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The kinetic simulation in vacuum electronics: uncovering the fundamental nature of non-Maxwellian distribution function effects

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Abstract. This paper presents main theoretical results obtained in vacuum explosive-emission electronics by using of the numerical methods aimed to the direct solution of Vlasov Poisson kinetic equations. It was shown that computational physical kinetics makes it possible to explain a number of important physical laws occurring in vacuum diodes unlike widely used Particle-in-Cell or hydrodynamic simulation. The kinetic approach makes it possible to take into account nonequilibrium non-Maxwellian effects mainly associated with the "tails" of distribution functions. The advantages of kinetic simulation are shown on two highly relevant problems of vacuum electronics are considered in details.

Keywords: vacuum electronics, physical kinetics, numerical simulation

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Материалы конференции
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Кинетическое моделирование в вакуумной электронике: раскрытие фундаментальной природы эффектов немаксвелловской функции распределения

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Аннотация. В статье представлены основные теоретические результаты, полученные в вакуумной взрывоэмиссионной электронике с использованием численных методов решения системы кинетических уравнений Пуассона-Власова. Было показано, что именно вычислительная физическая кинетика позволяет объяснить ряд важных физических закономерностей, происходящих в вакуумных диодах. Кинетический подход согласованно учитывает неравновесные немаксвелловские эффекты, определяемые т.н. «хвостами» функций распределения. Подробно показаны преимущества кинетического моделирования на примере двух актуальных задач вакуумной электроники.

Ключевые слова: вакуумная электроника, физическая кинетика, численное моделирование

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Introduction

For decades, vacuum electronics has remained an area of plasma physics attracting a great interest which is explained by its wide practical application [1]. From a theoretical point of view, vacuum electronics is a self-consistent description of space-charge limited (SCL) flows of charged particles. The SCL can be clearly explained by the example of a simple two-electrode system (vacuum diode) to which a certain voltage is applied. Even if the charges injection from cathode is assumed to be unlimited, the total current through the diode is always limited to

$$j_{SCL} = \frac{4\varepsilon_0}{9}\sqrt{\frac{2q}{m}}\frac{U^{3/2}}{D^2}, \tag{1}$$

w.r.t. the voltage U applied to the infinite parallel plates located at a distance D from each other (m is the electron rest mass, q is the electron charge, ε_0 is the vacuum dielectric permittivity). The expression of (1) is called the Child's law [2]. It involves many assumptions, e.g. zero initial electron velocities, electrons only emission and a steady-state approximation, however the Child's law provides an ideal theoretical understanding of the SCL phenomenon and the tendency limiting the maximum current that can be carried through the vacuum device. Subsequently [3], Child's law was significantly corrected to relax the approximations used in (1).

In the beginning of the computational plasma physics era, the particle-in-cell (PIC) method was introduced for studying many transient processes in plasma [4]. Another more fundamental and much more computationally difficult principle is based on simultaneous solution of Boltzmann equations together with the electromagnetic field equations. The primary objects of kinetic numerical simulation are the electron and ion distribution functions (EDF and IDF), which provide more information about the system of charged particles than the characteristics of macroparticles in PIC approach. As an example, a pair of one-dimensional (1D) problems of non-stationary current flow in planar vacuum diodes are considered below. The 1D makes it possible to exclude from consideration geometric factors, which are usually referred to for the phenomenological explanation of complex phenomena. Thus, physical kinetics is used in this paper to simulate problems that are simple in terms of computational performance, but at the same time, the methods reveal non-trivial features that can only be described using a more fundamental approach.

Materials and Methods

Let us denote f_e the EDF and f_i the single-charge IDF. Thus, in 1D formulation the EDF and IDF depend on spatial variable x, momentum p_x, and time variable t. Both distribution functions obey collisionless kinetic (Vlasov) [5] equations

$$\begin{cases} \dfrac{\partial f_e}{\partial t} + \dfrac{p_x}{m_e}\dfrac{\partial f_e}{\partial x} - qE_x\dfrac{\partial f_e}{\partial p_x} = 0 \\ \dfrac{\partial f_i}{\partial t} + \dfrac{p_x}{m_i}\dfrac{\partial f_i}{\partial x} + qE_x\dfrac{\partial f_i}{\partial p_x} = 0 \end{cases} \tag{2}$$

where m_e and m_i are rest masses of electron and ion, respectively.

Neglecting the magnetic field influence, the system of Vlasov equations (2) describes the electrons and ion motion in the electric field E_x. The electric field equation has to be solved mutually with (2) in order to obtain a self-consistent solution, so we complement (2) with 1D Poisson's equation for electric potential φ

$$\frac{\partial^2\varphi}{\partial x^2}=-\frac{q}{\varepsilon_0}\left(n_i-n_e\right),\quad E_x=-\frac{\partial\varphi}{\partial x},\quad n_{e,i}\left(x,t\right)=\int\limits_{-\infty}^{\infty}f_{e,i}\left(x,p_x,t\right)dp_x, \tag{3}$$

where ε_0 is dielectric permittivity of vacuum, n_e and n_i are electron and ion number densities, respectively.

The cathode is maintained at the point of $x = 0$ with the electric potential $\varphi = 0$, and the anode is at a distance D in the point $x = D$ with the electric potential $U(t)$. Thus, the Poisson's equation (3) can be solved in quadratures in the form of the following expressions:

$$\begin{gathered}E_x\left(x,t\right)=-\frac{U\left(t\right)}{D}-\frac{q}{\varepsilon_0}\int\limits_0^x\left[n_e\left(x',t\right)-n_i\left(x',t\right)\right]dx'+\frac{q}{\varepsilon_0 D}\int\limits_0^D\int\limits_0^x\left[n_e\left(x',t\right)-n_i\left(x',t\right)\right]dx'dx,\\ \varphi\left(x,t\right)=U\left(t\right)\frac{x}{D}+\frac{qx}{\varepsilon_0 D}\int\limits_0^D\int\limits_0^x\left[n_e\left(x',t\right)-n_i\left(x',t\right)\right]dx'dx-\frac{q}{\varepsilon_0}\int\limits_0^x\int\limits_0^{x'}\left[n_e\left(x'',t\right)-n_i\left(x'',t\right)\right]dx''dx'.\end{gathered} \tag{4}$$

The initial conditions for the equations (2)-(4) are chosen to be zero $f_e = f_i = 0$, $U = 0$. The system of equations (2)-(4) is solved numerically using semi-Lagrangian methods [5] at a rectangular uniform phase-space grid (x, p_x).

The "Anomalous" Electrons in Nanosecond Vacuum Diode

One of the novel physical phenomena regarding vacuum diodes became the effect of fast electrons generation with energies ε exceeding the maximum voltage U_{max} applied to the diode (multiplied by an elementary charge q) $\varepsilon > qU_{max}$. This group of electrons are also known as electrons with the "anomalously high" energies. Depending on experimental conditions fairly large groups of electrons with "anomalously" high kinetic energies can exist.

To elucidate the conditions for the "anomalous electrons" appearance, we posed the problem of a non-stationary current flow in planar vacuum diode with predominant electron emission from the cathode during the applying to it a voltage pulse $U(t)$ with a short leading edge t_{rise} and a constant amplitude of U_{max}. The term "short leading edge" suggests that the front duration is comparable to the electrons time of flight on their way from the cathode to the anode. By setting the cathode electron emission condition with the boundary condition at $x = 0$ for the EDF $f_e\left(x=0,p_x,t\right)=n_0/\sqrt{2\pi m W_0}e^{-\frac{p_x^2}{2mW_0}}$, where W_0 is the electron temperature, n_0 is emission electron number density, and $f_i = 0$, for short anode voltage rise time we obtain the relaxation oscillation of the collector current (Fig. 1) as it was shown in [6]. No oscillations are observed at the large lengths of the anode voltage rise time, and in the case of intermediate values (~ 0.5 ns), the oscillations have a lower amplitude and decay more intensively than at short rise times (below or equal to 0.1 ns). The establishment of a stationary current flow leads to that the collector current density becomes equal to j_{SCL}, regardless of the duration of the anode voltage pulse duration.

The collector current peak arises when the electron beam reaches the anode. The relaxation oscillations buildup is caused by the irregularity of charge inflow and outflow into the interelectrode gap. However, the computational physical kinetics allows to look at the process of the peak collector current density formation from the more fundamental point of view. Fig. 2 shows the comparison of the EDF density plots corresponding to the shortest anode voltage rise time ($t_{rise} = 0.1$ ns) in the case of stationary current flow (*a*) and at the time point corresponding to the maximum collector current density (*b*).

The electron beam reaches anode at this time point (Fig. 2, *b*) have mean energy exceeding qU_{max} by more that 20 percent with respect to steady-state value equal exactly to qU_{max}. If the rise time is relatively longer (here ~2 ns), the electron beam mean energy doesn't override qU_{max} value during the whole voltage pulse duration. Its EDF is similar to Fig. 2, *a*. The electrons with "anomalously high" energies form anode current density relaxation oscillations peaks.

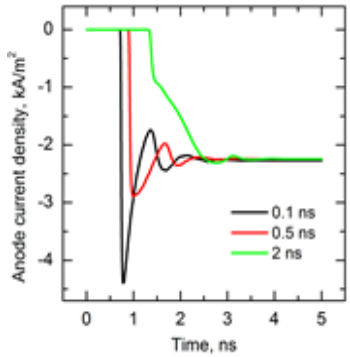

Fig. 1. The collector current density given for different risetimes (D = 1 cm, U_{max} = 2 kV)

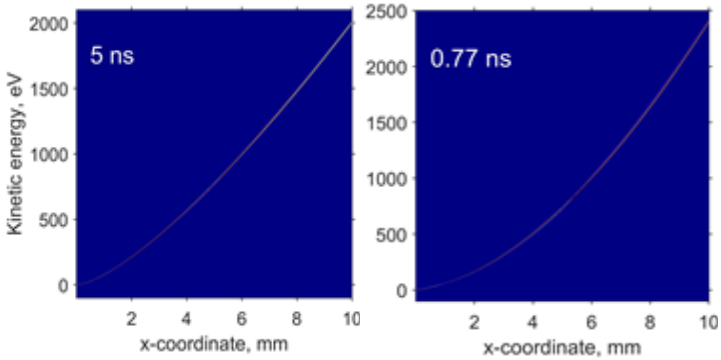

Fig. 2. The EDF density plots corresponding to a steady-state (t = 5 ns) and to the maximum of the collector current (t = 770 ps)

The "Anomalous" Ion Transport in Nanosecond Vacuum Diode

Another problem of vacuum electronics is connected to an explosive emission generating a dense quasi-neutral cathode plasma [7]. Explosive cathode plasma consists of electrons and a group of multiple-charged metal ions located in a thin near-cathode sheath. In various experiments it was convincingly shown that the critical role in the vacuum breakdown plays the existence of directed ion flows from cathode to anode [8]. Numerous experiments indicate that ions not only move towards the anode, but also their kinetic energies are exceeding the typical vacuum discharge combustion voltage (multiplied by q), e.g. see [9]. These ions are called the "anomalous", and their transfer processes in vacuum diodes are called the "anomalous ion transport", due to an ability to travel from cathode to anode. Here we give the simplest self-consistent explanation to this phenomenon from a standpoint of physical kinetics. The simplified case of a cathode plasma composition consisting of electrons and single-charged metal (carbon) ions is considered in planar 1D case here.

At t = 0 emission plasma enter the vacuum diode from the cathode as a nonequilibrium (T_i = 3 eV, T_e = 1 eV) quasi-neutral plasma with average number density of $n_e \approx n_i = n_0$. The regular plasma flow from cathode into the gap is modeled by simple boundary conditions

$$f_{e,i}\left(x=0, p_x, t\right) = n_0 / \sqrt{2\pi m q T_{e,i}}\, e^{-\frac{p_x^2}{2mqT_{e,i}}}.$$

Fig. 3 shows the EDF dynamics f_e at the initial stage of vacuum breakdown. The very first frame shows that the EDF looks similar to the case of stationary current flow in a vacuum diode (Fig. 2). Fig. 4 illustrates the phenomenon of the "anomalous ions acceleration" as is. Fast electron plasma component displacement is accompanied by slower ion dynamics also moving towards the anode. Fig. 5 shows the true electrodynamic cause of the "anomalous" ion acceleration. Starting from the first time points, electrons move towards the anode forming a region of negative electric potential near the cathode ("virtual cathode"). This forces ions to accelerate towards the anode. During the ion motion between the physical and the "virtual" cathode the displacement of a negative potential region occurs, and the cathode plasma starts to fill the near-cathode space. This process continues until the plasma fills the entire diode. These later stages of a vacuum breakdown are shown in details in [10] both for two- and multi-component cathode plasma.

The ion plasma component motion occurs at velocities that are significantly higher than the average value corresponding to a "virtual cathode" depth. One can say that "anomalous" ion acceleration is a motion of ions having an "anomalously" high energies. The IDF "tail" prolongs from near to thermal energies of several electron-volts up to 100 eV, while the instant electric potential minimum doesn't exceed −25 V. The reason of why ions are moving in the "anomalous transport" mode acquire higher energies is in the nonstationary influence of the electric field due to the formation and subsequent displacement of the "virtual cathode" region towards anode. The ions turn into the continuous electrodynamic acceleration regime. This physical mechanism is similar to the formation of runaway electron beams [11].

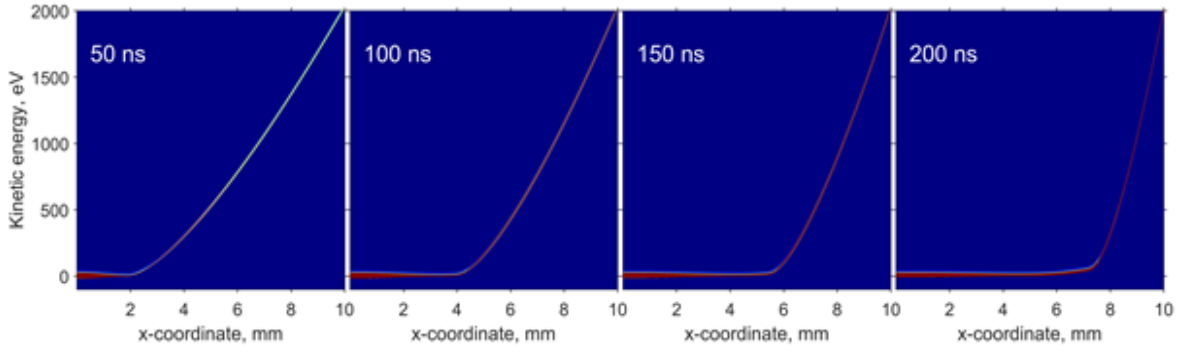

Fig. 3. The EDF f_e density plots computed for selected time steps. In arbitrary units normalized to boundary maximal value

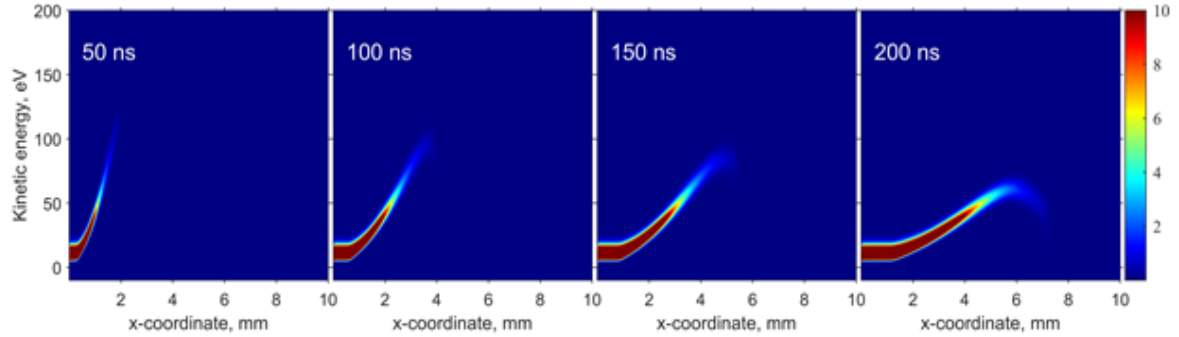

Fig. 4. The IDF f_i density plots computed for selected time steps. Plot is given in arbitrary units; the absolute values have to be multiplied by 10^{39} s/(kg·m^4)

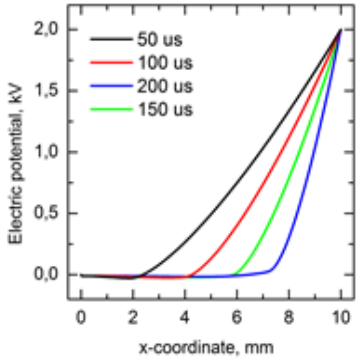

Fig. 5. The electrostatic potential evolution for selected time steps

The results of this paper convincingly show key advantages of computational physical kinetics with respect to the fundamental problems of vacuum electronics.

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