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Stability features of inhomogeneous steady-state potential distributions in diode with counter-streaming electron and ion flows

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Abstract. This paper continues studying stability features of steady states of a diode with counter-streaming electron and ion flows. In our recent paper, an integral-differential equation for the potential perturbation amplitude in the mode without potential barriers reflecting charged particles within the plasma was derived. Its exact solution was found for homogeneous steady-state field distribution. In this paper, we propose a semi-analytical method to solve the integral-differential equation for potential perturbation amplitude in the case of inhomogeneous steady-state solutions. It is based on the use of the piecewise linear approximation of the integral operator kernel and the variable coefficient as well as the potential perturbation distribution. A dispersion equation is obtained and five first dispersion branches are constructed. As a result, we have proved that all steady state potential distributions with the values of dimensionless inter-electrode gap up to $10\pi/\sqrt{2}$ are unstable. Numerical calculations of the potential perturbation development confirm analytical results.

Keywords: plasma diode, electron and ion flows, plasma instability

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Устойчивость неоднородного стационарного распределения потенциала в диоде со встречными потоками электронов и ионов

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Аннотация. Статья продолжает исследование устойчивости стационарных состояний диода со встречными потоками электронов и ионов. В предыдущей работе было получено интегро-дифференциальное уравнение для возмущения потенциала в режиме без отражения заряженных частиц от потенциальных барьеров. Для однородного распределения потенциала было найдено его аналитическое решение. В этой статье мы предлагаем полуаналитический метод решения этого уравнения в случае неоднородных распределений потенциала. С помощью этого метода мы исследовали устойчивость стационарных решений для длин диода вплоть до $10\pi/\sqrt{2}$ и показали, что все они неустойчивы. Численное моделирование эволюции возмущения подтвердило результаты, полученные аналитически.

Ключевые слова: плазменный диод, потоки электронов и ионов, плазменная неустойчивость

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Introduction

The study of the stability of steady states of plasma diodes with counter-streaming electron and ion flows is necessary to understand the mechanism of operation of a high-current plasma diode modulator, which is used to convert a constant voltage of several tens of volts accumulated on a series-connected thermionic energy converters into alternating voltage [1]. In such a diode the flow of electrons coming from the hot emitter and passing through the plasma region is strongly accelerated by a potential jump existing near the collector and ionizes caesium atoms. As a result, a stream of ions moving towards the emitter appears in the narrow collector layer. When studying the stability of steady-state solutions for the diode in the first approximation it can be assumed that the ion flow comes from the collector surface.

Steady-state solutions for such a diode were studied in detail in [2]. It was shown that there are several solutions for fixed inter-electrode gap. They are characterized by different values of the electric field strength at the left electrode. The representation of such solutions on the (ε_0,δ) plane, where δ and ε_0 are the dimensionless inter-electrode gap and electric field strength at the left electrode, respectively, is given in Fig. 4 in Ref.[2]. They belong to two modes: 1) all charged particles move in the inter-electrode gap without reflection and reach the opposite electrodes, 2) the potential distribution has extrema reflecting a portion of the particles. Thus, the problem of determining steady-states in a diode plasma can be considered solved.

Now, one needs consistently study the stability features of each type of steady-state solutions. It is important because non-linear oscillation can develop in plasma instead of stationary states as was reported in Refs. [1], [3]. In Ref.[4], stability features of steady-state solutions of the first type were examined. The method of expanding of potential distribution and charged particles densities into series in powers of small potential perturbation was used. It was assumed that the electrons fly through the interelectrode gap in less time than it takes for the ions to displace to the distance of the order of Debye-Hückel length. This allowed us to believe that perturbations develop with ionic velocities, i.e. to study ionic instability.

An integral-differential equation for the electric field perturbation amplitude was derived. This equation was solved analytically for a homogeneous steady-state solution (branch n_0 in Fig. 4 in [2]). The study of dispersion relation showed that there was a threshold in the magnitude of the inter-electrode gap, when exceeded an aperiodic instability developed in the diode plasma.

This paper examines stability features of steady-state solutions with inhomogeneous electric field distribution. For this purpose we have proposed a new original method for solving the equation for the field perturbation amplitude obtained in [4], which is suitable for studying stability of any potential distribution without potential barriers which can reflect charged particles. Applying this method, we have investigated stability features of the solutions belonging to the n_j branches with $j > 0$ (see Fig. 4 in [2]). In addition, we performed numerical calculations of the perturbation evolution for these steady-state solutions. It has been established that all inhomogeneous steady-state solutions of this type are unstable.

Solution of the equation for the electric field perturbation amplitude

We consider a diode of planar geometry. We assume that a mono-energetic flow of electrons enters the plasma from the left electrode with velocity $v_{e,0}$ and density $n_{e,0}$, while ions enter from the right electrode with a velocity $-v_{i,0}$ and density $n_{i,0}$. Charged particles move without collisions, and when reaching any electrode, they are absorbed. We assume also that electrons and ions enter into inter-electrode gap with the same kinetic energies, i.e. $W_0 \equiv mv_{e,0}^2/2 = Mv_{i,0}^2/2$ (here m and M are the electron and ion masses). As in Ref.[4] we assume that electrons fly through the inter-electrode gap in less time than it takes for ions to move the distance of the order of the Debye-Hückel length λ_D. This is equivalent to the condition $L/\lambda_D \ll \sqrt{(M/m)}$. In this case, the electrons "instantly" adjust to the existing electric field, and to study the time-dependent problem it is necessary to take into account only the ion movement effects.

For the consideration convenience, we turn to dimensionless quantities, choosing W_0 and $\lambda_D = [2\epsilon_0 W_0/(e^2 n_{e,0})]^{1/2}$ as units of energy and length; here e is the electron charge, and ϵ_0 is the vacuum permeability. The electron velocity is measured in units $v_{e,0}$, while ion one is measured in units $v_{i,0} = v_{e,0}\sqrt{(m/M)}$.

When studying the stability features of steady-state solutions, we track evolution of small potential perturbation presenting the potential distribution (PD) as

$$\eta(\zeta,\tau) = \eta_0(\zeta) + \tilde{\eta}(\zeta)\exp(-i\Omega\tau), \quad |\tilde{\eta}(\zeta)| << \eta_0(\zeta). \tag{1}$$

Here $\eta_0(\zeta)$, $\tilde{\eta}(\zeta)$ and Ω are the unperturbed PD, the potential perturbation amplitude, and the complex frequency respectively. In Ref.[4], integral-differential equation for $\tilde{\eta}(\zeta)$ was derived

$$\begin{aligned}\tilde{\eta}''(\zeta) &= -u_e^{-3}(\zeta)\tilde{\eta}(\zeta) + u_i^{-3}(\zeta)\int_\zeta^\delta dx \exp\{i\Omega[\sigma_i(\zeta)-\sigma_i(x)]\}\tilde{\eta}'(x) + \\ &+ i\Omega u_i^{-1}(\zeta)\int_\zeta^\delta dx u_i^{-3}(x)\int_x^\delta dy \exp\{i\Omega[\sigma_i(\zeta)-\sigma_i(y)]\}\tilde{\eta}'(y).\end{aligned} \tag{2}$$

Here $u_e(\zeta) = [1 + 2\eta_0(\zeta)]^{1/2}$ and $u_i(\zeta) = [1 - 2\eta_0(\zeta) + 2V]^{1/2}$ are the electron and ion undisturbed velocities corresponding to the mono-energetic particle velocity distributions, and $\sigma_i(\zeta) = \int_\zeta^\delta dx[u_i(x)]^{-1}$ is the ion time of flight from the right boundary to the point ζ.

Boundary condition for $\tilde{\eta}(\zeta)$ at the right boundary is $\tilde{\eta}(\delta) = 0$. On the other hand, $\tilde{\eta}'(\delta)$ is considered to be an arbitrary parameter.

A special case $\eta_0(\zeta) \equiv 0$ was considered in [4]. An analytical solution of Eq. 2 was found. In this paper, we study the stability features of inhomogeneous steady-state solutions without potential barriers so that all charged particles move in the inter-electrode gap without reflection and reach the opposite electrodes. The PDs typical for such solutions are shown in Fig. 1. As in the case of homogeneous solution we use Eq. 2 for this purpose. After calculating the first integral on the right-hand side of Eq. 2 in parts and substituting the result into the second term, we can rewrite this equation as

$$\begin{aligned}&\tilde{\eta}''(\zeta) + \left[u_e^{-3}(\zeta) + u_i^{-3}(\zeta)\right]\tilde{\eta}(\zeta) = \\ &= -i\Omega\int_\zeta^\delta dx\left[u_i^{-1}(x)u_i^{-3}(\zeta) + u_i^{-3}(x)u_i^{-1}(\zeta)\right]\exp\{i\Omega[\sigma_i(\zeta)-\sigma_i(x)]\}\tilde{\eta}(x) + \\ &+ \Omega^2 u_i^{-1}(\zeta)\int_\zeta^\delta dy u_i^{-1}(y)\exp\{i\Omega[\sigma_i(\zeta)-\sigma_i(y)]\}\tilde{\eta}(y)\int_\zeta^y dx u_i^{-3}(x) = \int_\zeta^\delta dx K(\zeta,x)\tilde{\eta}(x).\end{aligned} \tag{3}$$

The RHS of Eq. 3 is the Volterra integral operator with a degenerate kernel. To solve this equation, we propose the numerical-analytical method. It is as follows. We divide the entire interval $[0, \delta]$ into subintervals with boundaries at the ζ_k points: $0 = \zeta_0 < \zeta_1 < \dots < \zeta_{N-1} < \zeta_N = \delta$, and approximate Eq. 3 in each subinterval, replacing the functions $u_e^{-3}(\zeta)$, $u_i^{-3}(\zeta)$, $u_i^{-1}(\zeta)$, $u_e^{-3}(\zeta) + u_i^{-3}(\zeta)$ with their linear approximations. We solve the equations sequentially in intervals $[\zeta_{k-1},\zeta_k]$ starting from the right boundary. We designate an approximation for the perturbation amplitude $\tilde{\eta}(\zeta)$ in the interval $[\zeta_{k-1},\zeta_k]$ as $\tilde{\eta}_k(\zeta)$. The integral on the RHS of Eq. 3 is represented as

$$\int_\zeta^\delta dx K(\zeta,x)\tilde{\eta}(x) \approx \int_\zeta^{\zeta_k} dx \tilde{K}_k(\zeta,x)\tilde{\eta}_k(x) + \sum_{j=k+1}^{N}\int_{\zeta_j-1}^{\zeta_j} dx \tilde{K}_j(\zeta,x)\tilde{\eta}_j(x) \equiv F_k(\zeta) + H_k(\zeta), \tag{4}$$

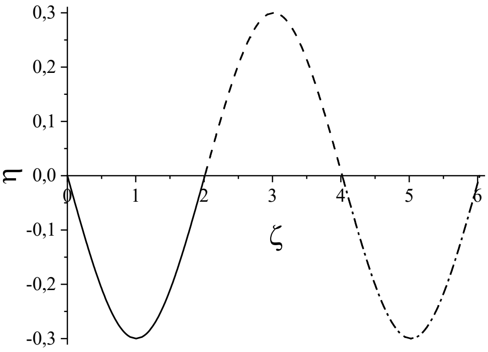

Fig. 1. PDs specific to inhomogeneous steady-state solutions: solid curve corresponds to n_1, solid + + dashed one to n_2 and solid + dashed + dash-dotted one to n_3 branches; $\varepsilon_0 = 0.453$

$$F_k(\zeta) = \int_{\zeta}^{\zeta_k} dx \tilde{K}_k(\zeta, x)\tilde{\eta}_k(x), \quad H_k(\zeta) = \sum_{j=k+1}^{N} \int_{\zeta_{j-1}}^{\zeta_j} dx \tilde{K}_j(\zeta, x)\tilde{\eta}_j(x). \tag{4}$$

Here the kernels $\tilde{K}_j(\zeta, x)$ are given by the same formulas as the kernel $K(\zeta, x)$, but instead of the functions $u_i^{-3}(\zeta)$, $u_i^{-1}(\zeta)$ and $\sigma_i(\zeta)$ their linear approximations on the interval $[\zeta_{j-1},\zeta_j]$ are used. Thus, the function $\tilde{\eta}_k(x)$ obeys the following equation

$$\tilde{\eta}_k''(\zeta)\left[R_k + Q_k(\zeta - \zeta_k)\right]\tilde{\eta}_k(\zeta) = F_k(\zeta) + H_k(\zeta). \tag{5}$$

$$\tilde{\eta}_k(\zeta_k) = y_k, \quad \tilde{\eta}_k'(\zeta_k) = z_k. \tag{6}$$

Here, R_k and Q_k are the coefficients of the linear approximation of the function $u_e^{-3}(\zeta) + u_i^{-3}(\zeta)$ on the kth interval. Note that by the moment the problem is solved on the interval $[\zeta_{k-1},\zeta_k]$, functions $\tilde{\eta}_j(x)$, $j = k+1,...,N$ have already been found. Therefore the term $H_k(\zeta)$ on the RHS of (5) is known. Due to the smallness of the interval $[\zeta_{k-1},\zeta_k]$, the term $F_k(\zeta)$ is small. Therefore, the solution to problem (5), (6) within each interval $[\zeta_{k-1},\zeta_k]$ can be obtained using iterations. Assuming the RHS to be known, one can find the solution to problem (5), (6). When $Q_k \neq 0$ its general solution is

$$\tilde{\eta}_k(\zeta) = c_k^1 Ai(\psi(\zeta)) + c_k^2 Bi(\psi(\zeta)) - \pi \int_{\psi_k}^{\psi(\zeta)} d\varphi\left[\tilde{F}_k(\varphi) + \tilde{H}_k(\varphi)\right]\left[Ai(\psi(\zeta))Bi(\varphi) - Bi(\psi(\zeta))Ai(\varphi)\right]. \tag{7}$$

Here $\psi(\zeta) = -\left[R_k + Q_k(\zeta - \zeta_k)\right]Q_k^{-2/3}$, $\psi_k = -R_k Q_k^{-2/3}$, $\tilde{F}_k(\psi) = Q_k^{-2/3}F_k(\chi_k(\psi))$,

$\tilde{H}_k(\psi) = Q_k^{-2/3}H_k(\chi_k(\psi))$, $\chi_k(\psi) = \zeta_k - \left(\psi Q_k^{2/3} + R_k\right)/Q_k$. When $Q_k = 0$ the general solution is

$$\tilde{\eta}_k(\zeta) = c_k^1 \cos\left(\sqrt{R_k}(\zeta - \zeta_k)\right) + c_k^2 \sin\left(\sqrt{R_k}(\zeta - \zeta_k)\right) - \int_{\zeta_k}^{\zeta} d\psi\left[F_k(\psi) + H_k(\psi)\right] \times$$
$$\times\left[\cos\left(\sqrt{R_k}(\zeta - \zeta_k)\right)\sin\left(\sqrt{R_k}(\psi - \zeta_k)\right) - \sin\left(\sqrt{R_k}(\zeta - \zeta_k)\right)\cos\left(\sqrt{R_k}(\psi - \zeta_k)\right)\right]\Big/\sqrt{R_k}. \tag{8}$$

The constants c_k^1 and c_k^2 in Eqs. (7), (8) are determined using conditions (6). Due to the weak dependence of the functions $F_k(\zeta)$ on $\tilde{\eta}_k(\zeta)$ the solution $\tilde{\eta}_k(\zeta)$ can be sequentially refined using the iterations on the base of solution (7) or (8).

Using the described algorithm for a given value of the inter-electrode gap δ, it is possible to calculate an amplitude of the potential perturbation $\tilde{\eta}_k(\zeta)$ for a number of values $\Omega = \omega + i\Gamma$, and, in particular, to build a dependence $\tilde{\eta}(0)$ on Ω. Having the boundary condition on the left boundary $\tilde{\eta}(0, \Omega)$ satisfied, one can determine the eigen-frequencies. (Note that the value of the $\tilde{\eta}'(\delta)$ does not affect the solution of the equation $\tilde{\eta}(0, \Omega) = 0$; the value $\tilde{\eta}'(\delta) = 1$ was used in the calculations). Thus, we calculate dependencies $\Omega(\delta)$, i.e. dispersion branches corresponding to steady-state branches n_k for $k > 0$.

We have implemented the described approach in the study of the aperiodic stability features of the n_k branches for $k > 0$ at zero electrode potential difference. The unperturbed PDs, $\eta_0(\zeta)$, for the solutions belonging to these branches at $\delta = 2, 4$ and 6 are represented in Fig. 1. The PDs for $\varepsilon_0 < 0$ can be obtained from the ones shown in the figure by reflection about ζ axis. The dependences of the aperiodic mode growth rate $\Gamma(\delta)$ for the n_1 and n_2 branches are shown in Fig. 2. It can be seen that the steady-state solutions relevant to these branches turn out to be unstable with respect to small aperiodic perturbations both for $\varepsilon_0 > 0$ and $\varepsilon_0 < 0$. We have also shown that n_j branches with $j = 3$, 4 and 5 are also unstable. At points $\varepsilon_0 = 0$, $\delta_k = \pi k\sqrt{2}$ the growth rate vanishes, which corresponds to the bifurcation at the points, found in Ref. [4] when studying the stability features of the solution corresponding to homogeneous steady states.

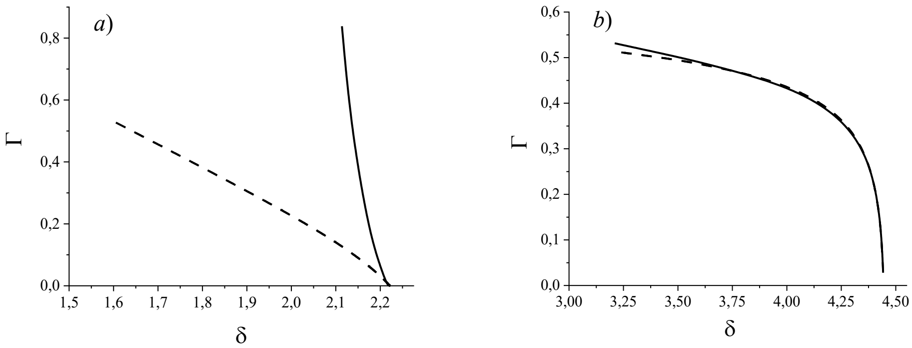

Fig. 2. Aperiodic mode growth rate Γ vs. δ for n_1 (*a*), n_2 (*b*) branches: solid (dashed) curves correspond to the values $\varepsilon_0 > 0$ ($\varepsilon_0 < 0$)

Thus, we have established that in a diode with counter flows of electrons and ions, steady states corresponding to the n_j branches with $j = 1,...,5$ cannot exist. It should be noted that investigation of the stability features of homogeneous steady-state solution by the developed method produces the same results as in Ref.[4].

Numerical study of the steady-state solutions stability features

We also studied the stability features of inhomogeneous steady-state solutions numerically using the code described in [4, 5]. In the simulations, as in the above analytical study, it was assumed that at each time moment τ_s the electrons instantly adjust to the electric field distribution existing at that moment. Therefore, in the calculations, the electron density was determined by stationary formulas. We simulated the time evolution of a small perturbation introduced to the electric field stationary distribution.

The calculation was carried out for two gap lengths: $\delta = 2.15$ and 2.2 and both signs of ε_0. In all cases, the instability developed. For $\delta = 2.2$ and $\varepsilon_0 > 0$ the value of the growth rate Γ turned out to be 0.060, while the analytical result is $\Gamma = 0.0616$. We also compared the shape of the aperiodic eigen-mode of the potential perturbation amplitude obtained analytically with that obtained in the simulation. These curves coincide within the calculation error. For $\delta = 2.15$ and $\varepsilon_0 > 0$ it was found impossible to obtain the value of Γ from the results of simulation, since the instability develops too quickly and the time interval in which the transient process is still observed, goes directly into the region of τ values, where the linear theory is no longer applicable. On the other hand, at $\varepsilon_0 < 0$, we obtained $\Gamma = 0.029$ for $\delta = 2.2$ and $\Gamma = 0.086$ for $\delta = 2.15$. Analytical calculation gives in these cases the values of 0.032 and 0.091, respectively.

After the process leaves the area where the linear theory is applicable, the PD extremum starts to decrease monotonically in absolute value in all cases considered. The rate of its change initially increases and then starts to decrease as the homogeneous solution is approached. In all cases, the evolution of the field distribution at last stage shows that it tends asymptotically to a homogeneous solution (Fig. 3). We found the values of the growth rate Γ when approaching to a homogeneous solution and compared them with those found analytically in [4]. For $\delta = 2.2$, numerical calculations gave $\Gamma = -0.0222$, while the analytical value of the growth rate was -0.0216. In the case of $\delta = 2.15$, $\Gamma = -0.074$ is obtained both numerically and analytically.

Thus, we have traced the evolution of the perturbed steady-state solutions from the n_1 branch to reaching a stable homogeneous stationary solution. In addition, we have validated both the analytical results and the numerical ones.

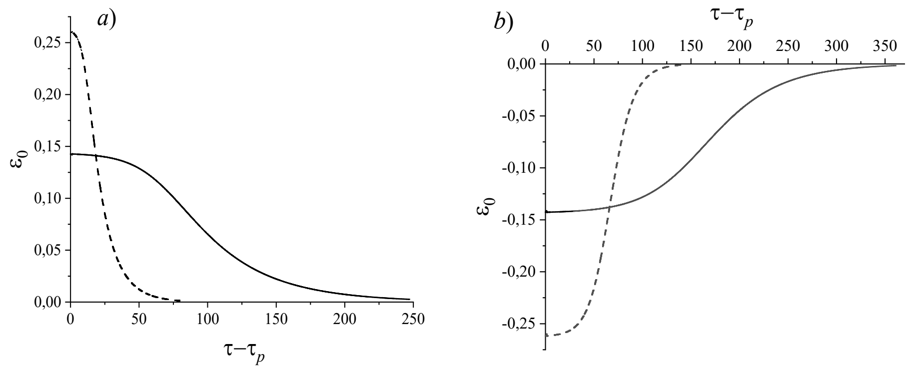

Fig. 3. The electric field strength at the left electrode ε_0 evolution during the process of perturbation development at diode lengths $\delta = 2.15$ (dashed lines) and 2.2 (solid lines) for steady-state solutions with $\varepsilon_0 > 0$ (*a*) and $\varepsilon_0 < 0$ (*b*)

Conclusion

We have studied the inhomogeneous steady-state solutions stability features for a diode with counter-streaming electron and ion flows for the mode without reflection of charged particles from potential barriers, using the equation for the amplitude of the electric field perturbation derived in [4]. To perform the investigation, we have developed an original semi-analytical method for solving this equation, which works for any undisturbed potential distribution. It is shown that the inhomogeneous steady-state solutions corresponding to the n_k branches with $0 < k \leq 5$ are unstable. The results of the theory are confirmed by numerical calculations using a high-precision numerical code. Based on our analysis we can suppose that in a diode with counter streams of electrons and ions, inhomogeneous stationary solutions in the mode without particle reflection cannot exist. Earlier [4], we found that for homogeneous steady-state solutions there is a threshold in the value of the inter-electrode gap δ_{th} above which the steady-state solutions become unstable; the steady-state solutions without reflection can exist only when $\delta < \delta_{th}$. It is shown that in the mode without particle reflection, the main instability mode is aperiodic.

The stability of some solutions with reflection of particles from potential barriers was studied numerically in our previous paper [5]. It turned out that for the solutions corresponding to the branch d_0 (see Fig. 4 in [2]), there is a threshold in δ, above which the solutions turn out to be unstable, and non-linear periodic oscillations develop. In this case, the main instability mode is oscillatory one. To obtain a complete picture of solutions that are implemented in a diode with counter-streaming electron and ion flows, it is necessary to develop a stability theory for the mode with reflection of charged particles from potential barriers. Besides, to describe the operation of real plasma switches, it is necessary to take into account also the scattering of ions by atoms, i.e. take into account the ion charge exchange [1, 3].

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