where the growth rate vanishes. For this purpose, the roots α_i, as well as the values Δ, Δ_A, Δ_1, and Δ_2 at $|\Gamma| << 1$, $\omega = 0$ were calculated. As a result, the dispersion Eq. (22) was reduced to the form

$$\tan\left(\delta/\sqrt{2}\right)=0. \tag{23}$$

The roots of this equation are $\delta_k = \sqrt{2}\pi k$, $k = 1, 2, 3,...$. The threshold of solution stability with respect to small aperiodic disturbances corresponds to the minimum value of k and is equal to

$$d=\sqrt{2}\pi\lambda_D. \tag{24}$$

This value turned out to be $\sqrt{2}$ times greater than the Pierce threshold. It should be noted that in the Pierce diode, during the development of the disturbance, positively charged particles (ions) are considered immovable. On the other hand, in a diode with counter flows of electrons and positrons, where the masses of positively and negatively charged particles are the same, all particles take part in the process of instability development. This leads to the fact that the instability threshold of such a diode differs from the Pierce one.

Conclusion

The stability features of steady-state solutions in a diode with counter flows of electrons and positrons in the mode when all charged particles reach the opposite electrode is studied. The equation for the electric field perturbation evolution is derived. For a steady-state solution with a homogeneous field distribution, its analytical solution is found and the dispersion equation is obtained. Solutions of this equation show that there is a threshold for the gap value, when exceeded, an aperiodic instability develops in the diode plasma.

Thus, we have taken the first step towards elucidating the nature of pulsar RF radiation. Further, it is necessary to study the stability features of inhomogeneous steady-state solutions, as well as solutions that have extremes on the potential distribution that reflect a portion of the flow of charged particles. After that, numerical calculations need to be carried out to understand in which states the process of instability development ends. This will allow us to determine the frequencies of the electric field oscillations in the electron-positron plasma, which may be associated with the RF radiation of pulsars.

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