Conference materials
UDC 519.6
DOI: https://doi.org/10.18721/JPM.161.153

Numerical simulation of waveguide couplers using the coupled mode theory for quantum gates implementation

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Abstract. The directional coupler formed by a system of two dual-mode optical waveguides is studied with the aim of being used as a switcher for a nonlinear optical CNOT quantum gate. The paper focuses on simulation of behaviour of electromagnetic radiation in regions of juxtaposition and separation, that surround the main coupling region and are composed of several circularly bent waveguides. The modes of bent waveguides are approximated as linear combinations of the guided and leaky modes in the straight waveguide with the same width and refractive indices. An advanced coupled mode theory is applied to describe the coupling between bent parts of the coupler. The system of differential equations for amplitude coefficients is solved with a finite difference method. The influence of signal distortions is analyzed. The results obtained are applied to correct the geometrical parameters of the coupler. The computational error of the whole device due to waveguide bends distortions is estimated to not exceed 5%.

Keywords: waveguides, quantum computing, coupled mode theory, CNOT quantum gate, bent waveguides, leaky modes

Citation: Lytaev A.A., Popov I.Yu., Numerical simulation of waveguide couplers using the coupled mode theory for quantum gates implementation, St. Petersburg State Polytechnical University Journal. Physics and Mathematics. 16 (1.1) (2023) 320–325. DOI: https://doi.org/10.18721/JPM.161.153

Материалы конференции
УДК 519.6
DOI: https://doi.org/10.18721/JPM.161.153

Численное моделирование сближения волноводов с помощью теории связанных волноводов для реализации квантовых вентилей

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Аннотация. Данная работа посвящена моделированию взаимодействия системы близко расположенных оптических волноводов с целью ее использования в качестве переключателя для реализации квантового CNOT вентиля. В работе проводится моделирование поведения электромагнитного излучения в областях сближения и удаления волноводов, смежных с прямым участком обмена энергиями, и состоящих из нескольких изогнутых волноводов. Моды изогнутых волноводов приближенно рассматриваются как линейные комбинации направляемых и вытекающих мод прямого волновода с теми же толщиной и коэффициентом преломления. Для описания взаимодействия между изогнутыми частями схемы применяется модифицированная теория связанных мод. Система дифференциальных уравнений решается с помощью метода конечных разностей. Изучается влияние искажений сигнала. Полученные результаты применяются для того, что скорректировать геометрические параметры

системы. В результате моделирования показывается, что вычислительная ошибка вентиля вследствие искажений сигнала на изгибах не превышает 5%.

Ключевые слова: волноводы, квантовые вычисления, теория связанных мод, CNOT квантовый вентиль, изогнутые волноводы, вытекающие моды

Ссылка при цитировании: Лытаев А.А., Попов И.Ю. Численное моделирование сближения волноводов с помощью теории связанных мод для реализации квантовых вентилей // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2023. Т. 16. № 1.1. С. 320–325. DOI: https://doi.org/10.18721/JPM.161.153

Introduction

The problem of physical implementation of the prototype of quantum computer is indeed an important question since the idea was proposed by Feynman [1], so it would be possible to solve a series of numerical problems with notable computational acceleration. Quick decoherence is a major obstacle encountered in this way (see, e.g., [2]). The optical model of quantum computations is an essential way to get around this hurdle since photons are known to be characterized with a relatively low interaction efficiency.

There are two main approaches to the optical model, namely linear optical and nonlinear. The former [3] is based on employing various linear optical devices, such as mirrors, beam splitters and phase shifters. In [4] it is shown that this approach allows to create a probabilistic CNOT gate with 1/9 probability of success. Such a low probability makes these gates hardly applicable for a practical computational scheme. The latter approach is described in [5]. It increases the probability of successful gate operation, but at the cost of long optical paths effectively making the gates susceptible to computational errors.

The idea described in [6] is an approach to curtail the optical paths by increasing the intensities of nonlinear interaction. The article introduces the optical scheme to implement the nonlinear CNOT gate that could be used to compute by using both single photon quantum states and classical quantum-like photonic states. In [7] we studied the latter case. Quantum bits are encoded by optical transverse modes of optical waveguides, namely TE_0 mode encodes $|0\rangle$ state and TE_1 encodes $|1\rangle$. The stronger is the TE_1 part of the signal in control waveguide the larger part of it is transferred into the upper arm of the MZI in the target waveguide and the stronger is the phase shift in nonlinear parts with intensity dependent refractive index, while TE_0 mode is pertained in the control waveguide. However, for the scheme to correctly work the coupling length L and the distance between coupled waveguides R are required to be matched in a specific way.

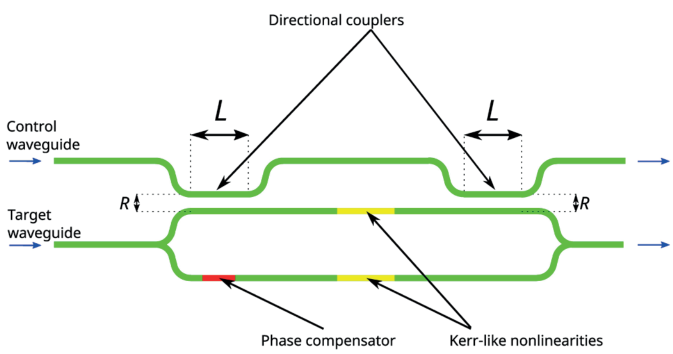

Fig. 1. Optical CNOT quantum gate

In [7] we applied a conventional coupled mode theory [8] to the straight part of the directional coupler and obtained an analytical solution for amplitude coefficients of the modes as trigonometric functions of spatial coordinate, so an explicit condition on the coupling coefficients ratio allowed us to adjust the geometrical parameters in a desired way. However, possible distortions to the state of quantum bit due to coupling and losses in the bent parts of the directional coupler were not considered. In this paper these parts of the directional coupler are studied. The field in the system is decomposed into a sum of bent waveguide modes and a modified coupled mode theory is applied to access and correct the distortions.

Modes of the bent waveguide

The spectrum of an ideal bent waveguide unlike that of the straight one does not include a discrete set of guided modes; it is fully continuous. However, not unlike the spectrum of straight waveguide radiation modes it can be approximated with a set of modes with complex propagation constants. This decomposition is not unique, in [9] one of the possible ways is discussed. The idea is to represent bent modes as a linear combination of modes of the straight waveguide with the same width and refractive indices. In the plane where the angular distance is zero the field in the bent waveguide can be represented as a linear combination of them of the straight waveguide. The angular distance accumulates the phase and attenuation with a propagation constant

$$\exp\left[-i\nu^j d\theta\right] E_b^j\left(x\right)|_{\theta=0} = E_b^j(x)|_{\theta=d\theta} = \sum_{n=1}^{N} a_n^j E_n\left(x\right)\exp\left[-i\beta_n\left(R_b+x\right)dz\right], \tag{1}$$

where E_b^j is the transverse field distribution of j-th bent mode, E_n is that of n-th straight mode, R_b is a radius of bending and ν_j is a propagation constant of the bent mode. It is possible to rewrite the equation (1) into the eigenvalue problem for eigenvalues ν_j and eigenvector a^j

$$(R_b B + D^{-1}CB)a^j = \nu^j a^j, \quad j = 0,1,2 \tag{2}$$

where

$$B_{ij} = \beta_i \delta_{ij}, \tag{3}$$

$$C_{ij} = \int_{-\infty}^{+\infty} \overline{E^i(x)} E^j(x) x dx, \tag{4}$$

$$D_{ij} = \int_{-\infty}^{+\infty} \overline{E^i(x)} E^j(x) dx. \tag{5}$$

In [9] the numerical experiment shows that the set of straight waveguide modes composed of all supported guided modes and one leaky mode of the lowest order is sufficient to get a valid approximation of the field in the bent waveguide with sufficiently large radius of curvature.

Snyder [10] shows that leaky mode propagation constants can be found from the same characteristic transcendental equation as that for guided modes, however it should be solved on the complex domain. The root with the real part closest to that of the guided mode of the highest order will represent a lowest order leaky mode. Thus, it could be found with a Newton-Raphson method initialized with the propagation constant of the TE_1 mode. The complex propagation constant describes a leaky mode, that behaves similar to the guided mode in the proximity of the waveguide core, however, is attenuated due to the negative imaginary part. The solution of (2) yields three propagation constants, each of them representing one of the bent modes. ν_0 and ν_1 are expected to have significantly smaller imaginary parts than that of ν_2 since bent TE_2 mode is formed primarily by the straight leaky TE_2 mode. Hence, the model describes two main sources of the signal loss: an attenuation of weakly radiative TE_0 and TE_1 modes and a loss of the signal at straight-to-bent waveguide transitions.

Simulation of coupling in juxtapositions and separations

In this paper regions of juxtaposition and separation that surround the straight part are divided into two bent parts and one straight inclined part (see Fig. 2).

In order to estimate losses in the bent parts and coupling in the whole region a modified coupled mode theory is employed. For simplicity only the coupling between the fields with the same z coordinate is considered, which serves as an approximation of the real field behaviour in the system, however, it is expected to suffice to estimate the distortions due to losses and coupling in the bent parts. Under this assumption it is possible to conduct the same derivation of differential equations for mode amplitude coefficients as it was done for the pair of straight waveguides in [7].

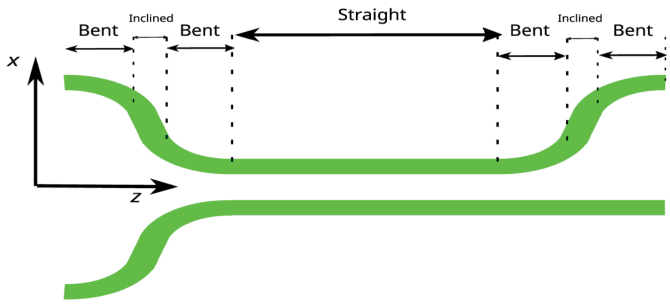

Fig. 2. The scheme of directional coupler

$$\frac{\partial A_{\nu_1 m_1}}{\partial z} c_{m_1} + \sum_{m_2=0}^{1} iA_{((\nu_1+1) mod\, 2) m_2} \tilde{D}_{m_2}^{m_1} e^{i(\beta_{m_1}-\beta_{m_2})z} = 0. \tag{6}$$

However, it should be noted that coupling coefficients are integrals of bent modes field distributions.

$$\tilde{D}_{m_1}^{m_2} = \int_{-\infty}^{+\infty} (\omega\varepsilon_0)\left(n_{core}^2 - n_{clad}^2\right) E_{m_1}^b(x) E_{m_2}^b(x) \cos(\alpha) dx, \tag{7}$$

where α is an angle between the direction of the local longitudinal axis of waveguide and the global one. By applying the decomposition (1) it is possible to express bent coupling coefficients in terms of the straight ones as

$$\tilde{D}_{m_1}^{m_2} = \sum_{m_3, m_4=0}^{2} a_{m_3}^{m_1} a_{m_4}^{m_2} D_{m_3}^{m_4} \cos(\alpha),\ m_1, m_2 = 0,1. \tag{8}$$

The coupled mode theory for inclined straight parts remains the same as for the straight except for multiplication by cos(α).

In order to account for distortions at straight-to-bent transitions a mode matching procedure is employed, the amplitude coefficients $A_{\nu m}$ can be expressed from them of bent waveguide modes $\tilde{A}_{\nu m}$ as

$$A_{\nu m} = \frac{\int_{-\infty}^{+\infty} (\tilde{A}_{\nu 0} \overline{E_0(x) E_m^b}(x) + \tilde{A}_{\nu 1} \overline{E_1(x) E_m^b}(x)) dx}{\int_{-\infty}^{+\infty} \overline{E_m(x) E_m}(x) dx}. \tag{9}$$

The modes of bent waveguides are not orthogonal strictly speaking, however, numerical calculations showed them to be small, thus for simplicity the reverse transition can be described with similar equations

$$\tilde{A}_{\nu m} = \frac{\int_{-\infty}^{+\infty} (A_{\nu 0} \overline{E_0^b(x) E_m}(x) + A_{\nu 1} \overline{E_1^b(x) E_m}(x)) dx}{\int_{-\infty}^{+\infty} \overline{E_m^b(x) E_m^b}(x) dx}. \tag{10}$$

The system of differential equations (6) is solved with the finite difference method.

Numerical results

The technique discussed above was applied to the system with refractive indices of waveguide core and cladding n_{clad} = 1.57 and n_{core} = 1.55 correspondingly, width of waveguide d =1.18 μm, radiation wavelength λ = 1.064 μm, radius of bending R_b = 100 μm and the minimal distance between the waveguides outside of the coupling zone R_f = 100 μm. The distance between waveguides in the coupling zone is taken from [7] as R = 2.36 μm. The resulting propagation constants are given in Table 1. The resulting behaviour of amplitude coefficients of signal modes is given in Figures 3–4.

Amplitude coefficients are normalized at they would be during the measurement at the output since the relation $|A_{\nu 0}|^2 + |A_{\nu 1}|^2 = 1$ must be fulfilled despite attenuation and losses at straight-to-bent transitions. It can be concluded that attenuation and straight-to-bent transitions affect both modes in a very similar way and hence do not distort the quantum states notably. However, additional coupling regions do increase the phase of periodic energy exchange between the

Table 1

Propagation constants

Propagation constant	TE_0	TE_1	TE_2
$\beta_{straight}$	9.234	9.156	9.049-0.241i
β_{bent}	9.264–0.005i	9.130–0.005i	9.044–0.236i

Notations: β_{straight} and β_{bent} are the propagation constants of given modes in the straight and bent waveguides respectively

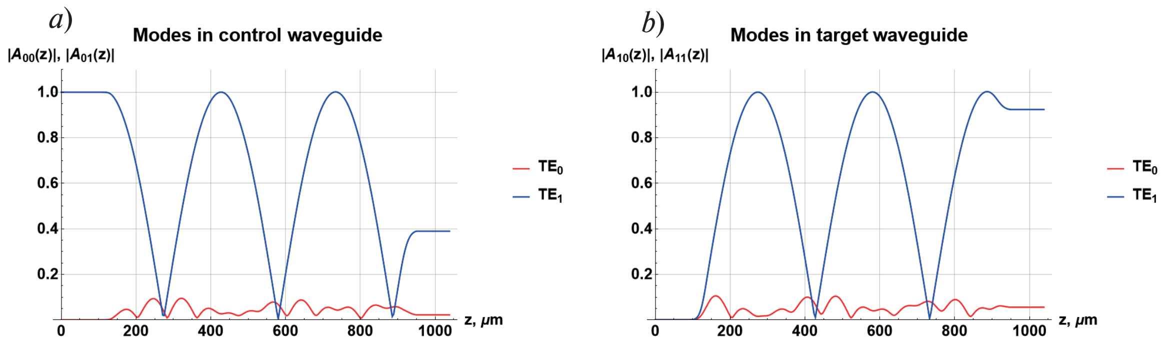

Fig. 3. Amplitude coefficient in (*a*) control waveguide (*b*) target waveguide

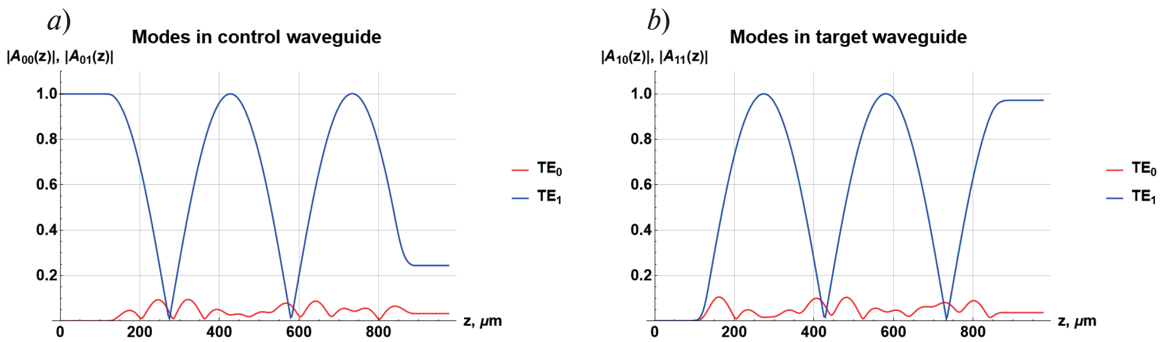

Fig. 4. Amplitude coefficient after corrections to parameter L in (*a*) control waveguide (*b*) target waveguide

waveguides, thus the computational error of the devise cab be reduced by modifying the coupling length L. The results are provided in Fig. 4.

The corrected coupling length L allows the device to transfer the TE_1 mode to the opposite waveguide with the error of 2%, thus the error of the whole CNOT gate can be estimated to be 5%.

Conclusion

The model of directional couplers based on the coupled mode theory discussed provides a tool to perform a computationally fast simulation of the behaviour of electromagnetic radiation within the system. Moreover, it allows to estimate the influence of distortions due to waveguide bending and straight-to-bent transitions. Application of this technique has shown that distortions do not affect the state of waveguide quantum bits notably, while the coupling at juxtapositions and separations do increase the phase of periodic energy exchange between the waveguides, however, it could be negated by decreasing the coupling length L. It was shown that the cumulative computational error due to distortions caused by bent parts and intermodal interaction does not exceed 5%, which could be coped with by application of quantum correction algorithms. However, despite both modes being attenuated similarly, the total attenuation of the signal in the computational scheme was found to be around 25%. That could be a source of potential difficulties for creating the device composed of a large number of CNOT gates applied sequentially.

Acknowledgments

The work was supported by Russian Science Foundation (grant number 22-11-00046).

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Received 28.10.2022. Approved after reviewing 09.11.2022. Accepted 09.11.2022.