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Electromagnetic fields of regular rotating electrically charged objects in nonlinear electrodynamics minimally coupled to gravity

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Abstract. We present a brief overview of the main properties of electromagnetic fields of regular rotating electrically charged objects in non-linear electrodynamics minimally coupled to gravity (NED-GR). The basic features of electromagnetic fields follow from the analysis of the regular solutions to the NED-GR dynamic equations. For NED-GR regular objects the Lagrangian inevitably branches at a single minimum of the field invariant F. The study of the asymptotic of the solutions of the field equations at $r \to 0$ reveals the fundamental features of the electromagnetic dynamics on the de Sitter vacuum disk ($r = 0$) in the deep interiors of rotating NED-GR objects. The disk has the properties of a perfect conductor and an ideal diamagnetic, zero magnetic induction, and is confined by a ring with a superconducting current, which replaces the Kerr ring singularity, serves as a non-dissipative source of electromagnetic fields of NED-GR regular objects and provides the origin of their intrinsic magnetic momenta.

Keywords: non-linear electrodynamics, Lagrangian, asymptotic of the solutions

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Материалы конференции
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Электромагнитные поля регулярных вращающихся электрически заряженных объектов в нелинейной электродинамике, минимально связанной с гравитацией

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Аннотация. Приведен краткий обзор основных свойств электромагнитных полей регулярных вращающихся электрически заряженных объектов в нелинейной электродинамике, минимально связанной с гравитацией (NED-GR). Основные черты электромагнитных полей следуют из анализа регулярных решений динамических уравнений NED-GR. Для обычных объектов NED-GR Лагранжиан неизбежно разветвляется в единственной точке минимума инварианта поля F. Изучение асимптотики решений уравнений поля при r, стремящемся к нулю, раскрывает основные черты электромагнитной динамики на вакуумном диске де Ситтера ($r = 0$) в глубоких недрах вращающихся объектов NED-GR. Диск обладает свойствами идеального проводника и идеального диамагнетика с нулевой магнитной индукцией и ограничен кольцом со сверхпроводящим током, заменяющим сингулярность кольца Керра, и, кроме того, служит недиссипативным источником электромагнитных полей для регулярных объектов NED-GR и обеспечивает возникновение их собственных магнитных моментов.

Ключевые слова: нелинейная электродинамика, Лагранжиан, асимптотика решений

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Introduction

Electrically charged objects related by electromagnetic and gravitational interactions are described in general setting by nonlinear electrodynamics coupled to gravity (NED-GR). Nonlinear electrodynamics (NED) was proposed by Born and Infeld in 1934 with the aim to describe electromagnetic field and particles in the unique common frame which provides finite values for physical quantities, and presents an appropriate model of the electron [1]. In their theory electromagnetic energy has made finite by imposing an upper limit on the electric field related to the electron size, but geometrical quantities remained singular.

The Born-Infeld program can be realized in the frame essentially including gravity. Source-free NED-GR equations admit the class of regular axially symmetric solutions asymptotically Kerr-Newman for a distant observer [2–4], which describe not only electromagnetic spinning solitons [5] (for a review see [6]) but also regular rotating electrically charged black holes.

Axially symmetric metrics are typically obtained from the spherical metrics of the Kerr–Schild class [7] by applying the Gürses–Gürsey formalism [8]. For this class of metrics the source terms have the algebraic structure [5]

$$T_t^t = T_r^r \ \ (p_r = -\rho). \tag{1}$$

In the Boyer–Lindquist coordinates the axially symmetric metric reads [8]

$$\begin{gathered} ds^2 = \frac{2f-\Sigma}{\Sigma}dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 - \frac{4af\sin^2\theta}{\Sigma}dtd\phi + \\ +\left(r^2 + a^2 + \frac{2fa^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2. \end{gathered} \tag{2}$$

Here a is the angular momentum, the Lorentz signature is [- + + +], $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 + a^2 - 2f(r)$, and $f(r) = r\int_0^r \tilde{\rho}(x)x^2dx$ where $\tilde{\rho}$ is the density profile of a related spherical solutions. The surfaces of constant r are the oblate confocal ellipsoids $r^4 - (x^2 + y^2 + z^2 - a^2)r^2 - a^2z^2 = 0$ which degenerate, for $r = 0$ to the equatorial disk $x^2 + y^2 \le a^2$, $z = 0$ bounded by the ring $x^2 + y^2 = a^2$, $z = 0$.

Spacetime can have at most two horizons defined by $\Delta(r) = 0$, and at most two ergospheres which are surfaces of a static limit $g_{tt} = 0$ [2]. Ergospheres confine ergoregions where $g_{tt} < 0$ which makes possible extraction of rotational energy. Geometrical structure of a regular rotating object with the metric (2) is determined by the weak energy condition (WEC) which requires $\rho > 0$ and $\rho + p_\perp \ge 0$. The density and pressures are related by [2]

$$\begin{gathered} \rho(r,\theta) + p_\perp(r,\theta) == \frac{r^2}{\Sigma}\left[2\left(\frac{r^2}{\Sigma} - 1\right)\tilde{\rho}(r) - \frac{r}{2}\tilde{\rho}'\right] = \\ = \frac{r|\tilde{\rho}'|}{2\Sigma^2}S(r,z);\ S(r,z) = r^4 - \frac{2a^2z^2}{r|\tilde{\rho}'|}(\tilde{\rho} - \tilde{p}_\perp). \end{gathered} \tag{3}$$

The prime denotes the derivative with respect to r. In the equatorial plane $r^2/\Sigma = 1$, and $(p_\perp + \rho) = -r\tilde{\rho}'(r)/2$. For regular spherical solutions satisfying WEC $\tilde{\rho}' \le 0$ and $r\tilde{\rho}'(r) \to 0$ at $r \to 0$. Equation of state takes the form $p = -\rho$ and describes the de Sitter vacuum in the co-rotating frame. The interior de Sitter vacuum disk is the generic feature of all regular rotating objects of this class [5, 2].

If the function $S(r,z)$ in (3) vanishes only on the disk $r = 0$, WEC is satisfied. This type of structure is shown in Fig. 1 (Left) together with the horizons r_+, r_- and the ergosphere. There can exist an additional surface of the de Sitter vacuum $p_\perp + \rho = 0$, S-surface which incorporates the de Sitter disk as a bridge [2]. Then WEC is violated in the cavities between the S-surface and the disk, filled with an anisotropic phantom fluid, $p_r = -\rho$; $p_\perp = w_\perp\rho$ with $w_\perp < -1$ [3]. This type of interiors is shown in Fig. 1 (Middle) for the case $\alpha < \pi\beta^2/8$, and in Fig. 1 (Right) for the case $\alpha > \pi\beta^2/8$, where $\alpha = a/m$ is the specific angular momentum and $\beta = q/m$ is the specific charge. The parameter r_q denotes the characteristic radius $r_q = \pi q^2/8m$ for the regularized Newtonian/Coulomb profile applied for pictures [2].

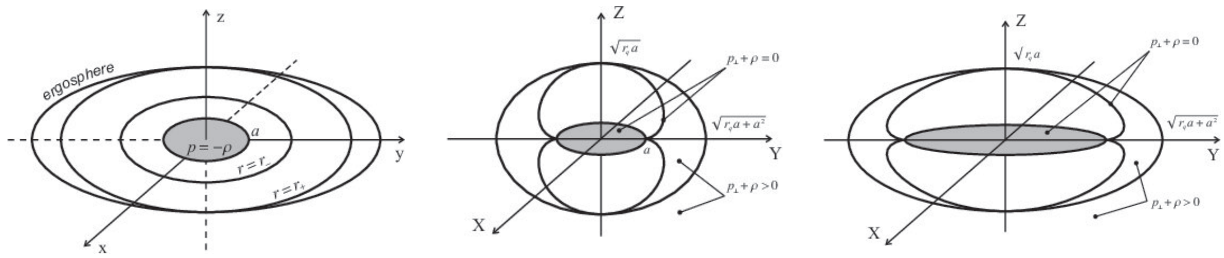

Fig. 1. Typical structure satisfying WEC (Left), and two cases of typical structure violating WEC: the cases with $\alpha < \pi\beta^2/8$ (Middle) and $\alpha > \pi\beta^2/8$ (Right)

NED-GR solutions for electrically charged objects belong to this class automatically since for any gauge-invariant Lagrangian L(F) stress-energy tensor of an electromagnetic field, $T^\mu_\nu = -2\mathcal{L}_F F_{\nu\alpha}F^{\mu\alpha} + 0.5\delta^\mu_\nu\mathcal{L}$ has the algebraic structure specified by (1) [9, 5]. In the Maxwell weak field limit the metric tends to the Kerr–Newman metric with $f(r) = mr - q^2/2$, where m is the mass of an object and q its electric charge.

The first problem encountered by regular electrically charged NED-GR objects, is the problem of their existence itself, which appears forbidden by "the nonexistence theorems" [10].

This existential problem is addressed in Section 2 where we outline the typical behavior of Lagrangians. Section 3 presents the basic features of regular solutions for electromagnetic fields in the limit $r \to 0$ and $r \to \infty$, and Section 4 contains conclusions.

Typical behavior of the Lagrangian for NED-GR regular objects

In the minimally coupled NED-GR the action is given (in geometrical units $c = G = 1$) by

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,[R - \mathcal{L}(F)];\ F = F_{\mu\nu}F^{\mu\nu}. \tag{4}$$

Here R is the scalar curvature, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field, and A_μ is the Maxwell limit of electromagnetic potential. The gauge-invariant electromagnetic Lagrangian $\mathcal{L}(F)$ should have the Maxwell limit, $\mathcal{L} \to F$, $\mathcal{L}_F = d\mathcal{L}/dF \to 1$ in the weak field regime. Variation with respect to A_μ and the Bianchi identities yield the dynamic field equations

$$\nabla_\mu(L_F F^{\mu\nu}) = 0;\ \nabla_\mu {}^*F^{\mu\nu} = 0, \tag{5}$$

where ${}^*F^{\mu\nu} = 1/2\eta^{\mu\nu\alpha\beta}F_{\alpha\beta}$, and the antisymmetric unit tensor is defined as $\eta_{0123} = \sqrt{-g}$.

In the spherically symmetric cases the only essential component of $F_{\mu\nu}$ describes a radial electric field $F_{01} = -E(r)$. Dynamical Eqs. (5) give $r^2\mathcal{L}_F F^{01} = q$ [10] where q is constant of integration identified as an electric charge. The field invariant F is given by

$$F = 2F_{01}F^{01} = -\frac{2q^2}{\mathcal{L}_F^2 r^4}. \tag{6}$$

For electrically charged NED-GR structures the density and pressures are given by [9]

$$\rho = -p_r = (\mathcal{L}/2 - F\mathcal{L}_F);\ p_\perp = -\mathcal{L}/2;\ p_\perp + \rho = -F\mathcal{L}_F. \tag{7}$$

WEC imposes two general constraints on the Lagrangian $\mathcal{L}(F)$. Equation (7) leads to

$$\mathcal{L}_F \geq 0;\ \mathcal{L}(F) \geq 2F\mathcal{L}_F. \tag{8}$$

The theorem of non-existence of electrically charged structures with the regular center [10] requires the Maxwell behavior at the center, $\mathcal{L} \to F$, $\mathcal{L}_F \to 1$ as $F \to 0$ [10]. The proof is that regularity of stress-energy tensor requires $|F\mathcal{L}_F| < \infty$ as $r \to 0$, while $F\mathcal{L}_F^2 \to -\infty$ by virtue of (6). It follows that $\mathcal{L}_F \to 0$, while $F \to 0$ which is not compatible with the Maxwell behavior.

In fact, regularity and WEC suggest existence of regular electrically charged structures without the Maxwell limit in the center. Regularity requires $\rho < \infty$. WEC requires $\rho \geq 0$ and $\rho + p_\perp \geq 0$ which leads to $\rho' \leq 0$ since the tangential pressure satisfies $\rho + p_\perp = -r\rho'/2$. As a result, the electromagnetic density $T_t^t = \rho$ achieves its maximum, and one cannot expect the weak field behavior in the region of the maximal density. De Sitter vacuum in the centers of regular solutions implies $p_\perp + \rho = 0$ which leads to $F\mathcal{L}_F = 0$ at $r = 0$. It follows, taking into account (8) and (6), that $\mathcal{L}_F \to +\infty$ while $F \to -0$ when $r \to 0$.

Conditions for a Lagrangian and its derivative in the regular center are thus

$$\mathcal{L}(0) = 2\rho(0);\ \mathcal{L}_F \to \infty. \tag{9}$$

The Maxwell asymptotic at $r \to +\infty$ imposes two conditions on Lagrangian in the limit $F \to -0$

$$\mathcal{L} \to F \to -0;\ \mathcal{L}_F \to 1. \tag{10}$$

The invariant F evolving between $F = -0$ at the center and at infinity, is not monotonic function, which leads unavoidable to branching of a Lagrangian [10, 9]. Lagrangian on its way from (9) to (10) must change its sign; according to (7), it is opposite to the sign of the pressure $p_\perp$ which can vanish only once for the case of one de Sitter vacuum scale [9].

As a result a Lagrangian has two branches, and the action takes the form [11]

$$S = S_{int} + S_{ext} = \\ = \frac{1}{16\pi}\left[\int_{\Omega_{int}} (R - \mathcal{L}_{int}(F))\sqrt{-g}\,d^4x + \int_{\Omega_{ext}} (R - \mathcal{L}_{ext}(F))\sqrt{-g}\,d^4x\right]. \tag{11}$$

Each region of the manifold, Ω_{int} and Ω_{ext}, is confined by the space-like hypersurfaces $t = t_{in}$ and $t = t_{fin}$ and by the time-like 3-surface at infinity, where electromagnetic fields vanish in the Maxwell limit. Internal boundary between Ω_{int} and Ω_{ext} is defined as a time-like hypersurface Σ_c at which the field invariant F achieves its minimum. In the case of the minimal coupling variation in the action (11) results in the dynamical Eqs. (5) in both regions Ω_{int} and Ω_{ext}, and in the boundary conditions on the surface Σ_c [11].

$$\int_{\Sigma_c} \left(\mathcal{L}_{F\,(int)} F_{\mu\nu\,(int)} - \mathcal{L}_{F\,(ext)} F_{\mu\nu\,(ext)}\right)\sqrt{-g}\,\delta A^\mu d\sigma^\nu = 0, \tag{12}$$

$$\mathcal{L}_{(int)} - 2\mathcal{L}_{F\,(int)} F_{int} = \mathcal{L}_{(ext)} - 2\mathcal{L}_{F\,(ext)} F_{ext}. \tag{13}$$

Eq. (6) defines in the first approximation the derivative $d\mathcal{L}_F/dF$ in the minimum $r = r_c$ of the invariant F by $\mathcal{L}_{FF} = -2\mathcal{L}_F/(Fr_c)$. In accordance with (8), $\mathcal{L}_F$ has the same finite limit as $F \to F_c + 0$ and $F \to F_c - 0$, while F changes its sign, so that $\mathcal{L}_{FF}$ tends to infinities of opposite signs, and a Lagrangian $\mathcal{L}(F)$ has the cusp at $F = F_c$.

According to (8), the Lagrangian $\mathcal{L}(F)$ is a monotonic function of F which decreases smoothly along the first branch from its maximal value $\mathcal{L}(0) = 2\rho(0)$ to its value in the cusp $\mathcal{L}_c$ at $F = F_{min} = F_c$, and then increases along the second branch from its minimal value $\mathcal{L}_c < 0$ to its Maxwell limit $\mathcal{L} \to F \to -0$ as F increases from F_c to $F \to -0$ as $r \to \infty$. Typical behavior of the Lagrangian and its derivatives dependently on F is shown in Fig. 2 [3].

Similar behavior of $\mathcal{L}(F)$ is generic for regular axially symmetric solutions where the invariant F evolves between $F = -0$ on the disk $r = 0$ [2] and in the Maxwell limit at $r \to \infty$ [12].

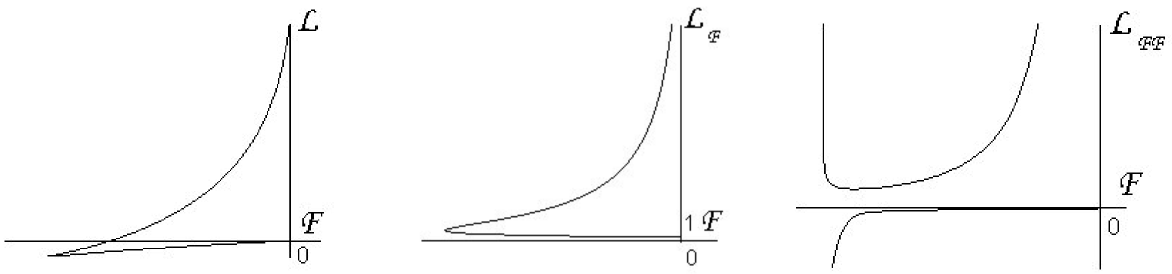

Fig. 2. Typical behavior of the Lagrangian (Left), Lagrangian derivative (Middle), and the second Lagrangian derivative (Right)

Electromagnetic fields

In the axially symmetric geometry the non-zero field components are F_{01}, F_{02}, F_{13}, F_{23}, related in the metric (2) by $F_{31} = a\sin^2\theta F_{10}$; $F_{23} = (r^2 + a^2)F_{02}$. As a result, Eqs. (5) form the system of four equations for two independent functions [2]

$$\frac{\partial}{\partial r}[(r^2+a^2)\sin\theta\mathcal{L}_F F_{10}] + \frac{\partial}{\partial\theta}[\sin\theta\mathcal{L}_F F_{20}] = 0;$$
$$\frac{\partial}{\partial r}\left[\frac{\mathcal{L}_F F_{31}}{\sin\theta}\right] + \frac{\partial}{\partial\theta}\left[\frac{\mathcal{L}_F F_{32}}{(r^2+a^2)\sin\theta}\right] = 0; \tag{14}$$

$$\frac{\partial F_{23}}{\partial r} + \frac{\partial F_{31}}{\partial\theta} = 0; \quad \frac{\partial F_{01}}{\partial\theta} + \frac{\partial F_{20}}{\partial r} = 0. \tag{15}$$

Solutions to this system should satisfy the compatibility condition [2]

$$\frac{\partial}{\partial r}\left(\frac{1}{\mathcal{L}_F}\frac{\partial\mathcal{L}_F}{\partial\theta}\right)\frac{\partial}{\partial\theta}\left(\frac{1}{\mathcal{L}_F}\frac{\partial\mathcal{L}_F}{\partial r}\right) + \frac{4a^2\sin^2(\theta)}{\Sigma^2}\frac{1}{\mathcal{L}_F^2}\left[r\frac{\partial\mathcal{L}_F}{\partial r} + \cot(\theta)\frac{\partial\mathcal{L}_F}{\partial\theta}\right]^2 = 0, \tag{16}$$

as the necessary and sufficient condition of compatibility of Eqs. (14)–(15) and the necessary condition for the existence of solutions [2].

Equations (14–15) and compatibility condition (16) are satisfied by the functions [5]

$$\Sigma^2(\mathcal{L}_F F_{01}) = -q(r^2 - a^2\cos^2\theta); \; \Sigma^2(\mathcal{L}_F F_{02}) = qa^2 r\sin 2\theta; \tag{17}$$

$$\Sigma^2(L_F F_{31}) = aq\sin^2\theta\,(r^2 - a^2\cos^2\theta); \;\; \Sigma^2(L_F F_{23}) = aqr(r^2+a^2)\sin 2\theta \tag{18}$$

in the weak field limit $\mathcal{L}_F = 1$, where they coincide with the solutions to the Maxwell-Einstein equations [13, 14, 15], and in the strongly nonlinear regime as the asymptotic solutions in the limit $\mathcal{L}_F \to \infty$ [2]. In terms of the field intensities defined as $E_j = \{F_{j0}\}$; $D^j = \{\mathcal{L}_F F^{0j}\}$; $B^j = \{{}^*F^{j0}\}$; $H_j = \{\mathcal{L}_F {}^*F_{0j}\}$; $j = 1, 2, 3$ [16, 5], the dynamical Eqs. (5) take the form of the source-free Maxwell equations

$$\nabla \cdot \mathbf{D} = 0; \ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}; \ \nabla \cdot \mathbf{B} = 0; \ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \tag{19}$$

The electric induction **D** and the magnetic induction **B** are related with the field intensities by $D^j = \varepsilon^j_k E^k$; $B^j = \mu^j_k H^k$ where ε^k_j and μ^k_j are the tensors of the electric and magnetic permeability given by [5]

$$\varepsilon^r_r = \frac{(r^2+a^2)}{\Delta}\mathcal{L}_F; \ \varepsilon^\theta_\theta = \mathcal{L}_F; \ \mu^r_r = \frac{(r^2+a^2)}{\Delta \mathcal{L}_F}; \ \mu^\theta_\theta = \frac{1}{\mathcal{L}_F}. \tag{20}$$

The relation connecting density and pressure with the electromagnetic fields reads [5]

$$p_\perp + \rho = 2\mathcal{L}_F \left(F_{10}^2 + \frac{F_{20}^2}{a^2 \sin^2\theta} \right). \tag{21}$$

According to (20) and (21), WEC should be satisfied for NED-GR structures, since $\mathcal{L}_F$ defines the electric permeability, which cannot be negative in electrodynamics of continuous media.

Applying (17, 18) in the limit $\mathcal{L}_F \to \infty$ we obtain

$$p_\perp + \rho = \frac{2q^2}{\mathcal{L}_F \Sigma^2}. \tag{22}$$

It follows that $L_F \to \infty$ at the S-surface, including disk where $\rho + p_\perp = 0$. This testifies for zero magnetic permeability and infinite electric permeability, i.e., for the properties of a perfect conductor and an ideal diamagnetic of the disk and S-surface. On these surfaces the magnetic induction **B** vanishes [17].

The surface current on the disk is obtained, with taking into account (17, 18) and (20), as [17]

$$j_\phi = -\frac{q}{2\pi a}\sqrt{1+q^2/a^2}\ \sin^2\xi \frac{\mu}{\cos^3\xi}, \tag{23}$$

where ξ is the intrinsic coordinate on the disk, $0 \le \xi \le \pi/2$. The magnetic permeability $\mu = 0$ on the disk by virtue of (20). As a result, the current j_ϕ is zero throughout the disk except the ring $\xi = \pi/2$, where both terms in the second fraction go to zero independently and the current on the ring can be any and amount to a non-zero value, which means that the basic condition for superconducting state [18] is satisfied.

The superconducting current (23) replaces the ring singularity of the Kerr–Newman geometry, represents a non-dissipative source of the exterior fields and of the intrinsic magnetic momentum [19], and can in principle provide a practically unlimited life time of an object [17].

Conclusions

Analysis of regular axially symmetric solutions to the NED-GR dynamic equations, which describe electrically charged regular rotating black holes and spinning electromagnetic solitons, allows us to reveal the fundamental features of electrically charged regular rotating NED-GR objects. They can have two types of interiors suggested by geometry, whose detailed properties are determined by behavior of electromagnetic fields and by the weak energy condition involving dependence on the electric permeability which is regulated by the basic requirements of electrodynamics of continuous media. Their obligatory basic features are as follows:

For regular solutions the electromagnetic invariant F is non-monotonic function evolving between $F = -0$ at $r = 0$ and at $r \to \infty$. This results in branching of a Lagrangian in the minimum of the invariant F. The basic generic feature of the regular electrically charged NEDGR structures is the existence of a characteristic surface separating regions described by different branches of Lagrangians in the non-uniform variational problem.

All NED-GR electrically charged regular rotating objects have in their deep interiors de Sitter vacuum disks $r = 0$ with the properties of a perfect conductor and an ideal diamagnetic, and zero magnetic induction.

De Sitter disk is confined by the ring with a superconducting current which serves as a non-dissipative source of electromagnetic fields of a NED-GR regular object, and provides the origin of its intrinsic magnetic momentum.

There can exist additional interior de Sitter vacuum S-surfaces with de Sitter disk as a bridge and with the properties of a perfect conductor and an ideal diamagnetic and zero magnetic induction over the whole surface.

Violation of WEC is prevented by the basic requirement of non-negativity of the electric permeability in electrodynamics of continuous media, which prefers the NED-GR structures without S-surfaces, and distinguishes admissible models for interiors of NED-GR objects with S-surfaces: shell-like models with the flat vacuum and zero fields in the cavities between S-surfaces and disks, and models with the de Sitter vacuum cores within S-surfaces, with the properties of a perfect conductor and an ideal diamagnetic and zero magnetic induction over the whole core [2, 3]. Such models require further research.

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