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Quasi-particle description of correlations and statistical memory effects in the discrete time dynamics of complex non-physical systems

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Abstract. We present the main provisions and ideas of the quasi-particle description concept of the discrete time dynamics of complex non-physical systems, developed within the frameworks of Memory Functions Formalism. The initial signal generated by a complex system is represented as time variations of the quasiparticle movement coordinate in one-dimensional space. The novelty of the proposed concept lies in the derivation of analytical expressions for chains of finite-difference equations relating time correlation functions and statistical memory functions for a sequence of derivatives of the temporal dynamics of the quasiparticle coordinates: velocity, acceleration, energy, energy flow. The proposed concept was tested for the study of correlations and statistical memory effects, as well as relaxation patterns, in the dynamics of neuromagnetic responses of healthy subjects and a patient with photosensitive epilepsy in response to a red-blue flickering stimulus. The results obtained allow establishing informationally significant sensors and the corresponding zones of localization of the human cerebral cortex for the analysis and diagnosis of photosensitive epilepsy.

Keywords: complex systems, time series analysis, Memory Functions Formalism, correlations, statistical memory effects, quasi-particle description, neuromagnetic responses, photosensitive epilepsy

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Материалы конференции
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Квазичастичное описание корреляций и эффектов статистической памяти в дискретной временной динамике сложных нефизических систем

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Аннотация. Представлены основные положения и понятия концепции квазичастичного описания дискретной временной динамики сложных нефизических систем, развиваемой в рамках формализма функций памяти. На основе техники проекционных операторов Цванцига–Мори получено аналитическое выражение цепочки конечно-разностных уравнений для набора исходных временных корреляционных функций и функций статистической памяти временных вариаций характеристик перемещения квазичастицы в одномерном пространстве. Проведена апробация предложенной концепции к исследованию корреляций и эффектов статистической памяти, а также релаксационных закономерностей в динамике нейромагнитных откликов здоровых испытуемых и пациента с фоточувствительной эпилепсией в ответ на красно-голубой мерцающий стимул.

Ключевые слова: сложные системы, анализ временных серий, формализм функций памяти, эффекты статистической памяти, квазичастичное описание, нейромагнитные отклики, фоточувствительная эпилепсия.

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Introduction

Currently, there is no strictly established definition of a complex system in the scientific literature. In most cases, a complex system is considered as an object consisting of an infinite (huge) number of interacting components. The interaction between the parts of the whole leads to unique properties that only complex systems possess. To describe the evolution and properties of complex physical and nonphysical (biological, social, economic, financial) systems, methods of statistical analysis of time signals are effectively used [1–3]. Conventionally, such methods can be divided into two large groups: methods of analysis in the time domain [4, 5] and spectral methods [6–8], which are used to study the periodic patterns of signals.

The study of the statistical memory effects in complex physical systems involves the use of the Hamiltonian [9]. The operation of statistical averaging is performed using the Gibbs distribution function or the quantum density operator. The evolution of such systems is described by integro-differential equations with infinitesimal time increments. In contrast, the stochastic dynamics of complex non-physical systems is characterized by discrete sequences of observations recorded by recording equipment with a certain discretization step [10, 11]. The discreteness of observational data leads to difficulties in the theoretical description of complex non-physical systems by classical statistical methods with continuous time. In addition, for such systems there is no possibility of representing the Hamiltonian.

In this paper, to study correlations and statistical memory effects of complex non-physical systems, we develop the main relations of the Memory Functions Formalism (MFF) for the case of a quasi-particle description. The initial discrete time dynamics of the experimental parameter is presented as a trajectory of the quasiparticle motion. In comparison with the previously obtained relations [6, 8, 12], in this work, for time sequences of quasi-derivatives (velocity, acceleration, energy, energy flow) of fixed parameters of complex systems, for the first-time systems of finite-difference equations are obtained that relate correlation functions and statistical memory functions. This approach allows extracting information not only from the original signals, but also from sequences of quasi-derivatives of a discrete type.

Methods. Quasi-particle description of complex systems within the framework of Memory Functions Formalism

Previously, in [6, 8, 12], the main provisions of the MFF were outlined, which is a finite-difference generalization of the Zwanzig-Mori formalism [13, 14]. The technique of Zwanzig and Mori projection operators is used in studying the dynamic properties of liquids and phase transitions, describing Brownian motion, calculating transport coefficients, and analyzing anomalous diffusion. Through the efforts of the Kazan scientific school under the guidance of Professor R.M. Yulmetyev, this technique was generalized to consider the discrete temporal dynamics of complex non-physical systems. The direct purpose of these methods is to study the correlations and statistical memory effects in the investigated non-physical complex systems. In this paper, we develop the MFF for the case of a quasiparticle description.

The dynamics of an experimentally recorded parameter of a complex system as a discrete time series is represented as $\{x_j\}$ of the variable X:

$$X = \{x(T), x(T+\tau), x(T+2\tau), \ldots, x(T+(N-1)\tau\}, x_j = X(T+j\tau). \tag{1}$$

where T is the initial moment of time, $(N-1)\tau$ is the total signal registration time, τ is the time step.

For clarity, the analysis of finite-difference quasi-derivatives of different orders of the original time sequence can be reduced to the following analogy. The sequence of observations x_j, recorded at time $t = T + j\tau$, can be considered as the "coordinates" of a quasi-particle with an "effective mass" m^*, moving in one-dimensional space.

The sequences of quasi-derivatives of a discrete type will be identified with a set of quantities characterizing the motion of a particle: "velocity" v_j, "acceleration" a_j, "kinetic energy" , "energy flow" q_j. To do this, we use the finite difference method.

For example, the sequence of velocity observations $\{v_j\}$would be defined as:

$$\nu = \left\{\nu_j\right\} = \left\{\nu(T), \nu(T+\tau), \nu(T+2\tau), \ldots, \nu\left(T+(N-2)\tau\right)\right\}, \nu_j = \nu(T+j2),$$
$$\nu_j = \frac{x_{j+1}-x_j}{\tau}. \tag{2}$$

By analogy, sequences of observations are found for acceleration $\{a_j\}$, energy $\{e_j\}$, energy flow $\{q_j\}$. Generalizing the obtained expressions, we represent the dynamics of the recorded parameter of a complex system in the form:

$$\xi = \left\{\xi(T), \xi(T+\tau), \xi(T+2\tau), \ldots\right\}, \ \xi = \left\{x, v, a, e, q\right\}. \tag{3}$$

For a discrete process ξ, we determine the average value $\langle\xi\rangle$, fluctuations $\delta\xi_j$, and absolute variance σ^2.

Next, we introduce a set of normalized time correlation functions (TCF) $X(t)$, $V(t)$, $A(t)$, $E(t)$, $Q(t)$ for sequences of observations $\{x_j\}$, $\{v_j\}$, $\{a_j\}$, $\{e_j\}$, $\{q_j\}$.

For example, for a sequence of observations of velocity $\{v_j\}$, TCF is presented as:

$$V(t)=\frac{1}{(N-m-1)\sigma_V^2}\sum_{j=0}^{N-m-2}\delta\nu(T+j\tau)\delta\nu(T+(j+m)\tau),$$
$$\sigma_V^2=\frac{1}{(N-1)}\sum_{j=0}^{N-1}\delta\nu^2(T+j\tau),\ 1\le m\le N-2. \tag{4}$$

When calculating the TCF, the effective mass is reduced by normalization.

The general expression for the TCF, according to expression (3), will have the form:

$$\Xi(t)=\frac{1}{Z}\frac{\sum_{j=0}^{Z-1}\delta\xi(T+j\tau)\delta\xi(T+(j+m)\tau)}{\sigma_\Xi^2}. \tag{5}$$

Using the Zwanzig–Mori projection operator technique, by analogy with [6, 8, 12], we write down the analytical expression for the chain of finite-difference equations for the set of initial TCF $\Xi(t)$ and obtain relations for higher-order memory functions $M_i^\Xi(t), i=1,2,...,$ where $\boldsymbol{\Xi}$ = $\{X, V, A, E, Q\}$:

$$\begin{cases}\dfrac{\Delta\Xi(t)}{\Delta t}=\lambda_1^\xi\Xi(t)-\tau\Lambda_1^\xi\sum\limits_{j=0}^{m}M_1^\Xi(j\tau)\Xi(t-j\tau),\\ \dfrac{\Delta M_1^\Xi(t)}{\Delta t}=\lambda_2^\xi M_1^\Xi(t)-\tau\Lambda_2^\xi\sum\limits_{j=0}^{m}M_2^\Xi(j\tau)M_1^\Xi(t-j\tau),\\ \dfrac{\Delta M_{n-1}^\Xi(t)}{\Delta t}=\lambda_n^\xi M_{n-1}^\Xi(t)-\tau\Lambda_n^\xi\sum\limits_{j=0}^{m}M_n^\Xi(j\tau)M_{n-1}^\Xi(t-j\tau).\end{cases} \tag{6}$$

Here λ_n^ξ are the eigenvalues of the Liouville quasi-operator $\hat{L}^\xi, \Lambda_n^\xi$ are the relaxation parameters having the dimension of the squared frequency.

The functions and parameters presented in expression (6) can be obtained using orthogonal dynamic variables $\mathbf{W}_n^\xi=\mathbf{W}_n^\xi(t)$. The variables $\mathbf{W}_n^\xi$ are related to the lower ones $\mathbf{W}_{n-1}^\xi$ by the following recursive relations:

$$\mathbf{W}_0^\xi=A_k^0(0),\ \ \mathbf{W}_1^\xi=\left(i\hat{L}^\xi-\lambda_1^\xi\right)\mathbf{W}_0^\xi, \mathbf{W}_2^\xi=\left(i\hat{L}^\xi-\lambda_2^\xi\right)\mathbf{W}_1^\xi-\Lambda_1^\xi\mathbf{W}_0^\xi,..., \tag{7}$$

where

$$A_k^0(0)=\{\delta\xi_0,\delta\xi_1,\delta\xi_2,...,\delta\xi_{k-1}\}.$$

To describe the statistical memory effects in complex non-physical systems, a frequency-dependent generalization is used:

$$\varepsilon_i^\xi(\nu)=\left\{\frac{\mu_{i-1}^\xi(\nu)}{\mu_i^\xi(\nu)}\right\}^{\frac{1}{2}}. \tag{8}$$

Here $\mu_i^{\xi}(\nu)$ is the power spectrum of the corresponding ith-order memory function $M_i^{\Xi}(t)$:

$$\mu_i^{\xi}(\nu) = \left| \Delta t \sum_{j=0}^{N-1} M_i^{\Xi}(t_j) \cos(2\pi\nu t_j) \right|^2 . \quad (9)$$

It should be noted that the finite-difference kinetic Eqs. (6) obtained above represent a generalization of the statistical theory of irreversible Zwanzig-Mori processes [13, 14] for the case of analyzing breaks in quasi-derivatives of different orders for the dynamics of complex systems.

Results and Discussion

Analysis of quasi-derivatives of the dynamics of evoked neuromagnetic activity in the human cerebral cortex

Approbation of the developed method is carried out on the example of the study of the human cerebral cortex neuromagnetic responses (MEG) [15, 16]. MEG signals were recorded by 61 superconducting quantum interference sensors (SQUID) under the influence of a red-blue flickering stimulus for a group of 9 healthy subjects and a patient with photosensitive epilepsy (PSE).

Fig. 1 and Fig. 2 show phase trajectories for dynamic variables $\{\mathbf{W}_0^{\xi}, \mathbf{W}_1^{\xi}\}$ of neuromagnetic responses recorded by the 10th sensor for a healthy subject and patient. In the case of a healthy subject, phase trajectories line up in quasi-periodic spirals. On all graphs, nuclei that are symmetrical with respect to the origin of coordinates are found. Similar structures were found for all members of the control group. In the case of a patient, an increase in the scale of phase trajectories is observed. For example, the stratification of phase trajectories for a patient in comparison with the control group is increased approximately 10 times for the combination $\{\mathbf{W}_0^{x}, \mathbf{W}_1^{x}\}$, 1000 times for $\{\mathbf{W}_0^{q}, \mathbf{W}_1^{q}\}$.

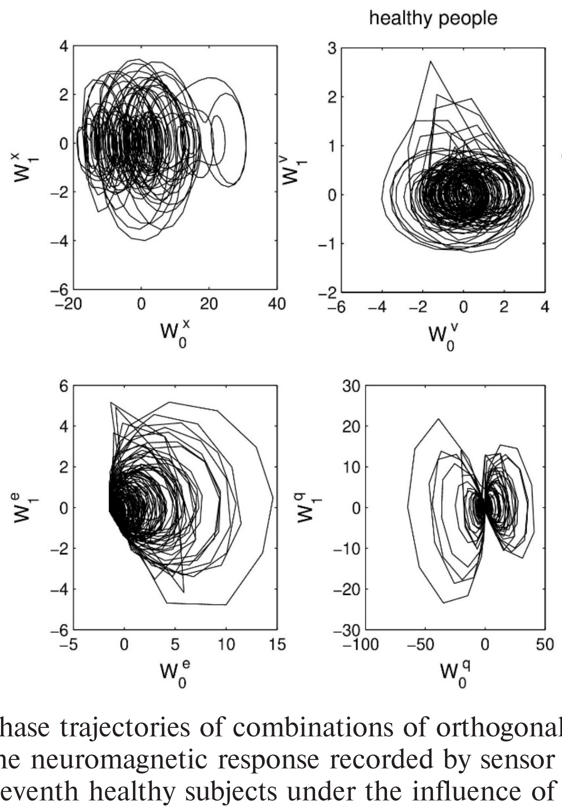

Fig. 1. Phase trajectories of combinations of orthogonal dynamic variables $\{\mathbf{W}_0^{\xi}, \mathbf{W}_1^{\xi}\}$ for the neuromagnetic response recorded by sensor 10 from the cerebral cortex of the seventh healthy subjects under the influence of a red-blue flickering stimulus. Calculation for variations of coordinate $\{x_j\}$, etc.

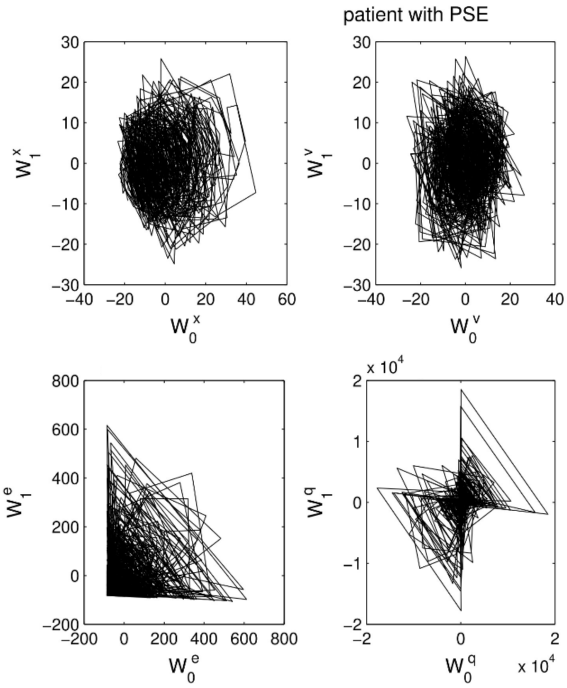

Fig. 2. Phase trajectories of combinations of orthogonal dynamic variables $\left\{\mathbf{W}_0^{\xi}, \mathbf{W}_1^{\xi}\right\}$ for the neuromagnetic response recorded by a 10-sensor from the cerebral cortex of a patient with photosensitive epilepsy when exposed to a red-blue flickering stimulus. Calculation for variations of coordinate $\{x_j\}$, etc.

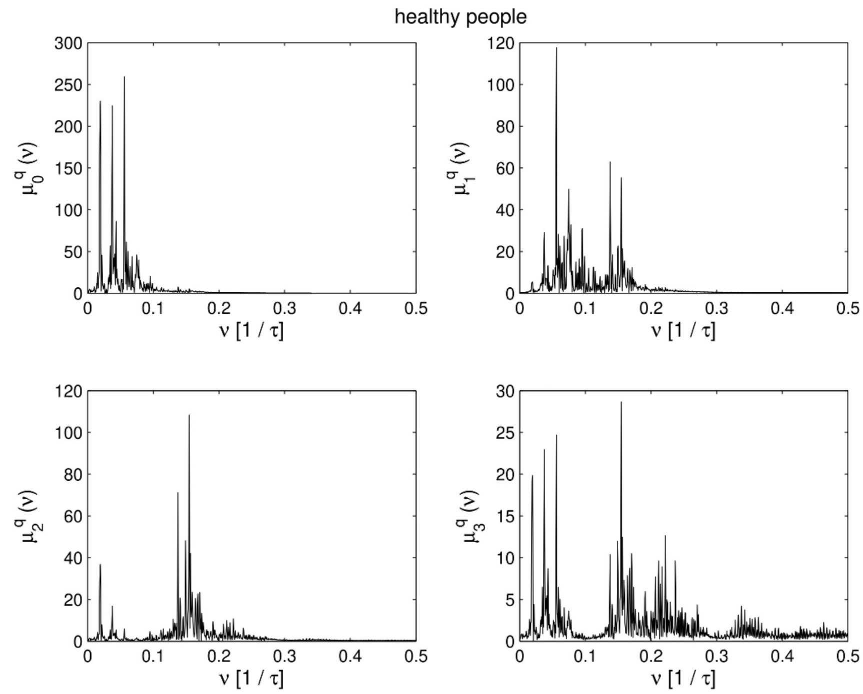

Fig. 3. Power spectra of the initial TCF and memory functions $\mu_i^q(\nu)$ for the neuromagnetic response recorded by sensor 10 from the cerebral cortex of the seventh healthy subjects under the influence of a red-blue flickering stimulus. Frequency dependences are obtained for time variations of the energy flow $\{q_j\}$

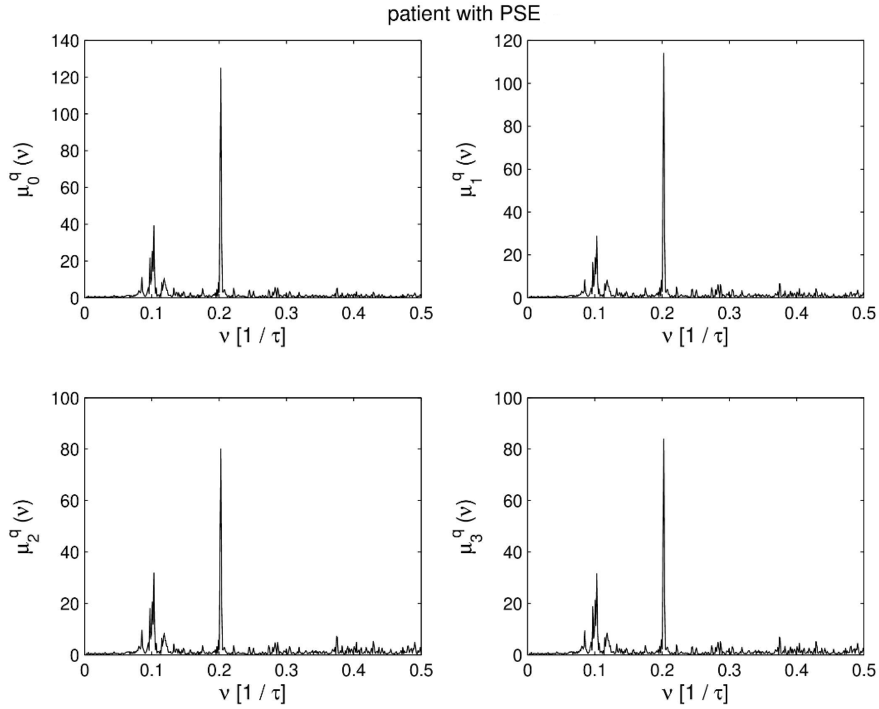

Fig. 4. Power spectra of the initial TCF and memory functions $\mu_i^q(\nu)$
for the neuromagnetic response recorded by the 10th sensor from the cerebral cortex
of a patient under the influence of a red-blue flickering stimulus.
Frequency dependences are obtained for time variations of the energy flow $\{q_j\}$

Fig. 3 and Fig. 4 show the power spectra of the initial TCF and memory functions $\mu_i^q(\nu)$ for time variations of the energy flow $\{q_j\}$ for neuromagnetic responses recorded by the 10th sensor for a healthy subject and patient. For the power spectrum of the initial TCF of a healthy subject in the region of medium and high frequencies, a fractal dependence $\mu_0^q(\nu) \sim 1/\nu^{\alpha}$. This indicates the self-similar nature of the MEG signals from healthy subjects. In the low frequency region up to 50 Hz, bursts are observed corresponding to physiological brain rhythms (**α**-, **β**-activity). The power spectra of the memory functions show additional periodic processes in the mid-frequency region. In the case of a patient (Fig. 4), the violation of the fractal dependence is noticeable for the power spectra of the TCF and memory functions over the entire frequency band. Bursts at frequencies of 50 Hz and 100 Hz are dominant. Suppression of normal brain rhythms is a diagnostic criterion for functional disorders in the activity of the patient's cerebral cortex. Exposure to a flickering stimulus provokes abnormal collectivity of neuron activity, for example, in the vicinity of sensor 10.

Fig. 5 shows the frequency dependences of the parameters $\varepsilon_1^e(\nu), \varepsilon_1^q(\nu)$ for the MEG signal of a healthy subject and a patient with PSE under the influence of a flickering stimulus. Visual analysis makes it possible to establish oscillations in the low-frequency region for a healthy subject with resonances characterized by brain rhythms. The values of the parameters $\varepsilon_1^e(0) \approx 4.3$, $\varepsilon_1^q(\nu) \approx 1.8$ for a healthy subject and $\varepsilon_1^{e,q}(0) \approx 1$ for the patient characterize the change in the statistical memory effects in the neuromagnetic activity of the cerebral cortex in pathology. The critical role of statistical memory effects serves as a diagnostic criterion for abnormal activity of neurons near the 10 sensor when exposed to a red-blue flickering stimulus.

Comparative analysis of kinetic λ_1^{ξ} and relaxation parameters Λ_1^{ξ} for MEG signals of a healthy subject and a patient establishes an increase in the decay rate of relaxation processes in pathology. This fact can be explained by the appearance of a significant number of high-frequency resonances in the functional activity of the patient's cerebral cortex in response to exposure to a flickering stimulus.

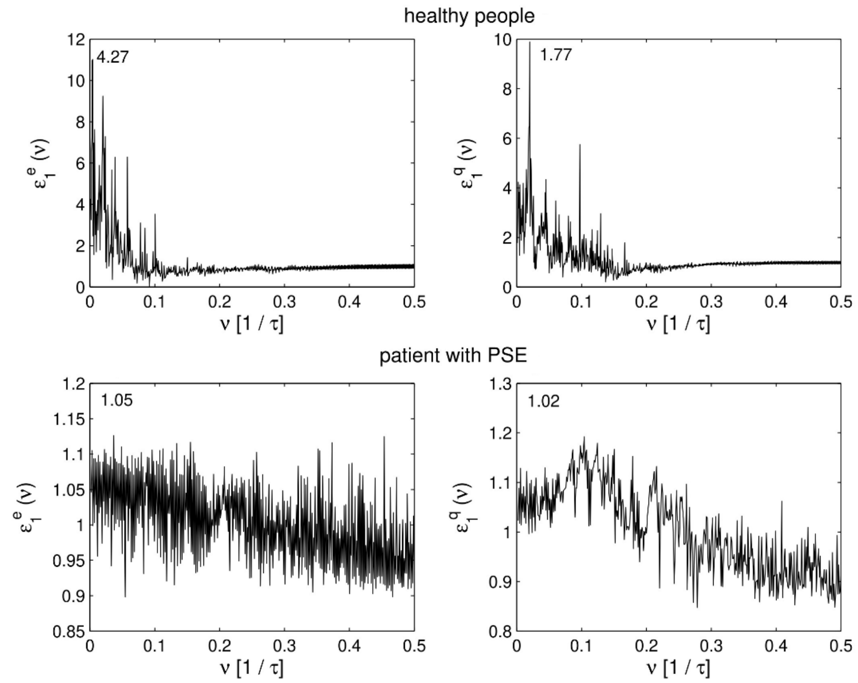

Fig. 5. Frequency dependence of the parameters $\varepsilon_1^e(\nu), \varepsilon_1^q(\nu)$ for the neuromagnetic response recorded by the 10th sensor from the cerebral cortex of the seventh healthy subject and a patient with photosensitive epilepsy when exposed to red-blue flickering stimulus. Frequency dependences are obtained for time variations of energy $\{e_j\}$ and energy flow $\{q_j\}$.
The values of the parameters at zero frequency are numerically presented

Conclusion

In this paper, we develop the main provisions of the Memory Functions Formalism for the case of studying finite-difference derivatives (velocity, acceleration, energy, energy flow) from initial time signals generated by complex systems of non-Hamiltonian nature. Within the framework of the concept of a quasiparticle description for time sequences of finite-difference derivatives, for the first-time systems of discrete type equations with model representations of statistical memory functions are obtained. The resulting equations determine the normalized time autocorrelation functions, statistical memory functions of different orders, construct their power spectra, calculate orthogonal dynamic variables, kinetic and relaxation parameters, and compare the relaxation times and the existence of memory in the dynamics of finite-difference quasi-derivatives of the original time series. Such a representation allows revealing additional information about spatiotemporal breaks in temporal signals, as well as correlations and memory effects at different levels of statistical description.

The concept was assessed for the analysis of neuromagnetic signals recorded from the cerebral cortex of healthy subjects and a patient with photosensitive epilepsy. The results obtained allow establishing informationally significant SQUIDs (localization of areas of the human cerebral cortex is determined accordingly) for the analysis and diagnosis of photosensitive epilepsy. The development of a quasi-particle description of complex non-physical systems provides effective tools for diagnosing neurological diseases [8, 17, 18], emotional [19, 20] and psychiatric [21, 22] human disorders based on the study of correlation and relaxation features of time dynamics [23–25].

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