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Properties of the squared Laguerre–Gaussian vortices

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Abstract. In this paper, a new type of optical vortex called the squared Laguerre–Gauss $(LG)^2$ vortex beam has been investigated. Theoretical conclusions and numerical experiment confirm that these beams are Fourier-invariant and retain their structure at the focus of a spherical lens. In the Fresnel diffraction zone, such a beam is transformed into a superposition of conventional LG beams, the number of which is equal to the number of rings in the $(LG)^2$ beam. The presented beams are structurally stable in the case of one intensity ring.

Keywords: optical vortex, topological charge, Laguerre–Gauss mode, Fourier invariance, Fourier transform, Fresnel diffraction

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Материалы конференции
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Свойства квадратичных вихрей Лагерра–Гаусса

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Аннотация. В данной работе исследовался новый тип оптического вихря, названный квадратным вихревым пучком Лагерра-Гаусса $(ЛГ)^2$. Теоретические выводы и численный эксперимент подтверждают, что эти пучки Фурье-инвариантны и сохраняют свою структуру в фокусе сферической линзы. В зоне дифракции Френеля такой пучок трансформируется в суперпозицию обычных пучков ЛГ, количество которых равно количеству колец в пучке $(ЛГ)^2$. Представленные пучки структурно устойчивы в случае одного кольца интенсивности.

Ключевые слова: оптический вихрь, топологический заряд, мода Лагерра-Гаусса, Фурье-инвариантность, преобразование Фурье, дифракция Френеля

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Introduction

A large number of studies and publications by scientists from all over the world are devoted to optical vortices [1–3], methods for their generation [4–5], and limited circles of applied problems where they are used [6–7]. One of the priority research areas is the search for new types of beams with certain properties [8–9]. For example, in [8], the authors proposed a new type of noncanonical optical vortex, called the "exponential-order phase vortex". In our study [9], a new type of Bessel beams was proposed, which have the property of Fourier invariance and, therefore, are called Fourier–Bessel beams.

However, despite the many methods for creating and describing new types of beams, the well-known Laguerre–Gauss (LG) beams do not lose their relevance [10–12]. Based on the LG modes, new types of optical beams are being developed with various useful properties [13–15]. The authors of [13], using LG modes, generated a vector beam with spatially dependent polarization in the cross section by means of nonlinear magneto-optical rotation. In [15], a new type of partially coherent beam with an unconventional correlation function, called the elliptic correlated Laguerre–Gauss Shell model (LGShM), was theoretically and experimentally investigated. The intensity of such beams in the far field (or in the focal plane) has an elliptical annular profile. Let us give examples of laser beams that differ from conventional Laguerre–Gauss beams and are not structurally stable, since they do not retain the shape of the intensity distribution during propagation.

It is also possible to give examples of laser beams that differ from conventional Laguerre–Gauss beams and are not structurally stable, since they do not retain the shape of the intensity distribution during propagation. For example, in [16] non-structurally stable Laguerre–Bessel–Gauss laser beams are considered. New exact solutions of the paraxial wave equation were obtained in the form of a product of Laguerre polynomials, Bessel functions and Gaussian functions. In [17], Laguerre–Gauss beams with radial and azimuthal polarizations are considered. Complete characteristics of the propagation of several types of azimuthally polarized Laguerre–Gauss beams through optical systems represented by complex ABCD matrices were obtained. In next paper Hermite-sinusoidal-Gaussian laser beams are considered [18]. Hermitian-sinusoidal-Gaussian solutions of the wave equation were obtained. In the limit of a large Hermite–Gaussian beam size, sinusoidal factors dominate and are reduced to ordinary rectangular waveguide modes. In the opposite limit, the rays reduce to the well-known Hermite–Gauss form. The beams that are described by the product of three Airy functions are considered in [19]. Propagation of the product of three Airy beams in a Fresnel zone is investigated numerically. It is shown that the Fourier image of this field has a cubic phase and a radially symmetric intensity with a super-Gaussian decrease.

It should be noted that LG beams and similar beams are of high practical importance for optical communications [20–22], micromanipulation [23], and atoms photoexcitation [24].

In this study, we have proposed a new type of optical beams, whose amplitude is proportional to the squared Laguerre polynomial. These beams extend the LG modes basis. Their theoretical and numerical research was carried out, which showed their Fourier invariance.

Materials and Methods

In this paper, numerical simulation of the generation and propagation of a Gaussian Laguerre beam in the square is implemented. The complex amplitude of this beam in the initial plane has the form:

$$E_{2,n,m}(r,\varphi)=E_2\exp\left(-\frac{r^2}{w^2}+i2n\varphi\right)\left(\frac{r}{w}\right)^{2|n|}\left[L_m^{|n|}\left(\frac{r^2}{w^2}\right)\right]^2, \tag{1}$$

where E_2 is the constant. For numerical simulation, author's Matlab scripts were used in which the Fourier integral and the Fresnel integral are realized.

Let us consider a beam, which we have called the squared LG beam or $(LG)^2$. Beam (1) is not a mode. It does not retain its structure in the Fresnel diffraction zone, but does it in the far field. It means, that the $(LG)^2$ beam is Fourier-invariant, and its complex amplitude at the focus of an ideal spherical lens with focal length f has the form:

$$\begin{aligned}E_{2n,m}(\rho,\theta)&=\frac{-iz_0}{f}E_2(-1)^n\exp(2in\theta)\int_0^\infty x^n\exp(-x)\left[L_m^{|n|}(x)\right]^2 J_{2n}\left(y\sqrt{x}\right)dx=\\&=\frac{-iz_0}{f}E_2(-1)^n\exp\left(2in\varphi-\frac{y^2}{4}\right)\left(\frac{y}{2}\right)^{2|n|}\left[L_m^{|n|}\left(\frac{y^2}{4}\right)\right]^2,\end{aligned}\tag{2}$$

where $x=(r/w)^2$, $y=kw\rho/f$, (ρ,θ) are polar coordinates in the Fourier plane. For obtaining Eq. (5), a reference integral was used from [2].

Comparing the complex amplitudes in the initial plane (1) and in the focus of the spherical lens (2) it can be seen that they coincide up to a constant. In the Fresnel diffraction zone, the $(LG)^2$ beam is a finite superposition of conventional LG beams, since the complex beam amplitude (1) at any z is calculated using the Fresnel trans-form and is equal to:

$$\begin{aligned}E_{2,n,m}(\rho,\theta,z)&=\frac{-iz_0}{z}(-1)^n\exp\left(\frac{ik\rho^2}{2z}+2in\theta\right)\int_0^\infty x^n\exp(-px)\left[L_m^{|n|}(x)\right]^2 J_{2n}\left(y\sqrt{x}\right)dx=\\&=\frac{-iz_0}{z}(-1)^n\exp\left(\frac{ik\rho^2}{2z}+2in\theta\right)\left(\frac{y}{2}\right)^{2|n|}\frac{\Gamma(|n|+m+1)}{\pi m!\,p^{2n+1}}\times\\&\times\exp\left(-\frac{y^2}{4p}\right)\sum_{s=0}^{m}\frac{(-1)^s}{(m-s)!}\frac{\Gamma(m-s+1)\Gamma(s+1/2)}{\Gamma(|n|+s+1)}\times\\&\times\left(\frac{p-2}{2}\right)^{2s}L_{2s}^{2|n|}\left(\frac{y^2}{2p(2-p)}\right),\end{aligned}\tag{3}$$

where $x=(r/w)^2$, $y=kw\rho/z$, $p=1-iz_0/z$, $\Gamma(x)$ is the Gamma function. For obtaining Eq. (6), a reference integral was used from [2].

The number of terms in the sum (3) coincides with the number of rings in the ordinary LG beam. It follows from (3) that for $m=0$ (the radial mode index is zero) the LG beam has one ring $\left(L_0^n(x)=1\right)$ and the sum in (3) reduces to one first term. Therefore, the ordinary LG beam with squared amplitude is conserved on propagation. It also follows from the general expression for the complex amplitude of structurally stable beams [3], which in the initial plane $z=0$ is given by

$$E(\xi,\eta,0)=\exp(-\xi^2-\eta^2)f(\xi+i\eta), \tag{4}$$

where $(\xi,\eta)=(x/w,y/w)$ are the transverse Cartesian coordinates normalized by the waist radius, and $f(.)$ is any entire analytic function of finite growth.

Next, we carried out numerical simulation of the $(LG)^2$ beam structure with a spherical lens using MATLAB templates. The author's script of the Fourier transform in the focal plane was implemented. After that, Fresnel diffraction was considered, which was realized through several Fourier transforms. The program was also implemented in the MATLAB package.

Results and Discussion

Fig. 1 shows the initial intensity and phase distribution for $(LG)^2$ with the following parameters: $\lambda = 532$ nm, $w = 0.5$ mm, $n = 5$, $m = 4$.

Focusing with a spherical lens is described by the Fourier transform. The simulation results for this beam in the focus are shown in Fig. 2.

Fig. 1 and 2 differ only by a constant and clearly demonstrate the Fourier invariance of $(LG)^2$ beams proved in the first section.

The simulation results at different distances are shown in Figs. 3–5.

Fig. 3 shows the intensity (*a*), its cross section (*b*) and the phase (*c*) of the $(LG)^2$ beam shown in Fig. 1, but at a half of the Rayleigh length. It can be seen in Fig. 3 that instead of five intensity rings (Fig. 1), seven intensity rings appear in the beam. In addition, the brightest ring is no longer the first one (Fig. 1). It is the second, instead.

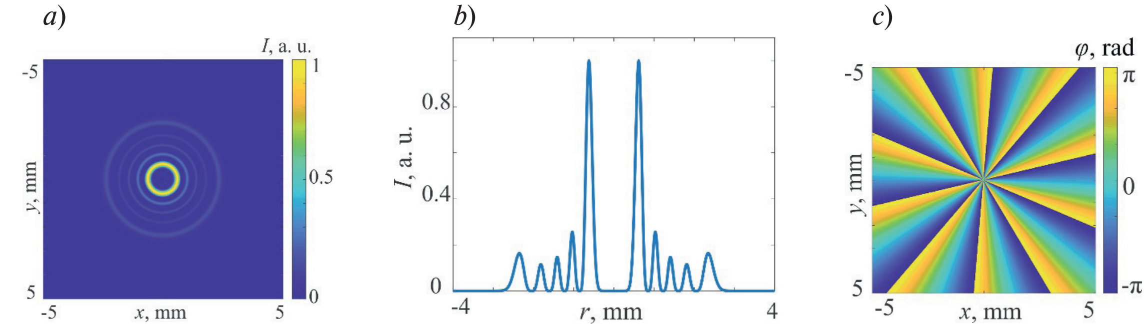

Fig. 1. Initial $(LG)^2$ beam: 2D intensity distribution (*a*); intensity cross section along the radius (*b*); 2D phase distribution (*c*)

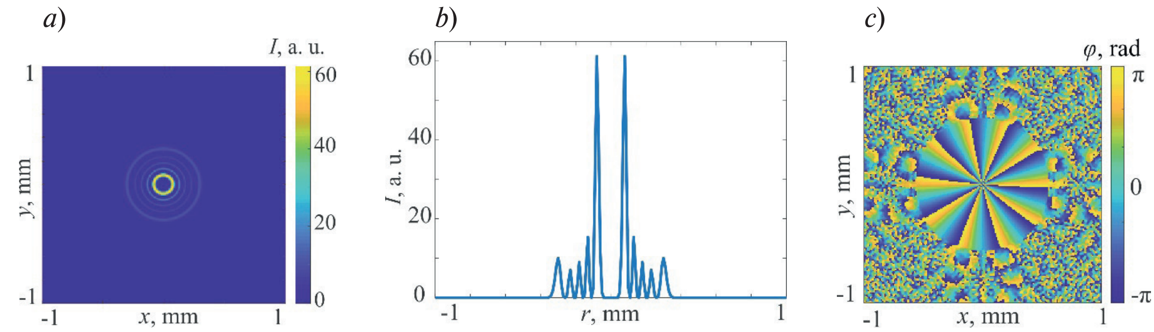

Fig. 2. Field at the focus of a spherical lens after focusing the initial beam from Fig. 1: 2D intensity distribution (*a*); intensity cross section along the radius (*b*); 2D phase distribution (*c*)

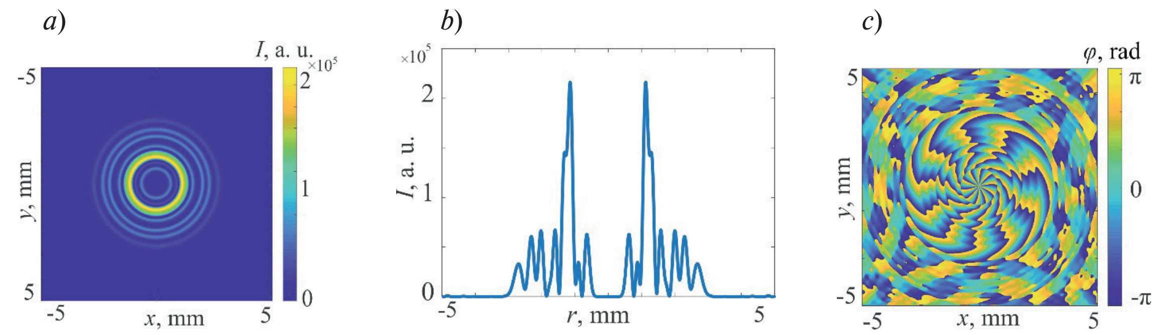

Fig. 3. Fresnel-transformed field of the initial $(LG)^2$ beam (*1*) at a distance $z = z_0/2$ ($z_0 \approx 1.476$ m): 2D intensity distribution (*a*); intensity cross section along the radius (*b*); 2D phase distribution (*c*)

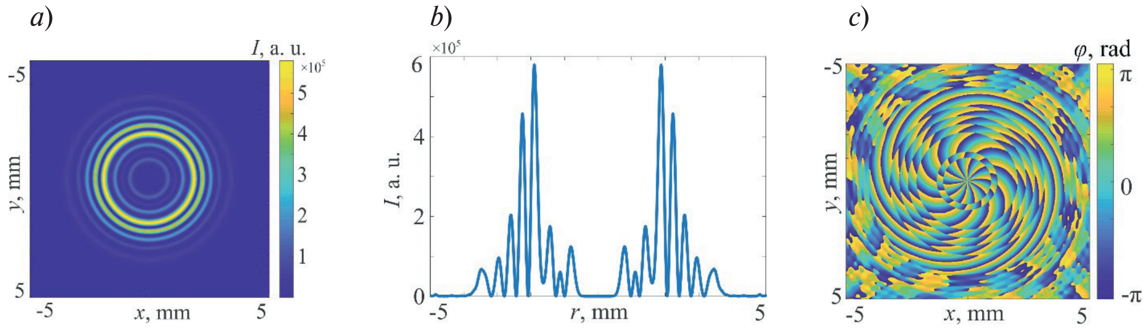

Fig. 4. Transformed field of the initial $(LG)^2$ beam (1) at a distance $z = z_0$ ($z_0 \approx 1.476$ m): 2D intensity distribution (*a*); intensity cross section along the radius (*b*); 2D phase distribution (*c*)

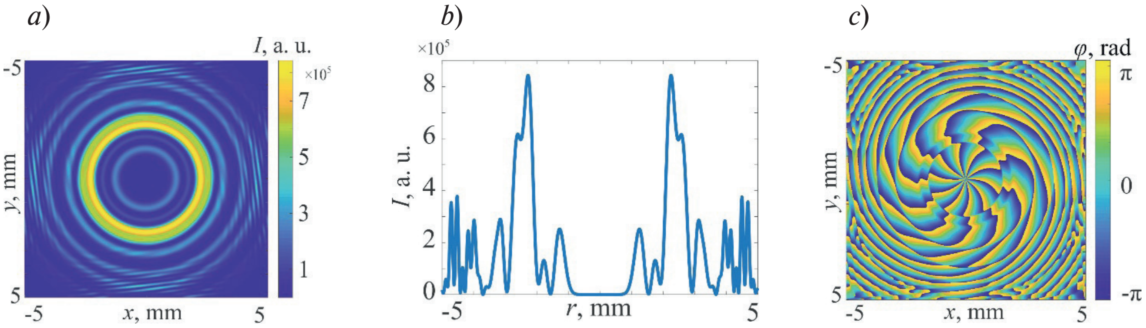

Fig. 5. Transformed field of the initial $(LG)^2$ beam (1) at a distance $z = 2z_0$ ($z_0 \approx 1.476$ m): 2D intensity distribution (*a*); intensity cross section along the radius (*b*); 2D phase distribution (*c*)

Fig. 4 shows the intensity (*a*), its cross section (*b*), and phase (*c*) of the same beam (Fig. 1), but at the Rayleigh distance. It can be seen in Fig. 4 that the beam has eight bright rings, but the energy distribution between them differs from that presented in Fig. 3.

Fig. 5 shows the same as Figs. 3–4, but at a distance of two Rayleigh lengths. There are still 7 rings in the intensity distribution.

Conclusion

In this paper, a new type of vortex beams was considered, which intersects with a family of well-known Laguerre–Gaussian (LG) beams. The complex amplitude of these beams is proportional to squared Laguerre polynomial and thus they are called squared Laguerre–Gaussian beams $(LG)^2$. Conventional LG beams with zero radial index and with even azimuthal index coincide with $(LG)^2$ beams. It is theoretically and numerically shown that the squared LG vortex beams are Fourier-invariant and retain their structure at the focus of a spherical lens or in the far diffraction field. In the Fresnel diffraction zone, such a beam is transformed into an axial superposition of conventional LG beams, the number of which is equal to the number of rings in the $(LG)^2$ beam. If there is only one ring, then the beam is structurally stable (propagation invariant). The results of this study can find their application in optical communications [16–18].

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