Original article
DOI: https://doi.org/10.18721/JPM.15209

ON CONJUGACY CLASSES OF THE F_4 GROUP OVER A FIELD *q* WITH CHARACTERISTIC 2

N. V. Yurova ✉
Nizhni Novgorod State Technical University named after R. E. Alekseev,
Nizhni Novgorod, Russia
✉ yurova1980@yandex.ru

Abstract: This article continues a series of papers devoted to solving the problem by which a non-identity conjugacy class in a finite simple non-abelian group contains commuting elements. Previously, this statement was tested for sporadic, projective, alternating groups and some exceptional groups. In this article, the validity of the above-mentioned statement for the series exceptional groups ${}^2F_4(q)$ has been verified. After some basic definitions two theorems were proved. The former said about the content of commuting elements in the group, the latter did about the presence of conjugation of a semisimple element with its inverse. Then classes of unipotent and mixed elements were considered. The investigative techniques used were recommended for testing the general hypothesis when dealing with other groups.

Keywords: Chevalley group, conjugacy classes, finite simple group, commuting element

Citation: Yurova N. V., On conjugacy classes of the F_4 group over a field q with characteristic 2, St. Petersburg Polytechnical State University Journal. Physics and Mathematics. 15 (2) (2022) 93–101. DOI: https://doi.org/10.18721/JPM.15209

Научная статья
УДК 512
DOI: https://doi.org/10.18721/JPM.15209

О КЛАССАХ СОПРЯЖЕННОСТИ В ГРУППЕ F_4 НАД ПОЛЕМ q С ХАРАКТЕРИСТИКОЙ 2

Н. В. Юрова ✉

Нижегородский государственный технический университет им. Р. Е. Алексеева,

г. Нижний Новгород, Россия

✉ yurova1980@yandex.ru

Аннотация. Данная статья продолжает цикл работ, посвященных решению проблемы, согласно которой нeединичный класс сопряженности в конечной простой неабелевой группе содержит коммутирующие элементы. Ранее это утверждение было проверено для спорадических, проективных, знакопеременных групп и ряда исключительных групп. В этой работе проверяется справедливость вышеупомянутого утверждения для серии исключительных конечных простых групп $^2F_4(q)$. После основных определений доказываются две теоремы: о содержании в группе коммутирующих элементов и о наличии сопряжения полупростого элемента со своим обратным. Затем рассмотрены классы унипотентных и смешанных элементов. Использованные в статье методы исследования рекомендовано применять для проверки общей гипотезы при рассмотрении других групп.

Ключевые слова: группа Шевалле, классы сопряженности, конечная простая группа, коммутирующий элемент

Для цитирования: Юрова Н. В. О классах сопряженности в группе F_4 над ролем q с характеристикой 2 // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2022. Т. 15. № 2. С. 93–101. DOI: https://doi.org/10.18721/ JPM.15209

Introduction

Investigations of left distributive quasigroups, i.e., binary systems $G(\circ)$ with the left-distributivity identity

$$x \circ (y \circ z) = (x \circ y) \circ (x \circ z) \text{ for } x, y, z \in G(\circ),$$

have led to the problem that should be solved via calculations based on the theory of finite groups.

The left-distributivity identity is appealing because the mapping

$$L_a = (x \to a \circ x)$$

in the binary system $G(\circ)$ containing it, is clearly an endomorphism, or even an automorphism if $G(\circ)$ is a quasigroup.

This issue is thoroughly explored in [1]. Consideration of left distributive quasigroups is equivalent to consideration of homogeneous spaces, i.e., a set containing cosets of a subgroup T in group Π. Turning to groupoids and weakening the axiom of right divisibility $x \circ a = a \Rightarrow x = a$, it generally follows that a groupoid cannot be represented by a homogeneous space.

Left distributive groupoids are often found in applications, manifesting close relationships to groups. Symmetric spaces can be given in differential geometry, characterizing the nodes from a topological standpoint [2,3]. Representations of finite groups by homogeneous spaces have been described by Erofeeva in several studies [4–6]. Apparently, this statement is equivalent to the hypothesis borrowed from pure group theory, assuming that a union of two conjugacy classes in a finite group always contains commuting elements. An even stronger assertion is formulated in [7], arguing that a non-identity class in a finite non-Abelian group must contain commuting elements.

This study continues to verify the hypothesis put forward in [7] that a non-identity class of conjugate elements in a finite simple group contains commuting elements. We have earlier tested some exceptional Lie-type groups and simple groups $SP_4(q)$ in [8–11]. In this paper, we intend to test a series of exceptional groups ${}^2F_4(q)$.

Basic definitions

General information about Chevalley groups is assumed to be known. This is discussed in sufficient detail in the monograph by Robert Steinberg [12]. Specific results on the structure of groups $F_4(q)$ and ${}^2F_4(q)$ are primarily given in Ken-ichi Shinoda's studies [13, 14], as well as in [15].

Here we generally follow the notations adopted by Shinoda [13, 14]. Let us point out some differences. The main field in group $F_4(q)$ is a field of q-elements, $q = 2^{2n+1}$; conversely, Shinoda uses the letter l instead of q, while $q = \sqrt{l}$. Further, the author introduces the notation $x_{i\pm j}$ for $\alpha = e_i \pm e_j$ instead of $x_\alpha(t)$, writing $x_{1\pm 2\pm 3\pm 4}$ for $\alpha = 1/2(e_1 \pm e_2 \pm e_3 \pm e_4)$.

Below we give the main definitions of groups $F_4(q)$ and ${}^2F_4(q)$. The type F_4 Dynkin diagram has the form shown in Fig. 1. The vertices 1, 2, 3, 4 of the graph in it correspond to the roots

$$e_2 - e_3;\ e_3 - e_4;\ e_4;\ \frac{1}{2}(e_1 - e_2 - e_3 - e_4),$$

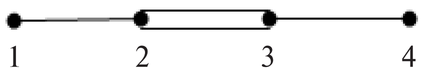

Fig. 1. Dynkin diagram:
graph vertices *1–4* correspond to the roots $e_2 - e_3$, $e_3 - e_4$, e_4, $(e_1 - e_2 - e_3 - e_4)/2$

where e_i ($i = 1,2,3,4$) is a system of orthogonal unit vectors in R^4.

The complete system of roots consists of vectors

$$\pm e_i; \pm e_i \pm e_j \ (i \neq j) \text{ and } \frac{1}{2}(\pm e_1 \pm e_2 \pm e_3 \pm e_4).$$

Group $F_4(q)$ is defined by the generators $x_\alpha(t)$, where α is the root, $t \in F_q$ is the finite field of q elements with certain ratios (see [12, p. 32]); notice the commuting form here:

$$\left(x_\alpha(t), x_\beta(t)\right) = \prod x_{i\alpha+J\beta}\left(c_{ij} t^i u^j\right)(\alpha+\beta \neq 0),$$

where c_{ij} are some constants from the main field.

The 'twisted' group ${}^2F_4(q)$ is constructed as follows. The so-called automorphism σ is selected in the Dynkin diagram (see Fig. 1). 1↔4; 2↔3. Apparently, it can be continued until some permutation of all roots, where the long root is mapped onto the short one, and vice versa. The so-called field automorphism Θ: $x \to x^{2^n}$, is selected in the field F_{q^2}. Now the automorphism σ over group $F_4(q)$ acts on the generators $x_\alpha(t)$ that are root subgroups in the following manner:

$$\sigma : x_\alpha(t) \to \begin{cases} x_{\sigma(\alpha)}(t^\Theta), \text{ if } \alpha \text{ is the long root,} \\ x_{\sigma(\alpha)}(t^{2\Theta}), \text{ if } \alpha \text{ is the short root.} \end{cases}$$

The subgroup of σ-fixed elements in group $F_4(q)$ is precisely the required group ${}^2F_4(q)$, which is simple at $n \geq 1$. A separate case is group ${}^2F_4(2)$, for which the field automorphism Θ is identical provided that $n = 0$. This group is not simple, but its commutator $({}^2F_4(2))'$ of index 2 in ${}^2F_4(2)$ is.

Analysis of the group ${}^2F_4(q)$

The following statements are crucial for subsequent consideration of group ${}^2F_4(q)$.

Statement 1. *The involution class in a finite simple group contains commuting elements.*

It is not necessary to prove this, since it follows from Glauberman's theorem from group theory (see [7] for more details).

Statement 2. *If the number of classes containing elements of this order n with centralizers of the same order is less than φ(n), the class contains commuting elements.*

Here φ(n) is the Euler function.

This statement can be assumed to be elementary; it is also proved in [7].

Let us now examine the corresponding group.

Theorem 1. *The non-identity conjugacy class in group ${}^2F_4(q)$ contains commuting elements.*

Proof. Consider the data in Table 1, where the element classes and their centralizers for group $({}^2F_4(2))'$ are given (the table is taken from Atlas [16]). For example, Table shows a representative of the conjugacy class $8A$, $8B^{**}$; this means that there are two classes $8A$ and $8B^{**}$ containing 8th-order elements with 32nd-order centralizers of the corresponding elements. The presence of commuting elements in the involution class is guaranteed by virtue of Statement 1, assuming that this holds true for any finite simple group. Statement 2 is applied for the remaining classes.

The theorem is proved.

Let us now examine the classes of semisimple elements.

Table 1

Representatives of conjugacy classes for group $({}^2F_4(2))'$

Element class	*Centralizer*
2*A*	10240
2*B*	1536
3*A*	102
4*A*	192
4*B*	128
4*C*	64
5*A*	50
6*A*	12
8*A*, 8B^{**}	32×2
8*C*, 8*D*	16×2
10*A*	10
12*A*, 12*B*	12×2
13*A*, 13B^{*}	13×2
16*A*, 16B^{**}, 16C^{*5}, 16D^{**5}	16×4

Theorem 2. *The semisimple element in group ${}^2F_4(q)$ is conjugate to its inverse.*

Proof. Notice that a semisimple element has an odd order, i.e., there are no involutions among semisimple elements. A semisimple element in the algebraic group $\bar{G}$, obtained from the group G by algebraic closure of the field F_q is conjugate to a Cartan element, i.e., an element of the form

$$h_{\alpha_1}(t_1)h_{\alpha_2}(t_2)...,$$

where α_1, α_2, α_3,... are simple roots.

We use ω to denote an element of the Weil group, which can be obtained by taking the product of reflections relative to four orthogonal roots, for example, relative to vectors e_1, e_2, e_3, e_4. Conjugating the Cartan element using ω converts it into an equality

$$h_{w(\alpha_1)}(t_1)h_{w(\alpha_2)}(t_2)... = h_{-\alpha_1}(t_1)h_{-\alpha_2}(t_2)... .$$

However, the element $h_{-\alpha}(t)h_{-\alpha}(t^{-1})$ commutes with elements of any root subgroup $x_\beta(u)$, and the center in the universal adjoint group of type F_4 is trivial [17]. Therefore, $h_{-\alpha}(t) = (h_\alpha(t))^{-1}$ is conjugate to $h_\alpha(t)$. Thus, a semisimple element is conjugate to the inverse in the algebraic group. Conjugacy in subgroup $^2F_4(q)$ of the algebraic group follows from the above observation on the triviality of the center [17,18].

Further reasoning here is as follows. The algebraic group F_4 is simply connected, the centralizer of the semisimple element is connected (see [17, p. 192, sentence 3.9]). Making a transition from the algebraic group G to G_σ, there is no splitting of the class of semisimple elements, i.e., the element from the group G_σ conjugate in G is also conjugate in G_σ (see [17], p. 171, 3.4 (c)). Conjugacy of a semisimple element to the inverse in an algebraic group is discussed above.

The theorem is proved.

Next, we consider the class of unipotent elements.

There are 18 classes of unipotents in $^2F_4(q)$, listed in [13] together with the order of centralizers for the corresponding elements. Here we present a fragment of this table.

Table 2

Selected classes of unipotent elements

u	$\|Z(u)\|$	$u \sim x$
u_1	$q^{12}(q-1)(q^2+1)$	x_3
u_2	$q^{10}(q^2-1)$	x_4
u_3	$2q^7(q-1)(q^2+1)$	x_{10}
u_4	$2q^7(q-1)(q^2+1)$	x_{10}
u_{11}	$4q^4$	x_{28}
u_{12}	$4q^4$	x_{28}
u_{13}	$2q^3$	x_{29}
u_{14}	$2q^3$	x_{29}
u_{15}	$4q^4$	x_{34}
u_{16}	$4q^2$	x_{32}
u_{17}	$4q^2$	x_{34}
u_{18}	$4q^2$	x_{32}

Note: u_i, x_i are the class representatives in subgroups $^2F_4(q)$ and in $F_4(q)$, respectively, $|Z(u)|$ is the order of the centralizer $Z(u)$ of the corresponding element.

The table compiled by Shinoda [13] does not provide classes x_5–x_{10} because the corresponding representatives are conjugate to the inverse ones as the centralizer order has a single value.

The orders of class representatives x_i are given in [15]. In particular, the representatives of classes u_1 and u_2 are involutions (x_3, x_4 have the order equal to 2). Consequently, they contain commuting elements, by virtue of Statement 1. The orders of elements u_{11} and u_{12}, equal to the order of elements x_{28}, are equal to 8, so, according to Statement 1, both classes contain commuting elements. The same reasoning applies for classes u_{13} and u_{14}, whose orders of elements are equal to 8, and for classes u_{15}–u_{18}, whose orders of elements are equal to 16.

Now the remaining step is to consider classes with the representatives

$$u_3 = x_2(1)x_{1-2}(1)x_4(1)x_1(1)x_{1+2}(1) \text{ and } u_4 = x_2(1)x_{1-2}(1)x_4(1).$$

Both classes are inverse to each other, which is easy to verify using the commutation formula or, alternately, taking into account that both classes merge to $F_4(q)$. Thus, it is sufficient to consider one of them, for example x_4.

We obtain the following equality:

$$u_4 = \alpha_5(1) = x_2(1)x_{1-2}(1)x_4(1).$$

This element lies in a subgroup generated by the root subgroups

$$x_{\pm 2}(t),\ x_{\pm(1-2)}(t),\ x_{\pm 1}(t).$$

This subgroup corresponds to the following Dynkin diagram in Fig. 2.

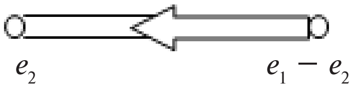

Fig. 2. Dynkin diagram:
e_2, $(e_1 - e_2)$ are the roots of the graph vertices

The Dynkin diagram is constructed by the well-known technique, based on the scalar product of the vertex roots. A graphic automorphism represents the vertices, the field automorphism remains the same as in group $F_4(q)$.

The twisted variation leads to the Suzuki ${}^2B_2(q)$ group, for which the hypothesis put forward in [7] has already verified in [9]. Thus, the class of conjugate elements u_3 must contain commuting elements even in the subgroup ${}^2B_2(q) \subset {}^2F_4(q)$.

Let us now examine the class of mixed elements.

Shinoda [13] compiled information about mixed elements into a table, which is given below (Table 3).

Table 3 [13]

Classes of mixed elements

N	Class representative	Centralizer order
1	$t_1x_1(1)x_{1+2}(1)$	$q^2(q-1)$
2	$t_1x_{1-2}(1)x_2(1)x_1(1)$	$2q^2(q-1)$
3	$t_1x_{1-2}(1)x_2(1)x_{1+2}(1)$	$2q(q-1)$
4	$t_2x_{1+2+3+4}(1)x_{1-4}(1)$	$q(q-1)$
5	$t_4x_{1+2+3-4}(1)x_{1+4}(1)$	$q^3(q+1)$
6	$t_4x_{\alpha_1}(1)x_{\alpha_2}(1)x_{1+2+3-4}(\tau_0)x_{\beta_1}(1)x_{\beta_2}(1)x_{1+4}(\tau_0^{2\Theta})$	$3q^2$
7	$t_4x_{\alpha_1}(\eta)x_{\alpha_2}(\eta^l)x_{1+2+3-4}(\tau_1)x_{\beta_1}(\eta^{2\Theta})x_{\beta_2}(\eta^{2\Theta l})x_{1+4}(\tau_1^{2\Theta})$	$3q^2$
8	$t_4x_{\alpha_1}(\eta^2)x_{\alpha_2}(\eta^{3l})x_{1+2+3-4}(\tau_2)x_{\beta_1}(\eta^{4\Theta})x_{\beta_2}(\eta^{4\Theta l})x_{1+4}(\tau_2^{2\Theta})$	$3q^2$
9	$t_5x_{1+2+3-4}(1)x_{1+4}(1)$	$q(q+1)$
10	$t_7x_1(1)x_{1+2}(1)$	$q^2(q-\sqrt{2q}+1)$
11	$t_7x_{1-2}(1)x_2(1)x_1(1)$	$2q(q-\sqrt{2q}+1)$
12	$t_7x_{1-2}(1)x_2(1)x_{1+2}(1)$	$2q(q-\sqrt{2q}+1)$
13	$t_9x_1(1)x_{1+2}(1)$	$q^2(q+\sqrt{2q}+1)$
14	$t_9x_{1-2}(1)x_2(1)x_1(1)$	$2q(q+\sqrt{2q}+1)$
15	$t_9x_{1-2}(1)x_2(1)x_{1+2}(1)$	$2q(q+\sqrt{2q}+1)$

Table 4

Structure of centralizers for semisimple elements

t	Centralizer order	Structure
t_1	$(q-1)q^2(q-1)(q^2+1)$	$Z_{q-1}{}^2B_2(q)$
t_2	$(q-1)q(q-1)$	$Z_{q-1}\mathrm{S}L_2(q)$
t_4	$3\frac{1}{3}q^3(q^2-1)(q^3+1)$	$Z_3U_3(q)$
t_5	$(q+1)q(q^2-1)$	$Z_{q+1}SL_2(q)$
t_7	$(q-\sqrt{2q}+1)q^2(q-1)(q+1)$	$Z_{q-\sqrt{2q}+1}{}^2B_2(q)$
t_9	$(q+\sqrt{2q}+1)q^2(q-1)(q+1)$	$Z_{q+\sqrt{2q}+1}{}^2B_2(q)$

Conclusion

Here the representatives of the classes are written as a product of semisimple and unipotent factors, i.e., $x = x_u \cdot u_s$ is a decomposition of the mixed element into a product of a semisimple factor x_s and a unipotent u_s. The factors commute if the decomposition is of the Jordan type, but this requirement is not always satisfied in Table 3 for representatives of classes containing mixed elements. It follows from the existence of the Jordan-type decomposition of the mixed element that it lies in the centralizer of the semisimple factor. In turn, it is easy to determine the structure of the centralizers of semisimple factors. Each such centralizer $Z(t)$ contains powers of t and, being a reductive group, has a semisimple factor. It is not necessary to consider the elements t_3, t_6, t_8, t_{10}, t_{11}, t_{12}, since their centralizers do not contain unipotents of semisimple elements. Inspecting the centralizers t_1, t_2, t_4, t_5, t_7, t_9, we find that they have semisimple parts of the orders. The structure of the centralizers of the semisimple elements considered is given in Table 4, where the factor of type Z_m is a cyclic group consisting of the powers of t and included in the center of the centralizer t. The second factors ${}^2B_2(q)$, ${}^2SL_2(q)$, $U_3(q)$ represent a Suzuki group, a linear group and a unitary group, respectively. Notably, these factors are simple groups. The presence of commuting unipotent factors lying in the same class is proved in [7] for the Suzuki group ${}^2B_2(q)$ and the linear group ${}^{2SL}{}_2(q)$. The commutator of the Sylow r-subgroup for an identity group lies in its center. If the unipotent lies in the center, conjugating it to an element from the Cartan subgroup gives a commuting element from the same conjugacy class. If the unipotent u is not from the center of the Sylow p-subgroup, then there is another unipotent u', not commuting with u. Elements u and $u'' = u' \cdot u \cdot (u')^{-1}$ lie in the same class and commute, since their commutator lies in the center of the Sylow p-subgroup. Elements x and $u' \cdot x \cdot (u')^{-1}$ clearly commute, lie in the same class, and are in fact different, since $u' \neq u''$. This concludes the consideration of mixed elements.

This study makes another step towards verifying the general hypothesis that a non-identity conjugacy class in a finite simple non-Abelian group contains commuting elements. We have described the methods for testing this hypothesis for the exceptional group ${}^2F_4$(q).

The research methods used in the study can be directly applied to verifying the general hypothesis for analysis of other groups.

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THE AUTHOR

YUROVA Nadezhda V.
Nizhni Novgorod State Technical University named after R.E. Alekseev
24, Minin St., Nizhni Novgorod, 603950, Russia
yurova1980@yandex.ru
ORCID: 0000-0003-2355-2820

СВЕДЕНИЯ ОБ АВТОРЕ

ЮРОВА Надежда Вячеславовна – *старший преподаватель Института транспортных систем Нижегородского государственного технического университета имени Р. Е. Алексеева.*
603950, Россия, г. Нижний Новгород, ул. Минина, 24
yurova1980@yandex.ru
ORCID: 0000-0003-2355-2820

Received 29.03.2021. Approved after reviewing 19.03.2022. Accepted 19.03.2022.
Статья поступила в редакцию 29.03.2021. Одобрена после рецензирования 19.03.2022. Принята 19.03.2022.