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Mathematical modeling of effects of plasma and gravitational inhomogeneities in the structure of electromagnetic signals

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Abstract. The three-dimensional algorithm of calculation of the trajectory characteristics of electromagnetic signals in a random-inhomogeneous space plasma placed in the gravity field of difficult configuration has been suggested. The influence of the gravity on the signal propagation has been taken into account by the use of efficient index of refraction. The inhomogeneities of plasma have been defined by the model of spatial correlation function of fluctuations of index of refraction. Results of mathematical modeling of lensing of electromagnetic signals in the gravity field from several space objects have been provided. It is shown that, a significantly different spatial structure of ray field can occur in picture plane of the observer depending on the properties of the gravitational field and parameters of random plasma inhomogeneities.

Keywords: mathematical modeling, lensing effect, geometrical optics approximation, gravitational field

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Математическое моделирование эффектов плазменных и гравитационных неоднородностей в структуре электромагнитных сигналов

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Аннотация. Предложен трехмерный алгоритм расчета траекторных характеристик электромагнитных сигналов в случайно-неоднородной космической плазме, помещенной в гравитационное поле сложной конфигурации. Влияние гравитации на распространение сигнала учтено путем введения эффективного показателя преломления вакуума. Неоднородности плазмы заданы моделью пространственной корреляционной функции флуктуаций показателя преломления. Приведены результаты математического моделирования линзирования электромагнитных сигналов в гравитационном поле от нескольких космических объектов. Показано, что в зависимости от свойств гравитационного поля и параметров случайных неоднородностей плазмы может возникать существенно разная пространственная структура лучевого поля в картинной плоскости наблюдателя.

Ключевые слова: математическое моделирование, линзирующий эффект, приближение геометрической оптики, гравитационное поле

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Introduction

As known [1], when interpreting data of measurements of electromagnetic signals of remote space sources, it is necessary to take into account the influence of inhomogeneities of medium, through which they have propagated. In particular, the plasma of the Solar system and interstellar space can significantly distort spatial-time structure of received signals. The electromagnetic signals propagating near massive space objects are also affected by the gravity fields of these objects at interstellar distances. Analysis of the collective influence of the plasma and gravity disturbance on the propagation of the signals of the space sources is important.

In complex gravity field created by system of stars, analytical calculation of the characteristics of received signals distorted by interstellar space is the great problem [2, 3]. Meanwhile, as follows from the theory of relativity [4, 5], the propagation of electromagnetic waves in the gravity field can be considered as task of waves in the Euclidian space. At the same time it is necessary to take into account, that gravity field changes in the certain way index of refraction of vacuum. Since the typical scales of such index of refraction are usually large is comparison to the wavelength the direction of wave propagation can be calculated with high accuracy in the ray approximation [6]. When the interstellar distances are considered then intersection of rays is possible leading to the effect of gravity lensing [7, 8]. One of these examples can be lensing of the radiation of the quasar on galaxies located on the propagation path. Random inhomogeneities of space plasma hide the effects of the gravity field and impose restrictions on wavelength of electromagnetic waves for observations of the gravity focusing. Thus effect of lensing of the gravity field in the electromagnetic waves propagation in random-inhomogeneous media can be realized for certain range of the electromagnetic scale.

Mathematical tool

To analyze the effect of influence of plasma and gravitational inhomogeneities on the trajectory characteristics of signals from remote space sources, a stochastic system of ray differential equations in the spherical case was used:

$$\frac{dR}{d\varphi} = R\cot\beta; \quad \frac{d\delta}{d\varphi} = \tan\alpha;$$
$$\frac{d\beta}{d\varphi} = \left(1+\sin^2\beta\tan^2\alpha\right)\left(\frac{1}{\tilde{n}}\frac{\partial\tilde{n}}{\partial\varphi}\cot\beta - \frac{R}{\tilde{n}}\frac{\partial\tilde{n}}{\partial R} - 1\right); \tag{1}$$
$$\frac{d\alpha}{d\varphi} = \left(1+\cot^2\beta\cos^2\alpha\right)\left(\frac{1}{\tilde{n}}\frac{\partial\tilde{n}}{\partial\delta} - \frac{1}{\tilde{n}}\frac{\partial\tilde{n}}{\partial\varphi}\tan\alpha\right);$$

where R, φ, δ are spherical coordinates of the ray, α, β are the angles of the refraction accordingly at the vertical and horizontal planes. is the efficient index of refraction of random-inhomogeneous medium in presence of the gravity field. Let us present as a sum:

$$\tilde{n} = n_0 + \tilde{n}_1, \tag{2}$$

where characterizes index of refraction of vacuum in conditions of influence of deterministic gravity field, describes the random inhomogeneities of space plasma. In the case of a single gravitational object, the function is represented as [9]:

$$n_0 = 1 + R_g / R, \tag{3}$$

where R_g is gravity radius of the space objects being researched.

The following model for is used at the research of propagation electromagnetic waves in the presence of several gravity objects:

$$\begin{gathered} n_0 = 1 + R_g / R + \\ + \sum_{i=1}^{N} A_i \exp\left[-b1_i \left(\varphi - \varphi 0_i\right)^2 - b2_i \left(\delta - \delta 0_i\right)^2 - b3_i \left(R - R0_i\right)^2 \right], \end{gathered} \tag{4}$$

where N is quantity of the additional objects, A_i, $\varphi 0_i$, $\delta 0_i$, $R0_i$, $b1_i$, $b2_i$, $b3_i$ are intensity, coordinates of localization and scales of inhomogeneity of index of refraction, accordingly, appearing due to influence of gravity object i.

To estimate collective influence of gravity field and random inhomogeneities of plasma on the propagation of the electromagnetic waves system (1) was solved by the perturbation method at . As a result the generating system of the equations was obtained (system (1) at). It describes trajectories of rays in the gravity field in absence of the influence of random plasma inhomogeneities.

Also system of equations for calculating of fluctuations of trajectory characteristics of electromagnetic waves was obtained. The second statistic moments of ray trajectories were modeled by this system. In particular, we have the following system of equations to calculating of deviation of refraction of δ in case of the Gaussian correlation function of plasma inhomogeneities:

$$\begin{gathered} \frac{d\sigma_\delta^2}{d\varphi} = \frac{\mu}{4}\sqrt{\frac{\pi}{Q}}\left(\frac{DP^2}{Q} + 16\left(D - \frac{K}{Q}\right)\left(\varphi J_1 - J_2\right)\right); \\ \frac{dJ_1}{d\varphi} = P^2; \ \frac{dJ_2}{d\varphi} = \varphi P^2. \end{gathered} \tag{5}$$

where $P(\varphi) = \left(\frac{1}{\cos^2\alpha_0} + \cot^2\beta_0\right)$; $D = \frac{\tan^2\alpha_0}{a_\varphi^2} + \frac{1}{a_\delta^2}$; $K = \left(\frac{1}{a_\varphi^2} - \frac{1}{a_\delta^2}\right)^2 \tan^2\alpha_0$; $Q = \frac{1}{a_\varphi^2} + \frac{1}{a_\delta^2}\tan^2\alpha_0 + \frac{R_0^2}{a_R^2}\cot^2\beta_0$, μ, a_φ, a_δ, a_R are intensity and scales of random plasma inhomogeneities, respectively. Characteristics α_0, β_0, R_0, δ_0 are defined by numerical integration of system (1) at $\tilde{n}_1 = 0$.

One can estimate the influence of the random inhomogeneities of plasma on the gravity lensing solving obtained system of equations (5) together with generating system (system (1) at $\tilde{n}_1 = 0$).

Results of mathematical modelling and their discussion

The generating system was numerically solved at the various parameters of index of refraction (4). The results of the calculations of density of points of arrival of ray trajectories on fix distance $R = R_k$ are shown on the picture plane of the observer in coordinates (δ, φ). For example, in Figs. 1, 2, accordingly, the results of modeling of ray trajectories and density of rays in picture plane of the observer in presence of two gravity objects are shown. The localization of the objects relatively source is schematically shown in Fig. 3. Parameters of objects amounted: R_g=1 cul, A_1=0.5,

$R0_1$ = 10 cul, $\delta 0_1$ = 0 rad, $\varphi 0_1$ = 0.6 rad, scales $b1_1$ = 0.5, $b2_1$ = 0.5, $b3_1$ = 0.5(1/cul^2). Here cul is conventional unit of length. Initial states: $R(\varphi$ = 50 = (0 cul, $\delta(\varphi = 0) = 0$. Angle α belonged to the range [—0.14; 0.14] rad with step 0.006 rad, angle β belonged to the range [—0.3; -0.21] rad and [0.21; 0.3] rad with step 0.004 rad. The calculated points of arrival or rays are provided in the picture plane (fig.2) for mentioned sectors of angles α, β around gravity objects. Also here the end angles φ are noticed, under which the ray arrives the distance R_k=50 cul. The lack of the values of φ from 0 till 2± rad occurs due to the impossibility of passing of rays through gravity objects. It is easy to see that the trajectory picture significantly distorts in gravity field of objects, and phenomenon of gravity focusing is possible. The closer an electromagnetic wave propagates near a gravitational object, the stronger the refraction effect. Also one can notice that the form of the trajectories of rays between two gravitational objects is non-monotone for some sector of the angles (Fig. 1). The results of the calculating of trajectories and density of distribution of the points of arrival of rays at picture plane in Euclidean space at n_0 = 1 are shown to estimate in Figs. 4, 5. In this situation the points of arrival of rays evenly distribute at the picture plane in researched sector of the angles α (0), β (0).

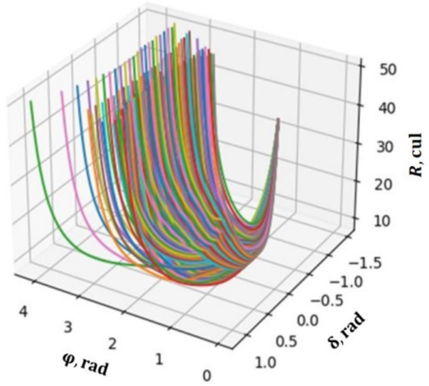

Fig. 1. Trajectories of propagation of electromagnetic waves
in gravity field of two space objects

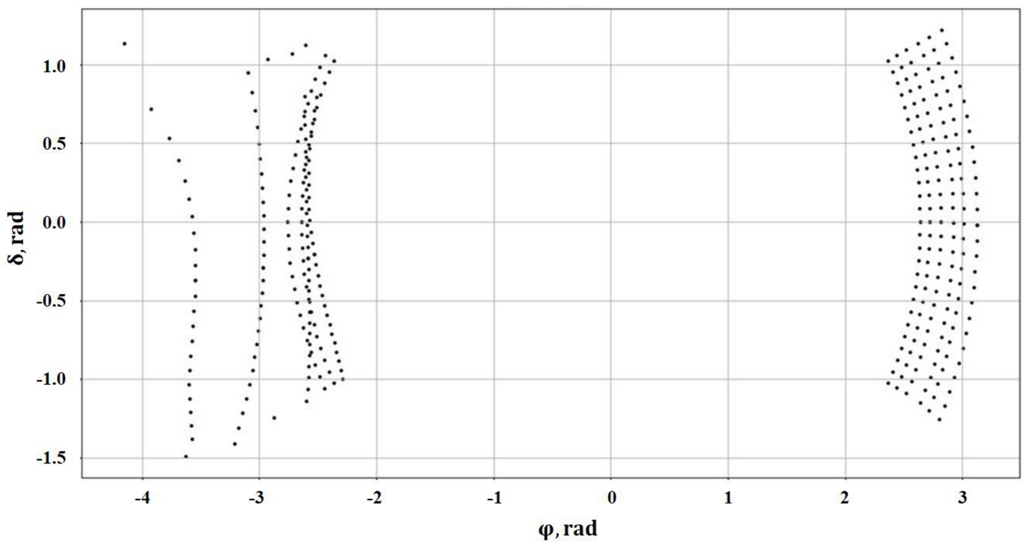

Fig. 2. The gravity lensing in the observer plane

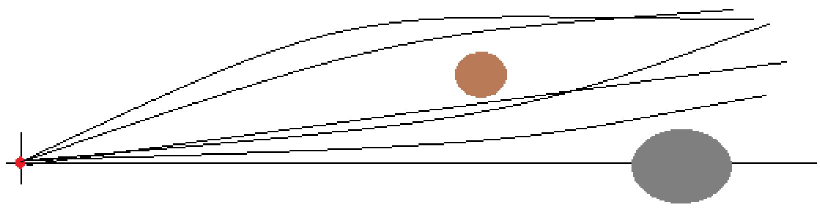

Fig. 3. Location of the source and gravity space objects. Side projection

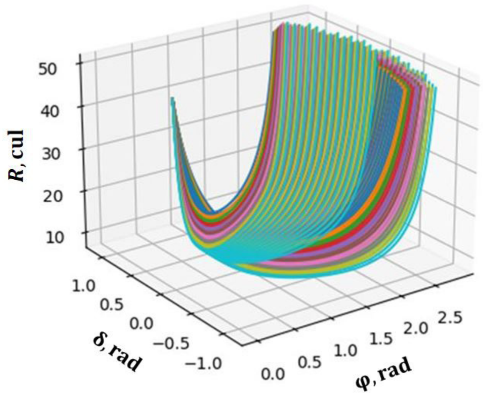

Fig. 4. The trajectories of electromagnetic waves in the absence of the gravity space object

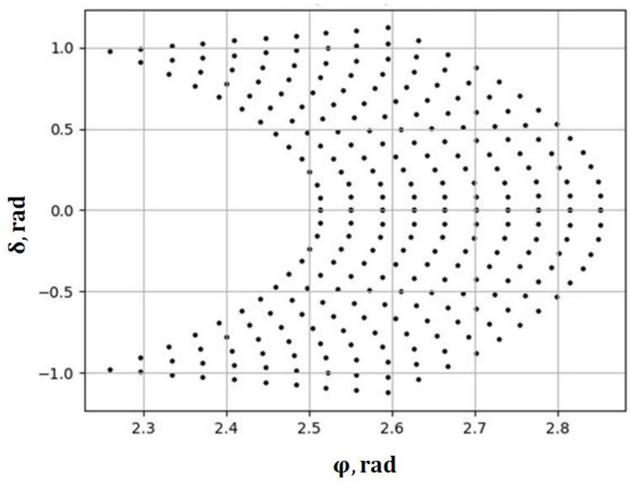

Fig. 5. Picture plane of the observer in the absence of the gravity space object

The results of the calculating of the fluctuations of the density of points of arrival of rays in picture plane are shown on fig.6. Here the calculated mean points of arrival of rays are shown for mentioned sectors of angles α, β in coordinates (δ, φ) for the same parameters of model (4) as on fig.2. Vertical lines demonstrate the standard deviations of obtained points of arrival in the picture plane triggered plasma random inhomogeneities. Parameters of plasma inhomogeneities amounted: $\mu = 10^{-6}$, $a_\varphi = 10^{-3}$, $a_\delta = 10^{-3}$, $a_R = 10^{-3}$ cul. One can notice that in the picture plane of the observer the statistical blurring of the gravity focusing appears. The smaller the path of propagation of electromagnetic radiation, the smaller the scatter of the point on the observer picture plane is observed.

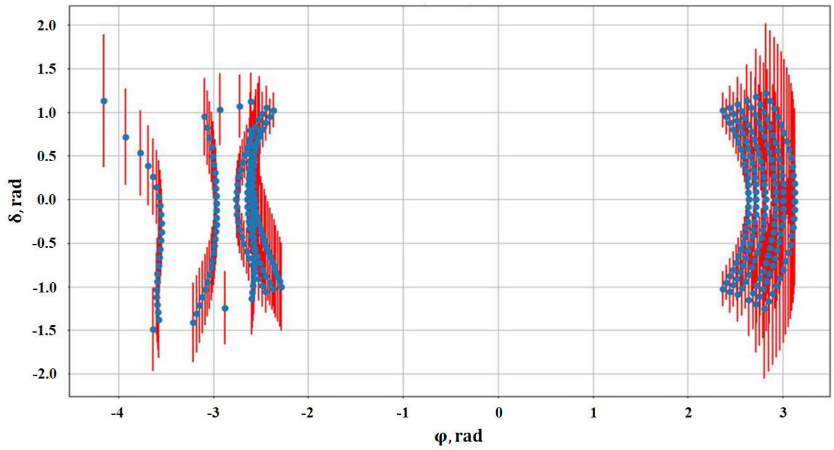

Fig. 6. Statistical blurring of gravity lensing in the observer plane

Conclusion

Mathematical modeling of the trajectory characteristics of the electromagnetic signals in difficult gravity field of several space objects has been performed by created three-dimensional algorithm. The random-inhomogeneous plasma has been taken into account in the calculations of this algorithm. The conditions of the appearance of the effect of the gravity lensing in picture plane of the observer have been showed. Calculation of standard deviation values of side deviations of rays under the influence of random inhomogeneities of plasma has been completed. Example of statistical blurring of effect of gravity focusing of rays on several space objects due to the refraction scattering on inhomogeneities of space plasma have been provided.

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