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Functional dependence of the notional area of field emission

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Abstract. Concepts and basic methods for extracting notional, formal and effective emission areas are considered. A functional dependence of the notional emission area on the field is obtained, taking into account the shape of the emitter. An analysis of the current-voltage characteristics in semi-logarithmic coordinates $\ln(I/U^k)$ vs $1/U$, called "k-power plot", is proposed, which makes it possible to take into account the shape of the emitter, to linearize the dependence, and, therefore, to obtain effective values of the field enhancement independent of the voltage range.

Keywords: field emission, effective parameters, emission area, field enhancement factor

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Функциональная зависимость условной площади полевой эмиссии

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Аннотация. Рассмотрены концепции и основные методы извлечения условной, формальной и эффективной площади эмиссии. Получена функциональная зависимость условной площади излучения от напряженности поля с учетом формы эмиттера. Предложен анализ ВАХ в полулогарифмических координатах $\ln(I/U^k)$ vs $1/U$, названный «график k-степени», позволяющий учесть форму эмиттера, линеаризовать зависимость и, следовательно, получить эффективные значения усиления поля, не зависящие от диапазона приложенного напряжения.

Ключевые слова: полевая эмиссия, эффективные параметры, площадь эмиссии, коэффициент усиления поля

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Introduction

Optimization of field emission sources is impossible without extracting reliable quantitative information about the main parameters of emission structures from current-voltage characteristics (IVC). As is known, emitters are characterized by three main parameters that are included in the main field emission equation. They are the work function φ, the field enhancement factor γ (FEF) and the emission area *A* (EA).

Most of the experimental works indicate the inadequacy of determining the work function from plotting the IVC in semi-logarithmic Fowler-Nordheim coordinates (FN-plot), based on visual observations of the geometry of potential emission sites according to scanning electron microscopy (SEM) [1–3]. Without considering adsorption processes, the work function for carbon nanostructures is assumed to be 4.6 eV.

The second parameter, FEF, is quite easily extracted from the processing (slope S^{fit}) of experimental and 3D modeling data in FN-plot and Murphy-Good (MG-plot) coordinates [4, 5]. Besides, its effective values weakly depend on the range of applied voltage. It is clear that in the case of a stable emitter geometry, its geometric field enhancement does not depend on the magnitude of the field itself. Given the notion that emission obeys a field emission (satisfies the field emission test), the S^{fit} value automatically determines the field on the emitter surface (more precisely, in the selected characteristic point "C").

Now the most discussed issue in the literature is how to determine the emission area (or area-like) parameter, both from the point of view of the conceptual apparatus and the search for the functional dependence of FEA on the applied voltage.

The concept of notional and effective emission parameters

It is worth to note that the theory of field emission was developed for a fundamentally flat case (planar case). First, it concerns the derivation of the formula for the transparency of the potential barrier when the Schrödinger equation in Cartesian coordinates is used in one-dimensional form along the chosen coordinate axis. Secondly, a flat surface of a solid body is considered to derive Sommerfeld's constant, when the flow of particles through the energy space with energy components normal and parallel to the surface is considered.

Another problem concerns the concept of emission area. In theoretical premises, there is no talk about the dimensions of the emitter surface (there is no concept of "emission area"). The particle flux is indeed defined in terms of the surface area of a unit quantity. In this case, the number of particles is determined by the volume and velocity of the outflow from this volume, that is, their kinetic energy. However, the ratio of the number of particles (states) to the outflow area leads to a reduction in the area. Therefore, only the energy characteristics of the particles remain in the expression for the density. As a result, the current density is determined by the energy window, the area in the energy space by the field at a given field value, which acts as a parameter or constant. The concept of current density is akin to the concept of substance density, that is, a value that does not depend on the size of the object, for example, on the size of the considered body surface.

The lack of a clear understanding of emission area leads to problems in the perception of the concept of current density, causing confusion between local (theoretical) and macroscopic current densities. In radio engineering the current density is a value determined from the flowing current divided by the cross-sectional area of the conductor. On the contrary, in field emission the area of emission will be recovered by the size of the surface having the same magnitude of the electric field. As a result, this surface will provide the emission current measured by the instruments.

In general, it is believed that the emission area serves to formally link the magnitude of the emission current and the theoretical current density. Among the two dozen variants of writing the theoretical types of current density [6], the structure of the formula stands out, called "kernel", in which there are no functional dependencies in the pre-exponential factor, with the exception of the pre-exponential field factor to the power of *k* ($k = 2$):

$$J_k = \mathrm{a}\varphi^{-1}F^2\exp\left(-\nu \mathrm{b}\varphi^{3/2} / F\right), \tag{1}$$

where *a* and *b* are the first and second Fowler-Nordheim constants, *v* is the barrier "correction factor" or first barrier factor, φ is the work function.

Taking into account such representations, the total current density can be represented as:

$$J = \lambda J_k \ , \tag{2}$$

where in the multiplier λ is the pre-exponential correction factor (various physical corrections are taken into account, e.g., temperature contribution $\pi p/\sin(\pi p)$, p is the Swanson-Bell parameter, the correcting barrier factor τ or the second barrier factor, etc.).

Two main definitions of the emission area are used in the literature – notional A_n and formal A_f:

$$I \equiv A_n J, \tag{3}$$

$$I \equiv A_f J_k. \tag{4}$$

As can be seen from eq. (3), the notional area fully corresponds to the definition of the emission area, since it describes a surface with the same fields and temperature (all this in a flat case). The formal area, on the other hand, formally includes all functional dependences on the field, temperature, and so on. A number of theoretical papers consider only the situation λ = 1, assuming $I = A_n J_k$.

In the experiment, we are dealing with measured quantities - current and voltage. At this stage, it is useful to introduce the concept of effective parameters – FEF α_{eff} ($F = \alpha U$) and emission area A_{eff}. These parameters are extracted from the processing of the current-voltage characteristic in a certain range of applied voltages (fields). The main way to extract these parameters is to process dependences in semilogarithmic coordinates. Thus, the effective values of FEF and emission area depend on the extraction method (most often along the trend line) and on the range (number of points) of the data being processed. The effective parameters obtained in such way, as shown in the ref. [7], coincide with the given notional values $A_n = A_{eff}$ and $\alpha = \alpha_{eff}$ only in two cases (and only in the planar case): using the current density equation (Shrednik equation) in the ref. [8] and processing in quadratic coordinates FN plot, and the equation in the form of MG kernel and processing in MG plot coordinates, respectively:

$$\ln\left(I / U^2\right) = \ln\left(R^{ES,Fit}\right) + S^{ES,Fit}\left(1/U\right), \tag{5}$$

$$\alpha_{ES,eff} = -0.95 \mathrm{b} \varphi^{3/2} / S^{ES,Fit}, \tag{6}$$

$$A_{ES,eff} = R^{ES,Fit} / \left[\tau^{-2} a \varphi^{-1} \exp\left(1.03\eta\right) \alpha^2_{ES,eff}\right], \tag{7}$$

$$\eta = b\varphi^{3/2} / F_R = bc_S^2 \varphi^{-1/2}, \tag{8}$$

where c_S is Schottky constant, $F_R = \varphi^2 c_S^{-2}$ is reference field or barrier removal field, $\tau^2 \approx 1.1$, $v = 0.95 - 1.03f^2$, $f = F/F_R$ is scaled barrier field.

$$\ln\left(I / U^{2-\eta/6}\right) = \ln\left(R^{MG,Fit}\right) + S^{MG,Fit}\left(1/U\right), \tag{9}$$

$$\alpha_{MG,eff} = -\mathrm{b} \varphi^{3/2} / S^{MG,Fit}, \tag{10}$$

$$A_{MG,eff} = R^{Fit} / \left[\tau^{-2} a \varphi^{-1} F_R^{\eta/6} \exp\left(\eta\right) \alpha^{2-\eta/6}_{MG,eff}\right], \tag{11}$$

where $\tau^2 = 1$, $v = 1 - f + (f/6)\ln f$.

Earlier, to take into account the curvature of dependences when plotting in the FN plot coordinates, it was proposed to use correction factors r_t and s_t ("RS" method) at point t, as shown in [9]:

$$\alpha_{eff} = -s_t \mathrm{b} \varphi^{3/2} / S^{fit}, \tag{12}$$

$$A_{eff} = R^{fit} / \left[\mathrm{a} \varphi^{-1} \alpha^2 r_t\right]. \tag{13}$$

However, this method turned out to be very cumbersome for processing experimental data.

Of all the methods, the MG plot method seems to be the most productive since it rectifies the theoretical dependence.

Functional dependence of the emission area

Real field emitters have a curved surface. There are a number of models that are used to describe the most commonly used field emitters. These models include the semi-ellipsoid model for describing metal emitters (tungsten, molybdenum tips), the hemisphere on a cone for some spiked silicon structures, and the hemisphere on the cylinder model (HCP), which most closely resembles carbon nanotubes in shape.

The question arises about the correctness of the description of emitters having the above shapes, using the effective values of the area and FEF. These effective values turn out to be very strongly dependent on the increase in the applied voltage.

The idea is, as before, to use the basic formula for the field emission for the field at a selected point on the surface (at the apex of the tip), and for the notional emission area to set a functional dependence on the value of the field at this point.

Thus, the curved surface of the emitter is replaced by a flat disk lying on the apex of the tip, the area of which increases according to a certain law with an increase in the applied voltage.

In a number of theoretical works, such functional dependences were obtained, for example, for a hemisphere on a plane [10], a paraboloid [11], a semiellipsoid [12, 13]. However, for processing the IVC in semilogarithmic coordinates, the proposed analytical formulas are practically not applicable.

In the refs [14, 15] studies of the power-law dependence of the pre-exponential voltage factor for emitters of various shapes were carried out. In the work [14], three-dimensional modeling of electrostatic fields was carried out and the model I–V characteristics of emitters of various shapes were obtained. This made it possible to determine the characteristic shifts in the degree of pre-exponential filed factor. Characteristic and stable shift values k_A were found, depending on the shape of the emitter tip. For example, for nanotubes, the shift values are, depending on the IVC processing method, in the range $k_A = 0.5–0.7$. In [15], a hypothesis was put forward about the direct proportionality of the notional emission area to the voltage in the corresponding degree $A_n \sim U_k$.

Based on 3D modeling of electrostatic fields for the tip in the form of HCP, we obtained the dependence $A_n = I/J_k$ for given values of the field at the apex of the tip $f_a = F/F_R$. Next, we used the expression for the dimensionless notional area g_n for a hemisphere with radius r_a: $g_n = A_n / [2\pi r_a^2]$. The dependence of g_n on f_a was plotted in an extended range of fa values from 0.1 to 1. Plot (A) in Fig. 1, a shows the calculated dependence of the notional area $A_n = [2\pi r_a^2]\, g_n$.

To determine the power dependence, a solution was sought in the form of a simple approximation formula of the form $y = cx^d$. As a result of the approximation, the value $b = 0.62$ was obtained. According to the normalization conditions, the maximum value $g_n = cf_a^{0.62}$ will take at $f_a = 1$, hence $c = g_{n,\max}(1) = 0.552$. The power dependence of the notional emission area on the dimensionless field at the apex of the tip will take the form (B) in Fig.1, *a*.

As a result, simple dependences of the notional area on the applied voltage were obtained, allowing the processing of the I–V characteristics in semi-logarithmic coordinates (Fig. 1, *b*):

$$A_n = \left[2\pi r_a^2\right] g_n(1) f_a^{k_A} = A_{n,max} f_a^{k_A}, \tag{14}$$

$$I = [2\pi r_a^2 g_n(1)(\alpha U)^{k_A} / F_R^{k_A}] a\varphi^{-1} (\alpha U)^{2-\eta/6} F_R^{\eta/6} \exp(\eta) \exp\left(-b\varphi^{3/2}/(\alpha U)\right), \tag{15}$$

$$I = [A_{n,\max}] \alpha^{2-\eta/6+k_A} a\varphi^{-1} F_R^{\eta/6-k_A} \exp(\eta) U^{2-\eta/6+k_A} \exp\left(-b\varphi^{3/2}/(\alpha U)\right), \tag{16}$$

$$\ln\left(I/U^{2-\eta/6+k_A}\right) = \ln\left(R^{kpower,Fit}\right) + S^{kpower,Fit}(1/U), \tag{17}$$

$$\alpha = -b\varphi^{3/2} / S^{kpower,Fit}, \tag{18}$$

$$A_c = a\varphi^{-1} F_R^{\eta/6-k_A} \exp(\eta), \tag{19}$$

$$R^{kpower,Fit} = A_{n,max} A_c \alpha^{2-\eta/6+k_A} = A_{n,max} A_c \alpha^{k_t}, \tag{20}$$

$$A_{n,max} = R^{kpower,Fit} / \left(A_c \alpha^{k_t}\right), \tag{21}$$

$$r_a = \sqrt{R^{kpower,Fit} / [2\pi g_n(1) A_c \alpha^{k_t}]}. \tag{22}$$

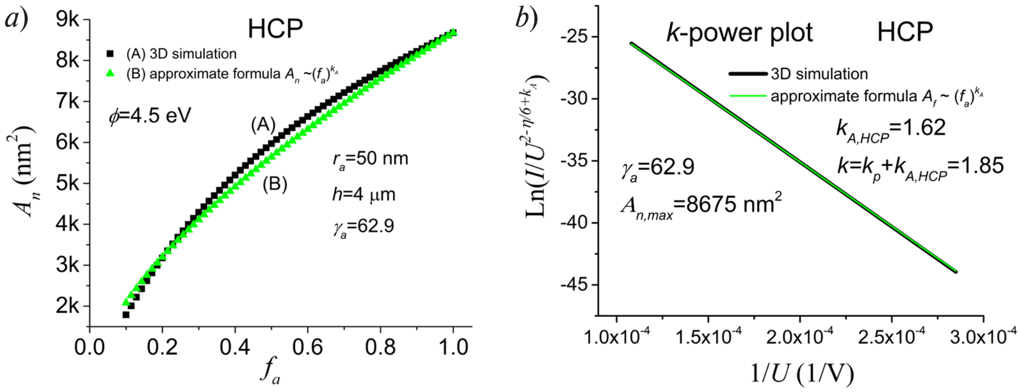

Fig.1. Functional dependences of the notional area on the field: calculated and approximate dependences (*a*), determination of the main emission parameters in k-power plot coordinates (*b*)

Conclusion

A functional dependence of the notional emission area on the field is obtained. Analysis of the I-V characteristics in semi-logarithmic coordinates $\ln(I/U^k)$ vs $1/U$, called “k-power plot”, allows you to linearize the dependence and obtain FEF that is independent of the voltage range. The second parameter, intercept R^{fit}, makes it possible to determine the maximum emission area under the condition of the field of complete removal of the barrier. Which, in turn, allows you to determine the radius of the emitter rounding.

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