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Study of mechanical resonance frequencies in tapered nanowires

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Abstract. In this paper, the transcendental equation was obtained in the framework of the Euler–Bernoulli beam theory, which allows obtaining the values of resonant frequencies for any tapered nanowire. Calculations of the frequencies of the first few resonances of mechanical oscillations for nanowires with various conicity (the average radius and length of the nanowire remained constant) were performed. It was established that the frequencies of the first three modes increase with an increase in the conicity angle, while the frequency of the fourth mode ($n = 4$) is nearly constant and independent of the conicity angle. It was also established that the frequencies of the higher order ($n > 4$) modes decrease with an increase in the conicity angle. The ratios of the resonance values of the first few modes can be used to clarify the conicity value, which is necessary when determining the Young's modulus of tapered nanowires.

Keywords: nanowires, tapered nanowires, mechanical resonances frequencies, Young's modulus

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Механические резонансы в конических нитевидных нанокристаллах

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Аннотация. В данной работе в рамках теории Эйлера–Бернулли получено трансцендентное уравнение, позволяющее получить значения резонансных частот для любых конических нитевидных нанокристаллов. Проведены расчеты частот первых нескольких резонансов механических колебаний для нитевидных нанокристаллов различной конусности (средний радиус и длина нитевидных нанокристаллов при этом оставались постоянными). Установлено, что частоты первых трех мод увеличиваются с увеличением угла конусности, а частота четвертой моды ($n = 4$) практически постоянна и не зависит от угла конусности. Установлено также, что частоты мод высших порядков ($n > 4$) уменьшаются с увеличением угла конусности. Соотношения резонансных значений первых нескольких мод могут быть использованы для уточнения значения конусности, которое необходимо при определении модуля Юнга конических нитевидных нанокристаллов.

Ключевые слова: нитевидные нанокристаллы, конические нитевидные нанокристаллы, частоты механических резонансов, модуль Юнга

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Introduction

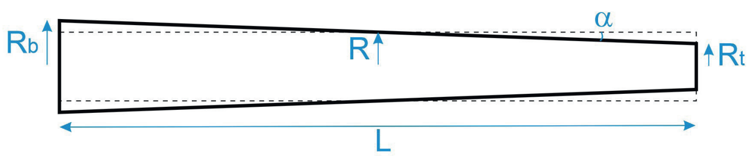
Studies of transport, optical and mechanical properties of semiconductor nanowires (NW) have been of interest recently. A feature of thin nanowires is their ability to withstand without breaking quite large mechanical stresses and deformations (up to 10%) [1]. Semiconductor nanowires can be used in sensor devices [2], triboelectric nanogenerators [3] and flexible microelectronics devices. Thus, the study of the mechanical properties of nanowires, the measurement of their elastic modules [4], as well as their resonant frequencies of mechanical oscillations, is an actual task today.

In this work, the calculation (based on the Euler–Bernoulli beam theory) of resonant frequencies values for tapered nanowires will be performed. The dependence of the resonant frequencies values on the conicity parameter of the nanowire will be investigated, while the average radius and the length of the nanowire will be fixed. Calculations of resonant frequencies for some typical tapered shaped III-V nanowires will be performed. Based on the measurement of several resonant frequencies, the approach will be proposed to clarify the value of the conicity parameter, which is necessary for measuring the Young's modulus of tapered nanowires.

Results and Discussion

Figure 1, a shows a scheme of tapered nanowire oscillation. The figure shows the main geometric parameters describing the conical nanowire (R_b is radius at the base of NW, R_t is radius at the top of NW, R is average radius of NW, L is length of the nanowire, α is angle of conicity). Figure 1, *b* shows a schematic image of the oscillating nanowire (mode n = 2). It should be noted here, that the tapered nanowire, as well as a cylindrical one, has an endless set of oscillation modes and resonant frequencies. In this work, we will consider the case of the oscillations of nanowire, when the base of the nanowire is fixed, and the top end of NW is free. This corresponds to common situation when the nanowire was grown epitaxially on the substrate.

a)



b)

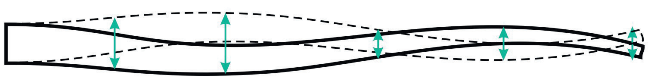


Fig. 1. Scheme of tapered nanowire (R_b is radius at the base of NW, R_t is radius at the top of NW, R is average radius of NW, L is length of the nanowire, α is angle of conicity) (*a*); schematic image of the oscillating nanowire (mode n = 2) (*b*)

Below is the time-dependent Euler- Bernoulli equation of the oscillating nanowire together with the boundary conditions for nanowire with the fixed base and the free upper end.

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 u(x,t)}{\partial x^2}\right) = -\mu(x)\frac{\partial^2 u(x,t)}{\partial t^2}, \tag{1}$$

$$u(x=0,t)=0, \tag{1.a}$$

$$\frac{\partial u}{\partial x}(x=0,t)=0, \tag{1.b}$$

$$\frac{\partial^2 u}{\partial x^2}(x=L,t)=0, \tag{1.c}$$

$$\frac{\partial}{\partial x}\left(x^4\frac{\partial^2 u}{\partial x^2}\right)(x=L,t)=0, \tag{1.d}$$

where E is the nanowire Young's modulus, $I(x) = \pi R^4(x)/4$ is the second area moment of inertia of the tapered nanowire, L is the length of NW, $\mu(x)$ is the mass per unit length $\mu(x) = \rho\pi R^2(x)$, $R(x)$ is the local radius of the nanowire $R(x) = \alpha x$, α is the conicity angle of NW. Dependence on time is taken into account as follows: $u \sim \exp[i\omega t]$, which allows one to obtain the next equation

$$\frac{\partial^2}{\partial x^2}\left(x^4\frac{\partial^2 u(x)}{\partial x^2}\right)=\frac{4\rho}{E\alpha^2}x^2\omega^2 u(x). \tag{2}$$

The general solution of this equation can be expressed through the Bessel functions (J_2, Y_2, I_2, K_2) in the following form.

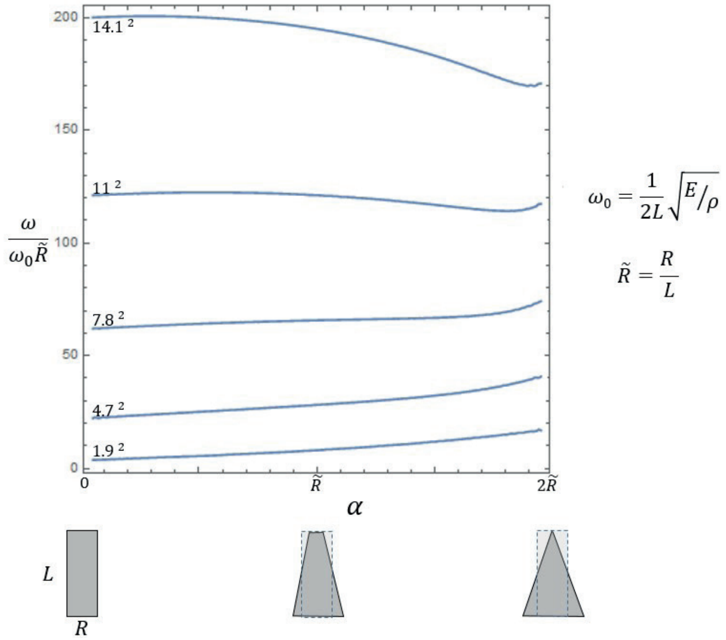


Fig.2 Calculated values of resonant frequencies for the first five oscillation modes depending on the conicity coefficient α for nanowire with R/L=0.1

$$u(\xi)=A\xi^{-1}J_2\left(2\sqrt{\xi}\right)+B\xi^{-1}Y_2\left(2\sqrt{\xi}\right)+C\xi^{-1}I_2\left(2\sqrt{\xi}\right)+D\xi^{-1}K_2\left(2\sqrt{\xi}\right). \tag{3}$$

Here ξ is the dimensionless position at the nanowire $\xi = 2x\omega(\rho/E)^{1/2}/\alpha$.

By using the expression (3) in the boundary conditions, one gets a system of the following four linear equations:

$$AJ_2\left(2\sqrt{b}\right)+BY_2\left(2\sqrt{b}\right)+CI_2\left(2\sqrt{b}\right)+DK_2\left(2\sqrt{b}\right)=0, \tag{4.1}$$

$$AJ_3\left(2\sqrt{b}\right)+BY_3\left(2\sqrt{b}\right)-CI_3\left(2\sqrt{b}\right)+DK_3\left(2\sqrt{b}\right)=0, \tag{4.2}$$

$$AJ_3\left(2\sqrt{t}\right)+BY_3\left(2\sqrt{t}\right)+CI_3\left(2\sqrt{t}\right)-DK_3\left(2\sqrt{t}\right)=0, \tag{4.3}$$

$$AJ_4\left(2\sqrt{t}\right)+BY_4\left(2\sqrt{t}\right)+CI_4\left(2\sqrt{t}\right)+DK_4\left(2\sqrt{t}\right)=0. \tag{4.4}$$

Here $b = (\omega/\omega_0)(R/L\text{-}\alpha/2)/\alpha^2$ is the dimensionless position of bottom (fixed) end of the nanowire and $t = (\omega/\omega_0)(R/L+\alpha/2)/\alpha^2$ is the position of top (free) end of the NW. This system of equations (4.1-4.4) has non-trivial solutions when the determinant of the corresponding matrix is equal to zero. Thus, by setting the determinant of the matrix to zero, one can get an equation whose most compact form is given below:

$$\{J_2 I_3\}(t)\{Y_3 K_4\}(b)+[J_2 K_3](t)[Y_3 I_4](b)-\{Y_2 I_3\}(t)\{J_3 K_4\}(b)-[Y_2 K_3](t)[J_3 I_4](b)=1/\pi\sqrt{bt}. \quad (5)$$

“Commutator-like” and “anticommutator-like” brackets are introduced here:

$$\begin{aligned}\{F_n G_m\}(x) &= F_n(x)G_m(x)+F_m(x)G_n(x),\\ [F_n G_m](x) &= F_n(x)G_m(x)-F_m(x)G_n(x).\end{aligned} \quad (6)$$

Using equation (5), one can numerically obtain the resonant frequencies for various conicity angles of NW. Figure 2 shows the results of the calculation of resonant frequencies for the first five oscillation modes depending on the conicity coefficient. It should be noted that the calculated values of the resonant frequencies on the left side of the graph ($\alpha = 0$) exactly correspond to the resonant frequencies for a non-tapered cylindrical nanowire. It can be seen that the frequencies of the first three modes increase with an increase in the conicity angle, while the frequency of the fourth mode ($n = 4$) is nearly constant and non-depended on the conicity angle. The frequencies of the higher order ($n > 4$) modes also decrease with an increase in the conicity angle.

Conclusion

Since the values of resonant frequencies can be experimentally determined with a high degree of accuracy, it becomes possible to refine the value of the conicity angle α from the ratios of the frequencies of the first several resonances (ω_2/ω_1 and ω_3/ω_1). This is actual for the accurate determination of the Young's modulus in sufficiently thin conical NWs.

REFERENCES

1. **Wang Y., Wang L., Joyce H., Gao Q., Liao X., Mai Y., Tan H., Zou J., Ringer S., Gao H., Jagadish C.,** Super deformability and Young's modulus of GaAs nanowires, Adv. Mater. 23 (2011) 1356–1360.

2. **Alekseev P., Sharov V., Dunaevskiy M., Kirilenko D., Ilkiv I., Reznik R., Cirlin G., Berkovits V.,** Control of conductivity of $In_xGa_{1-x}As$ nanowires by applied tension and surface states, Nano Letters. 19 (2019) 4463–4469.

3. **Alekseev P., Sharov V., Geydt P., Dunaevskiy M., Lysak V., Cirlin G., Reznik R., Khrebtov A., Soshnikov I., Lähderanta E.,** Piezoelectric current generation in wurtzite GaAs nanowires, Phys. Stat. Sol. RRL. 12 (2018) 1700358.

4. **Dunaevskiy M., Geydt P., Lähderanta E., Alekseev P., Haggrén T., Kakko J., Jiang H., Lipsanen H.,** Young's modulus of wurtzite and zinc blende InP nanowires, Nano Letters. 17 (2017) 3441–3446.

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