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Dynamics of the uncertainty value of quadratures for bosonic quantum states

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Abstract. In this work we consider the time evolution of the mean values of the first and second moments of the quadrature operators for an arbitrary bosonic quantum state in a single mode transmitted through an optical fiber channel. We utilize the density matrix formalism and the open quantum systems theory and investigate Lindblad master equation in order to derive expressions for the dynamics of mentioned field observables. Obtained expressions contain terms characterized by high frequency oscillations. For the purpose of elimination of these terms we find the envelope functions for the values of the first and second moments of the quadrature operators.

Keywords: quantum optics, open quantum systems theory, quadratures, single mode

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Динамика неопределенности квадратур бозонных квантовых состояний

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Аннотация. В работе рассматривается временная эволюция первых и вторых моментов квадратур одномодовых фотонных квантовых состояний, передаваемых по оптоволоконному каналу. При использовании формализма матриц плотности и теории открытых квантовых систем выведены выражения, описывающие динамику указанных выше полевых наблюдаемых. Для исключения из анализируемых выражений членов, характеризующихся высокочастотными осцилляциями, были найдены огибающие значений.

Ключевые слова: квантовая оптика, теория открытых квантовых систем, квадратуры, одномодовый случай

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Introduction

One of the main constraints on the technological utilization of the unique quantum features such as superposition or squeezing lies in decoherence: the detrimental influence of environment leads any quantum system to the loss of its beneficial quantum features [1]. A theory that may be employed to investigate the evolution of quantum systems considering decoherence is open quantum systems approach [2]. Within this theory different methods are being used; in our research we focus on solving a master equation for a density matrix of a quantum state.

Materials and Methods

In order to give a description of a nonunitary dynamics of a bosonic quantum state study the Liouville master equation that is a special case of the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) master equation [3]:

$$\begin{cases} \dfrac{\partial}{\partial t}\hat{\rho}(t) = -i\left[\hat{H},\hat{\rho}(t)\right] + \hat{\Gamma}\hat{\rho}(t) \\ \hat{\rho}(t)\big|_{t=0} = \hat{\rho}_0 \end{cases}, \tag{1}$$

where Hamiltonian of the system is

$$\hat{H} = \omega\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right), \tag{2}$$

here $\hat{a}$, $\left(\hat{a}^\dagger\right)$ are the annihilation (creation) operator, ω is the frequency of the optical mode, and relaxation superoperator acting on a density matrix of a state has the form:

$$\hat{\Gamma}\hat{\rho}(t) = -\frac{\gamma}{2}(n_T+1)\left(\hat{a}^\dagger\hat{a}\rho(t) + \rho(t)\hat{a}^\dagger\hat{a} - 2\hat{a}\rho(t)\hat{a}^\dagger\right) - \frac{\gamma n_T}{2}\left(\hat{a}\hat{a}^\dagger\rho(t) + \rho(t)\hat{a}\hat{a}^\dagger - 2\hat{a}^\dagger\rho(t)\hat{a}\right), \tag{3}$$

where γ denotes the thermalization rate, n_T denotes the mean number of thermal photons.

The explicit solution to this equation may be found, for example, with the use of SU(1,1) algebra formalism [4], Jordan mapping [5]. In this investigation we act by the quadrature operators [6]:

$$\hat{q} = \frac{1}{\sqrt{2}}\left(\hat{a}^\dagger + \hat{a}\right), \qquad \hat{p} = \frac{i}{\sqrt{2}}\left(\hat{a}^\dagger - \hat{a}\right), \tag{4}$$

of interest on the master equation, apply the trace operation and then solve the resulting equation to obtain the time-dependence of a mean value of an operator [7, 8].

Results and Discussion

Firstly, utilizing the method described in the above, we obtain evolutionary equations for the first moment of quadratures operators:

$$\langle q\rangle = e^{-\frac{\gamma}{2}t}\left(\langle q_0\rangle\cos\omega t + \langle p_0\rangle\sin\omega t\right),$$
$$\langle p\rangle = e^{-\frac{\gamma}{2}t}\left(\langle p_0\rangle\cos\omega t - \langle q_0\rangle\sin\omega t\right), \tag{5}$$

where q_0, p_0 are the mean values of the quadrature operators at the initial time moment:

$$\langle q_0\rangle \equiv Tr\left\{\hat{q}\hat{\rho}_0\right\},$$
$$\langle p_0\rangle \equiv Tr\left\{\hat{p}\hat{\rho}_0\right\}. \tag{6}$$

Eqs. (5) show that the time dependence of both quadrature operators' mean values has the form of high frequency (optical frequency ω) damped oscillations. However, the part of steady-state oscillations does not provide the essential information concerning the dynamics of the quadratures. Thus we find the envelope function:

$$f_{env.}(t) = \pm\sqrt{\langle q_0\rangle^2 + \langle p_0\rangle^2}\, e^{-\frac{\gamma}{2}t}, \tag{7}$$

that has an identical form for both $q(t)$ and $p(t)$.

Secondly, keeping in mind that some quantum states, for example, squeezed vacuum states, possess unique qualities which can be observed through the use of the second order of an operator, we derive the equations for the squares $\langle q^2\rangle$, $\langle p^2\rangle$:

$$\langle q^2\rangle = \frac{1}{2}\left(e^{-\gamma t}\left(c_0 + a_0\cos 2\omega t + b_0\sin 2\omega t - d_0\right) + d_0\right),$$
$$\langle p^2\rangle = \frac{1}{2}\left(e^{-\gamma t}\left(c_0 - a_0\cos 2\omega t - b_0\sin 2\omega t - d_0\right) + d_0\right), \tag{8}$$

and variances $\Delta q(t)$, $\Delta p(t)$ of the quadratures:

$$\Delta q = \frac{1}{2}\left(e^{-\gamma t}\left(C_0 + A_0\cos 2\omega t + B_0\sin 2\omega t - d_0\right) + d_0\right),$$
$$\Delta p = \frac{1}{2}\left(e^{-\gamma t}\left(C_0 - A_0\cos 2\omega t - B_0\sin 2\omega t - d_0\right) + d_0\right), \tag{9}$$

where coefficients are

$$\begin{aligned} a_0 &= \langle q_0^2\rangle - \langle p_0^2\rangle, & A_0 &= \Delta q_0 - \Delta p_0, \\ b_0 &= \langle qp_0\rangle + \langle pq_0\rangle, & B_0 &= \langle qp_0\rangle + \langle pq_0\rangle - 2\langle q_0\rangle\langle p_0\rangle, \\ c_0 &= \langle q_0^2\rangle + \langle p_0^2\rangle, & C_0 &= \Delta q_0 + \Delta p_0, \\ d_0 &= 2n_T + 1, \end{aligned} \tag{10}$$

Δq_0, Δp_0 are the mean value variances of the quadrature operators at the initial moment of time:

$$\Delta q_0 \equiv \langle q_0^2\rangle - \langle q_0\rangle^2,$$
$$\Delta p_0 \equiv \langle p_0^2\rangle - \langle p_0\rangle^2, \tag{11}$$

and $\langle q_0^2\rangle, \langle qp_0\rangle$ and $\langle pq_0\rangle$, $\langle p_0^2\rangle$ are the constituents of the covariance matrix:

$$\begin{bmatrix} \langle q_0^2\rangle & \langle qp_0\rangle \\ \langle pq_0\rangle & \langle p_0^2\rangle \end{bmatrix} \equiv \begin{bmatrix} Tr\left\{\hat{q}^2\hat{\rho}_0\right\} & Tr\left\{\hat{q}\hat{p}\hat{\rho}_0\right\} \\ Tr\left\{\hat{p}\hat{q}\hat{\rho}_0\right\} & Tr\left\{\hat{p}^2\hat{\rho}_0\right\} \end{bmatrix} \tag{12}$$

It can be seen that the dynamics of both the squares and variances incorporates high frequency (optical frequency) damped oscillations likewise. Thus, we proceed to determine the envelope function of the dynamics. We find bend points for multiplier of $e^{-\gamma t}$ from, for example, Eqs. (9):

$$\frac{\partial}{\partial t}\left(C_0 + A_0\cos 2\omega t + B_0\sin 2\omega t - d_0\right) = 0,$$
$$\frac{\partial}{\partial t}\left(C_0 - A_0\cos 2\omega t - B_0\sin 2\omega t - d_0\right) = 0, \tag{13}$$

After a simplification we obtain the following envelope functions of the dynamics of the squares F_{env}:

$$F_{env.}(t) = \frac{1}{2}\left(\left(c_0 \pm \sqrt{a_0{}^2 + b_0{}^2} - d_0\right)e^{-\gamma t} + d_0\right), \tag{14}$$

and variances Δf_{env} of the quadratures:

$$\Delta f_{env.}(t) = \frac{1}{2}\left(\left(C_0 \pm \sqrt{A_0{}^2 + B_0{}^2} - d_0\right)e^{-\gamma t} + d_0\right). \tag{15}$$

Obtained expressions are of a more utility considering technical realisation of the quadratures detection [9].

Conclusion

Utilizing the method of solving GKSL master equation for a mean value of a particular operator we obtain the expressions of time evolution for the first and second moments of the quadrature operators for an arbitrary bosonic quantum state in a single mode transmitted through an optical fiber channel. Moreover, we find the envelope functions for the obtained expressions for the purpose of detection that are of a more utility considering technical realisation.

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