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MODELING OF NONSTATIONARY FILTRATION IN RESERVOIRS WITH NATURAL FRACTAL FRACTURES

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Abstract. The article discusses a problem of naturally fractured reservoir modeling because some reservoirs exhibit the flow processes characterized by anomalous kinetics that do not obey Gaussian statistics. For this reason, the classical approach to the well test interpretation, its generalizations, and their transformation to a more complex model taking into account the fractal structure of fracture networks have been considered. The obtained results indicated the validity of the application of the fractal model to the interpretation of the well-test data where a power-law time dependence of the producing bottom-hole pressure was observed. Moreover, some symptoms were formulated whereby someone could identify the fractal well-stream behavior using the well-test data. Finally, the main issues for further studies the authors refer to the determination of the fractal parameters and the fractal model validation in laboratory experiments.

Keywords: permeability, power law, fractured reservoir, anomalous diffusion, fractal dimension

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МОДЕЛИРОВАНИЕ НЕСТАЦИОНАРНОЙ ФИЛЬТРАЦИИ В КОЛЛЕКТОРАХ С ЕСТЕСТВЕННЫМИ ФРАКТАЛЬНЫМИ ТРЕЩИНАМИ

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Аннотация. В статье рассмотрена проблема моделирования естественно трещиноватых коллекторов, процессы протекания в которых обладают аномальной кинетикой (не подчиняются статистике Гаусса). Рассмотрен переход от классической модели интерпретации данных гидродинамических исследований скважин (ГДИС) к модели, учитывающей фрактальную структуру трещин. Полученные результаты указывают на правомерность применения последней модели для интерпретации результатов ГДИС, согласно которым наблюдается степенная зависимость забойного давления на скважине от времени. Сформулированы также признаки, с помощью которых можно выявить фрактальный режим притока к скважине по данным ГДИС. К основным вопросам для дальнейшего исследования авторы относят определение параметров фрактальности и валидацию модели на лабораторных экспериментах.

Ключевые слова: проницаемость, степенной закон, трещиноватый коллектор, аномальная проводимость, фрактальная размерность

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Introduction

Simulation of hydrocarbon production processes largely depends on estimating the parameters characterizing the filtration and storage capacity in a reservoir, that is, porosity and permeability. Errors in assessing these properties generally lead to incorrect decisions in field development, well intervention design and control, ultimately reducing the volumes of reservoir fluid produced.

The most common methods for finding the reservoir parameters are coring (collecting rock samples extracted from a well), well logging and well testing. The advantage of well testing over other approaches is that it affects a much larger area of the reservoir, about hundreds of meters (up to the external boundary where the pressure is equalized to the reservoir level), while the radius of surveys carried out by other methods typically does not exceed tens of centimeters from the wellbore wall.

Simulating the properties of certain reservoir beds, especially in carbonate rocks, can be challenging due to the presence of natural fractures. While the behavior of such fractured systems can be unpredictable, simulation is still based on classical models that do not account for some significant effects.

This paper presents hydrodynamic studies of wells in such reservoirs.

It has been established [1] that the pressure versus time dependence on the wall of a vertical well in an infinite porous reservoir (with intergranular porosity) with a constant fluid flow rate is approximated by a logarithmic function, including for double-porosity models used to simulate naturally fractured reservoirs.

However, this approach does not allow interpreting the data obtained from well tests for naturally fractured reservoirs for which the dependence of bottom-hole differential pressure on time follows a power law; furthermore, the power law exponent can be a fractional number [2].

This study considers radial fluid flow in a fractal system. We compared classical testing models with a fractal model for the case of fluid filtration (flow) to the well via a system of natural fractures [3], formulating several unsolved problems related to application of the fractal model.

Classic well test models

According to monograph [1], the general approach to determining the filtration and storage capacity characteristics in conventional porous reservoirs can be extended to naturally fractured reservoirs. This concerns, in particular, comprehensive well test analysis.

Simulation of fluid flow in a porous medium. The basic model describing the unsteady process of pressure redistribution in a reservoir where filtration (flow) of fluid occurs in the pore space (intergranular porosity) in well tests is the diffusivity equation [3].

This equation describes the process of fluid filtration to a well making several assumptions:

the direction of fluid flow is a horizontal plane;

Darcy's law is fulfilled in the reservoir, with filtration occurring for one phase with low compressibility (for example, a mixture of oil with water);

gravitational effects are negligible;

the parameters of the reservoir and fluid do not depend on pressure.

The equation follows from a combination of the main laws governing fluid filtration in the pore space: Darcy's law (conservation of momentum), the continuity equation (conservation of mass), and the equations of state (compressibility) of the fluid and the reservoir:

$$\begin{cases} \mathbf{v} = -\dfrac{k}{\mu} grad\left(p\right) \\ -\nabla \cdot \left(\rho \mathbf{v}\right) = \dfrac{d\left(\varphi\rho\right)}{dt} \\ c_f = \dfrac{1}{\rho}\dfrac{\partial \rho}{\partial p} \\ c_r = \dfrac{1}{\varphi}\dfrac{\partial \varphi}{\partial p} \end{cases}, \qquad (1)$$

where **v** is the vector of the fluid filtration rate; p, Pa, is the fluid pressure in the pores; k, m^2, is the permeability of the porous medium; φ, %, is the porosity; ρ, kg/m^3, is the density of the reservoir rock; μ, Pa·s, is the dynamic viscosity of the fluid; c_f, 1/Pa, is the compressibility of the liquid; s_r, 1/Pa, is the compressibility of the reservoir rock.

Substituting Darcy's law and compressibility equations into the fluid continuity equation, subsequently neglecting the second-order term $(\partial p/\partial r)^2$ yields the diffusivity equation:

$$\Delta p = \frac{\mu\varphi c_t}{k}\frac{\partial p}{\partial t} \tag{2}$$

and its particular case for cylindrical symmetry of the system (plane-radial fluid flow):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\mu\varphi c_t}{k}\frac{\partial p}{\partial t}, \tag{3}$$

where c_t, 1/Pa, is the total compressibility of the system, $c_t = c_f + c_r$.

For convenience, the diffusivity equation can be written using dimensionless coordinates. The index D here and below denotes dimensionless variables:

$$p_D = \frac{2\pi kh\left(p_i - p\right)}{q\mu};\ t_D = \frac{kt}{\mu\varphi c_t r_w^2};\ r_D = \frac{r}{r_w}, \tag{4}$$

where p_i, Pa, is the reservoir pressure; q, m^3/s, is the fluid flow rate through the well wall; h, m, is the reservoir thickness (the height of the cylinder in the plane-radial problem); r_w, m, is the well radius.

The dimensionless diffusivity equation takes the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\partial p}{\partial t}, \tag{5}$$

or

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r} = \frac{\partial p}{\partial t}. \tag{6}$$

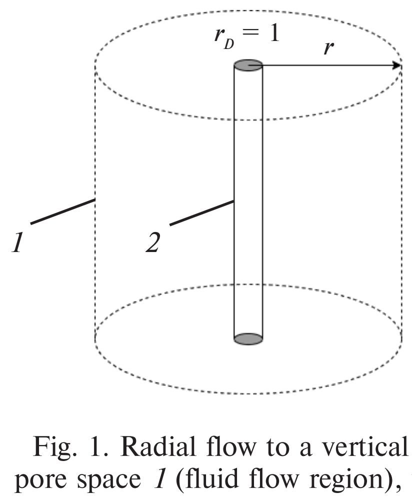


Fig. 1. Radial flow to a vertical well:
pore space *1* (fluid flow region), well *2*.
The pore space is unbounded for an infinite reservoir

The resulting equation is used to interpret the well test data. For example, one of the estimated parameters is the transmissibility coefficient $\varepsilon = kh/\mu$, m^3/(Pa·s). The estimates of ε (well testing interpretation), h (seismic exploration, etc.), and μ (fluid sampling) can be used to estimate the permeability of the reservoir k.

The procedure for estimating ε can be illustrated by the problem of radial flow in an infinite reservoir, unperturbed at the initial moment of time:

$$p_D\Big|_{t_D=0} = 0; \tag{7}$$

while

$$\lim_{r_D\to\infty} p_D = 0; \tag{8}$$

the constant flow rate at the well wall (Fig. 1) is expressed as

$$\left.\frac{\partial p_D}{\partial r_D}\right|_{r_D=1} = -1. \tag{9}$$

The solution of the diffusivity equation with such initial and boundary conditions was obtained in [5] using the Boltzmann substitution:

$$y = \frac{r_D^2}{4t_D}. \tag{10}$$

This reduces the problem to an ordinary differential equation with respect to y, yielding the solution to the equation:

$$p_D\left(r_D, t_D\right) = \frac{1}{2}\int_y^\infty \frac{e^{-z}}{z}dz = -\frac{1}{2}\mathrm{Ei}\left(-\frac{r_D^2}{4t_D}\right), \tag{11}$$

where Ei is the integral exponential function:

$$\mathrm{Ei}\left(-x\right) = -\int_x^\infty \frac{e^{-u}}{u}du, \tag{12}$$

approximated for small values of the parameter using the Euler constant γ and the logarithmic function:

$$-\mathrm{Ei}\left(-x\right) \approx \ln\left(\frac{1}{x}\right) - \gamma. \tag{13}$$

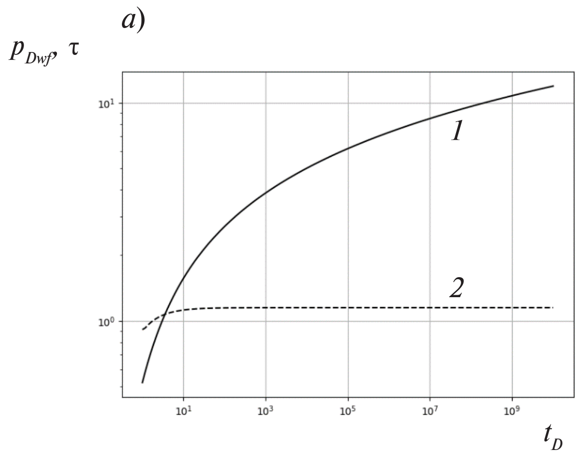
Thus, the approximate solution for dimensionless pressure p_D in the reservoir takes the form of a logarithmic function, i.e.,

$$p_D\left(r_D, t_D\right) \approx \frac{1}{2}\ln\left(\frac{4}{e^\gamma}\frac{t_D}{r_D^2}\right), \tag{14}$$

while the error in the approximate value of p_D for $t_D/r_D^2 > 5$ does not exceed 2% [6]).

The main interest lies in the bottom-hole pressure p_{wf}. This quantity, measured in the region of the well wall penetrating the reservoir, along which filtration of fluid occurs, takes the form

$$p\left(r_w, t\right) = p_{wf}\left(t\right) \approx p_i - \frac{Q}{4\pi\varepsilon}\ln\left(\frac{4}{e^\gamma r_w^2}\frac{k}{\mu\varphi c_t}t\right). \tag{15}$$



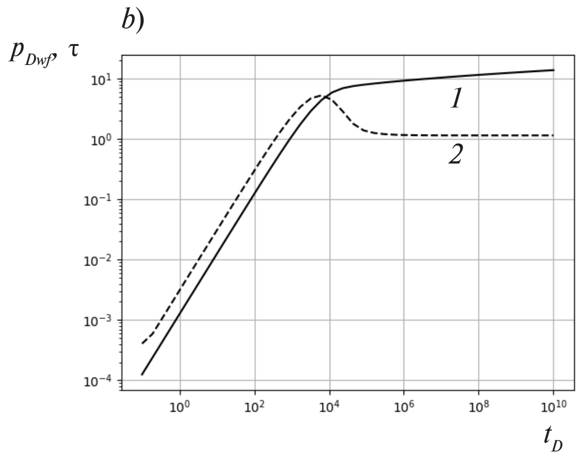


Fig. 2. Example dependences of dimensionless bottom-hole pressure p_{Dwf} (*1*) and numerical approximation of its derivative $\tau = dp_{Dwf}(t_D)/d(\ln t_D)$ (*2*) on dimensionless time t_D, satisfying the diffusivity equation, without (*a*) and with taking into account (*b*) the skin factor and the wellbore effect ($S = 2$, $C_D = 800$); double logarithmic coordinates were used

Thus, measuring the bottom-hole pressure $p_{wf}(t)$ at a fluid flow rate Q, we can graphically estimate the transmissibility coefficient by plotting the derivative $dp_{wf}(t)/d(\ln t)$.

For ease of interpretation, well test data are customarily represented as two graphs: $p_{wf}(t)$ and $\tau = dp_{wf}(t)/d(\ln t)$ in double logarithmic coordinates. For convenience, data can also be reduced to a dimensionless form (similar to Eq. (4)) for approximation, e.g., by the solution (14) (Fig. 2,*a*).

In fact, the obtained dependences of pressure and its derivative on time have a characteristic 'hump' (Fig. 2,*a,* Fig. 2,*b*) in the initial periods of time due to the following effects:

errors caused by numerical differentiation (Fig. 2,*a,* Fig. 2,*b*);

change in the permeability of the bottom-hole zone (in the vicinity of the reservoir near the well) due to its pollution or, conversely, through artificially generating permeable fractures (determined by the coefficient S that is the skin factor [6]) (Fig. 2,*b*);

effect of wellbore storage due to compression and/or expansion of the fluid in the well (depending on the dimensionless wellbore storage coefficient C_D [6]) (Fig. 2,*b*).

These effects are taken into account by adjusting the boundary conditions and solving the equation by numerical methods or using the Laplace transform. The effects are simulated by substituting the boundary condition on the well wall with two third-kind boundary conditions (see the equality (16), (17) below).

The influence of permeability of an infinitesimal region near the bottom-hole zone is taken into account by increasing ($S > 0$) or decreasing ($S < 0$) the pressure on the well wall (see condition (16)) proportionally to the product of the skin factor and the flow rate on the well wall:

$$p_{D_{wf}} = p_D\big|_{r_D=1} - S\left(r_D\frac{\partial p_D}{\partial r_D}\right)_{r_D=1}. \quad (16)$$

The wellbore storage effect is that once the well is started up, the initial flow rate at the surface (at the wellhead) is triggered by expansion of the fluid previously filling the wellbore. As a result, the flow rate in the bottom-hole (at the point of direct contact between the well and the reservoir) changes more slowly over time than the flow rate at the surface. The wellbore storage effect is taken into account using the boundary condition relating the change in bottom-hole pressure to time at a constant fluid flow rate on the well wall (17):

$$C_D\frac{\partial p_{D_{wf}}}{\partial t_D} - \left(r_D\frac{\partial p_D}{\partial r_D}\right)_{r_D=1} = 1. \quad (17)$$

In this case, the transmissibility coefficient ε is also estimated by the value of the function $\tau = dp_{Dwf}(t_D)/d(\ln t_D)$ (plotted in double logarithmic coordinates) in regions where the derivative curve reaches a plateau (see Fig. 2,*b*), i.e., where radial flow evolves.

Simulation of fluid flow in a fractured medium. The diffusivity equation is used to describe the process of fluid flow through pores (a reservoir with primary intergranular porosity). However, reservoirs with secondary voids, i.e., containing fractures and vugs, should be described by more complex models, most including two types of porosity [1]. They simulate fluid filtration in two spaces related by the interflow equation. There are different types of such models [7–9], including the Warren–Root dual-porosity model (‘sugar cubes’) [10].

According to the Warren–Root model, fractures form a continuous and uniform network oriented parallel to the principal axis of permeability (Fig. 3), while the fracture width remains constant.

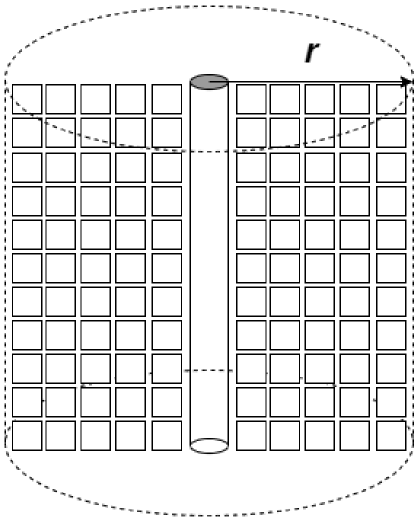


Fig. 3. Multi-block model of simultaneous symmetric radial fluid flow through the pore space (matrix) and the fracture system (Warren–Root model)

The same as other dual-porosity models, this one considers the parameters k, c_t, φ of the reservoir and the filtering fluid as well as the variables p and v in Eqs. (2)–(6) separately for the matrix space and the fracture space. Each quantity is denoted by the subscripts *ma* and *f*, respectively. For example, $p_{D_{ma}}$ is the dimensionless pressure in the matrix (pore) space, p_{D_f} is the dimensionless pressure in the fracture space.

Similar to how the standard diffusivity equation is formulated, Darcy’s laws, continuity equations accounting for fluid interflow from one space to another, compressibility equations of the reservoir’s solid phase, and also the equation of fluid compressibility which is the same for both spaces are combined for the matrix space and the fracture space.

Two new parameters are introduced into the model to relate the two spaces:

ω (storativity ratio) that is a parameter characterizing the storage capacity of the fracture system;

λ (matrix/fracture interaction parameter) is a dimensionless parameter that characterizes the fluid’s capacity for interflow between porous regions in dimensionless form, i.e., the intensity of fluid interflow from matrix to fractures.

The Warren–Root model assumes that the final system of equations for the case of cylindrically symmetric radial flow to a vertical well has the form:

$$\begin{cases} \omega\left(\dfrac{\partial p_{Df}}{\partial t_D}\right)+\left(1-\omega\right)\left(\dfrac{\partial p_{Dma}}{\partial t_D}\right)=\dfrac{1}{r_D}\dfrac{\partial}{\partial r_D}\left(r_D\dfrac{\partial p_{Df}}{\partial r_D}\right) \\ \left(1-\omega\right)\left(\dfrac{\partial p_{Dma}}{\partial t_D}\right)=\lambda\left(p_{Df}-p_{Dma}\right). \end{cases} \tag{18}$$

If $\omega = 1$, the reservoir is fractured, with negligible matrix porosity. The first equation in the system is reduced to the diffusivity equation with respect to p_{Df}; the case $\omega = 0$ concerns a reservoir with negligible influence of fractures, the case of $0 < \omega < 1$ a reservoir with a dual porosity.

Similar to Eq. (6), the following boundary conditions are adopted for the problem of plane-radial flow to a vertical well with a constant flow rate in an infinite reservoir:

$$p_{D_f}\Big|_{t_D=0}=p_{D_{ma}}\Big|_{t_D=0}=0, \tag{19}$$

$$\lim_{r_D\to\infty}p_{D_f}=\lim_{r_D\to\infty}p_{Dma}=0, \tag{20}$$

$$\left.\frac{\partial p_{D_f}}{\partial r_D}\right|_{r_D=1}=-1. \tag{21}$$

The expression for the bottom-hole pressure for the system of equations (18) was obtained by Warren and Root [1, 10]:

$$p_{D_f}\left(1,t\right)=p_{D_{f_{wf}}}\left(t\right)=\frac{1}{2}\left(const+\ln\left(t_D\right)+\mathrm{Ei}\left[-\frac{\lambda t_D}{\omega\left(1-\omega\right)}\right]-\mathrm{Ei}\left[-\frac{\ddot{e}t_D}{\left(1-\omega\right)}\right]\right). \tag{22}$$

Given small values of the dimensionless time t_D, expression (22) is approximated using a logarithmic function:

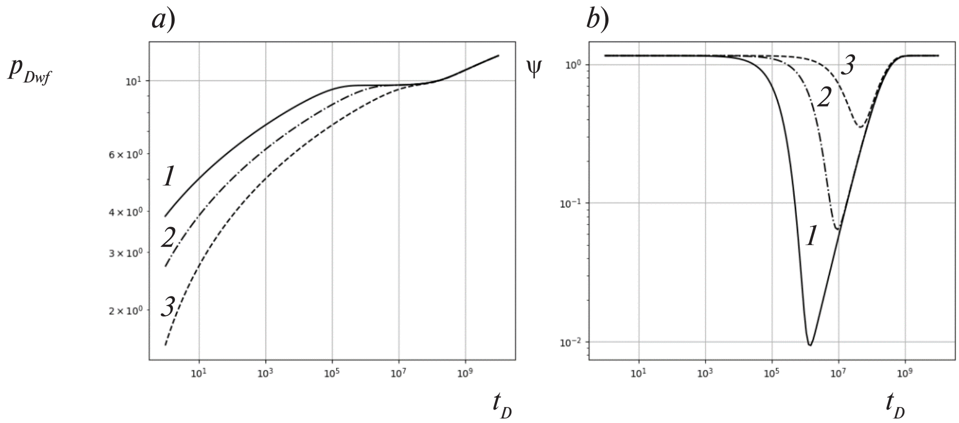


Fig. 4. Examples dependences of dimensionless pressure (*a*) and its derivative $\psi = d(p_{Dwf}(t_D))/d(\ln t_D))$ (*b*) versus dimensionless time t_D, satisfying the diffusivity equation for different values of the fracture storativity ratio ω: 0.001 (*1*); 0.01 (*2*); 0.1 (*3*)

$$p_{D_f}\left(t_D, t_D \to 0\right) \approx \frac{1}{2}\left(const + \ln\left(t_D\right) + \ln\left(\frac{1}{\omega}\right)\right). \quad (23)$$

Similarly, expression (20) is approximated with a logarithmic function for large values of the dimensionless time t_D (quasi-steady flow evolves, the entire reservoir functions as an equivalent homogeneous one):

$$p_{D_f}\left(r_D = 1, t_D \to \infty\right) \approx \frac{1}{2}\left(const + \ln\left(t_D\right)\right). \quad (24)$$

Dependences of pressure and its derivative $\psi = d(p_{Dwf}(t_D))/d(\ln\ t_D))$ on time are similar to expression (15), with the difference that there is a dip in the plateau on the curve describing the derivative of pressure with respect to the time logarithm (Fig. 4).

This *V*-shaped dip corresponds to the transition phase of fluid flow, when the time dependence of pressure in fractures is approximately constant (this corresponds to an early stage of supplying fluid from matrix blocks to the fracture system).

Thus, the same as the diffusivity equation (14), the dual-porosity models do not account for the possibility that the dependence of pressure on time may follow a power law. The reason for this is that the spaces of fractures and pores in the model have an integer dimension: they are one-, two-, or three-dimensional.

However, the classical diffusivity model and the Warren–Root model can be generalized by introducing the concept of fractional (fractal) dimension.

Fractal well test model

Problem statement. One the ways for describing the complexity and non-smoothness of objects is modeling their fractal properties. Fractals are sets whose Hausdorff–Besicovitch dimension is strictly greater than the topological dimension (see monograph [11]), that is, than the standard Lebesgue dimension assuming that a segment is a one-dimensional body, a sphere is two-dimensional, etc.

Dynamical systems possessing fractal properties are characterized by ‘anomalous’ behavior primarily associated with the fractal property that the dependence of its integral properties on the scale at which the object is considered [12] follows a power law, and, furthermore, with the resulting concept of fractional dimension of space. This behavior is observed by many authors for different branches of mathematics and physics: fractional differentiation and integration [13, 14], thermodynamics and heat transfer [15], elasticity theory [16], fluid dynamics [17], electrodynamics [18], molecular dynamics [19] and statistical physics [20], quantum mechanics [21], percolation theory [22], mechanics [23] and geometry [17, 24–26] of porous media, random function theory [27], and many others [17].

In particular, it is proposed in [3] to consider a similar approach for modeling plane-radial fluid flow to a vertical well through a system of natural fractures with fractal properties (Fig. 5).

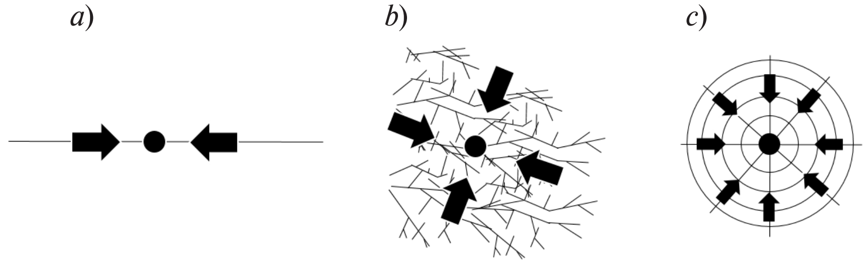


Fig. 5. Different pathways of fluid flow to the well: along a one-dimensional fracture (*a*), along a fractal system of fractures (*b*), along a two-dimensional network of fractures (*c*)

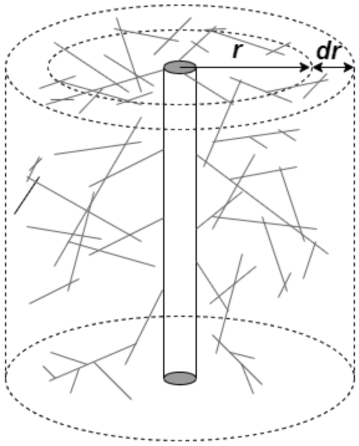


Fig. 6. Fractal system of natural fractures surrounding a vertical well and an annulus of the same cylinder (r, $r + dr$) with a thickness dr

As before, we will consider the flow to the well in a system whose averaged characteristics have cylindrical symmetry (Fig. 6).

Let us assume that fluid flow in such a fractured space occurs over a set of ‘sites’ (states) with the same volume V_s, and $n(r)dr$ is the number of sites in the Euclidean shell enclosed by (r, $r + dr$) (see Fig. 6). In this case, the function $n(r)$ is the density of states characterizing how tightly the fracture sites are ‘packed’ in the Euclidean space $\mathbb{R}^N$. The density of states is understood in statistical physics as a quantity determining the number of states per unit of energy or wave vector [28].

The mathematical expression for the fractal dimension of a system of natural fractures is that the mass fraction taken by fractures in a cylinder of radius R depends on that radius by a power law. The exponent is the mass dimension D_m of the fractal system, which characterizes the ability of a fractal medium to fill space. The dimension D_m is similar to the well-known Hausdorff–Besicovitch dimension or the Minkowski dimension [12].

Since the volume V_s of each fracture segment is the same, the function $n(r)dr$ also gives the mass fraction of the fracture system in the cylinder [3]:

$$Mass = \int_0^R n(r)\,dr \propto R^{D_m}. \tag{25}$$

Thus, the number of fractures contained in a circle with an increase in its radius r grows more slowly than the mass fraction of these fractures starting from a certain time:

$$n(r) \propto r^{D_m - 1}. \tag{26}$$

Similar fractal characteristics are observed in rocks [30–33], some porous materials, polymers, colloidal systems, aerogels, sandstones [11, 34–37].

In addition, it is pointed out in [17] that many porous media exhibiting fractal properties are homogeneous, i.e., a vertical well may act as a cylinder axis.

According to [19], the magnitude of the flux through the surface of a circle is proportional to its radius to a degree equal to $D_m - 1 - \theta$, which is also true for the fluid filtration rate v through the surface of the circle:

$$v(r) \propto \frac{r^{D_m - 1 - \theta}}{\mu}\frac{\partial p}{\partial r}. \tag{27}$$

These dependences were used in [3] and [39] to obtain the expressions were for permeability k and porosity φ of the fractal fracture system contained in a circular segment with radius r:

$$k(r) = k_w \left(\frac{r}{r_w}\right)^{D_m - \theta - d}, \tag{28}$$

$$\varphi(r) = \varphi_w \left(\frac{r}{r_w}\right)^{D_m - d}, \tag{29}$$

where d is the Euclidean dimension of space ($d = 2$ in the given statement); θ is the spectral exponent of the fractal network, or anomalous conductivity index ; r_w, k_w, φ_w are, respectively, the values of the well radius, permeability and porosity, measured at the well wall.

The quantity θ appears because the flow equation in fractal fractures (given below) was obtained similar to the diffusion equation for fractal objects [19] (Fick's equation coincides with the diffusivity equation), for which θ has a specific physical meaning. In this case, the diffusivity coefficient χ at a point is inversely proportional to the distance to the origin r to a degree equal to the anomalous conductivity index:

$$\chi \propto r^{-\theta}, \tag{30}$$

which is similar to the phenomenon of anomalous diffusion, when the assumptions underlying Einstein's law and the model of Brownian motion are not fulfilled, i.e., the square of the distance traveled by a particle ceases to be proportional to the travel time [40].

Notably, $k(r)$ and $\varphi(r)$ in expressions (28), (29) are not local permeabilities and porosities at a point but rather the average permeability and porosity in a cylinder with radius r [41]. The following proportions are also fulfilled for them:

$$k \propto r^{D_m-\theta-d}, \tag{31}$$

$$\varphi \propto r^{D_m-d}. \tag{32}$$

A similar dependence for the conductivity of a fractal system on the coordinate r was earlier observed by Sokolov, considering random walks on fractal grids in percolation theory [22].

Generalized diffusivity equation. The obtained power-law dependences of permeability $k(r)$ and porosity $\varphi(r)$ allow deriving a generalized diffusivity equation for fractal fractured systems.

The equation of radial flow to the well along fractal fractures (no flow along the matrix) is obtained by introducing dimensionless variables, similar to expressions (4),

$$p_D = \frac{2\pi k_w h\left(p_i - p\right)}{q\mu};\ t_D = \frac{k_w t}{\mu\varphi_w c_t r_w^2};\ r_D = \frac{r}{r_w}, \tag{33}$$

and substituting $k(r)$ and $\varphi(r)$, similar to the formulation of the classical diffusivity equation [3]:

$$\frac{1}{r_D^{D_m-1}}\frac{\partial}{\partial r_D}\left(r_D^{D_m-1-\theta}\frac{\partial p_D}{\partial r_D}\right) = \frac{\partial p_D}{\partial t_D}, \tag{34}$$

i.e.,

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{D_m - 1 - \theta}{r_D}\frac{\partial p_D}{\partial r_D} = r_D^{\theta}\frac{\partial p_D}{\partial t_D}. \tag{35}$$

Thus, the dimensionless variables p_D, t_D, r_D were obtained similarly to the formulation of expressions (4).

Let us consider several cases.

1. $\theta = 0$, $D_m = 2$; this means that there is no anomalous conductivity, and the fractal dimension coincides with the Euclidean one; then Eq. (33) is reduced to the diffusivity equation for plane-radial flow to the well (see Eqs. (5), (6)); this corresponds to the case shown in Fig. 5,*c*.
2. $\theta = 0$, $D_m = 1$; Eq. (35) is reduced to the diffusivity equation for linear (one-dimensional) flow to the well (see Eq. (5)), see Fig. 5,*a*.
3. $1 < D_m < 2$; flow evolves along a plane fractal system of fractures: it is transitional between linear and plane-radial, see Fig. 5,*b*.
4. $2 < D_m$; flow evolves along a fractal system of fractures; it is transitional between plane-radial and spherical flow.

The diffusivity equation with initial and boundary conditions (7)–(9) [3] was solved by introducing a substitution similar to the Boltzmann substitution

$$y = \frac{r_D^{\theta+2}}{\left(\theta+2\right)^2 t_D}, \tag{36}$$

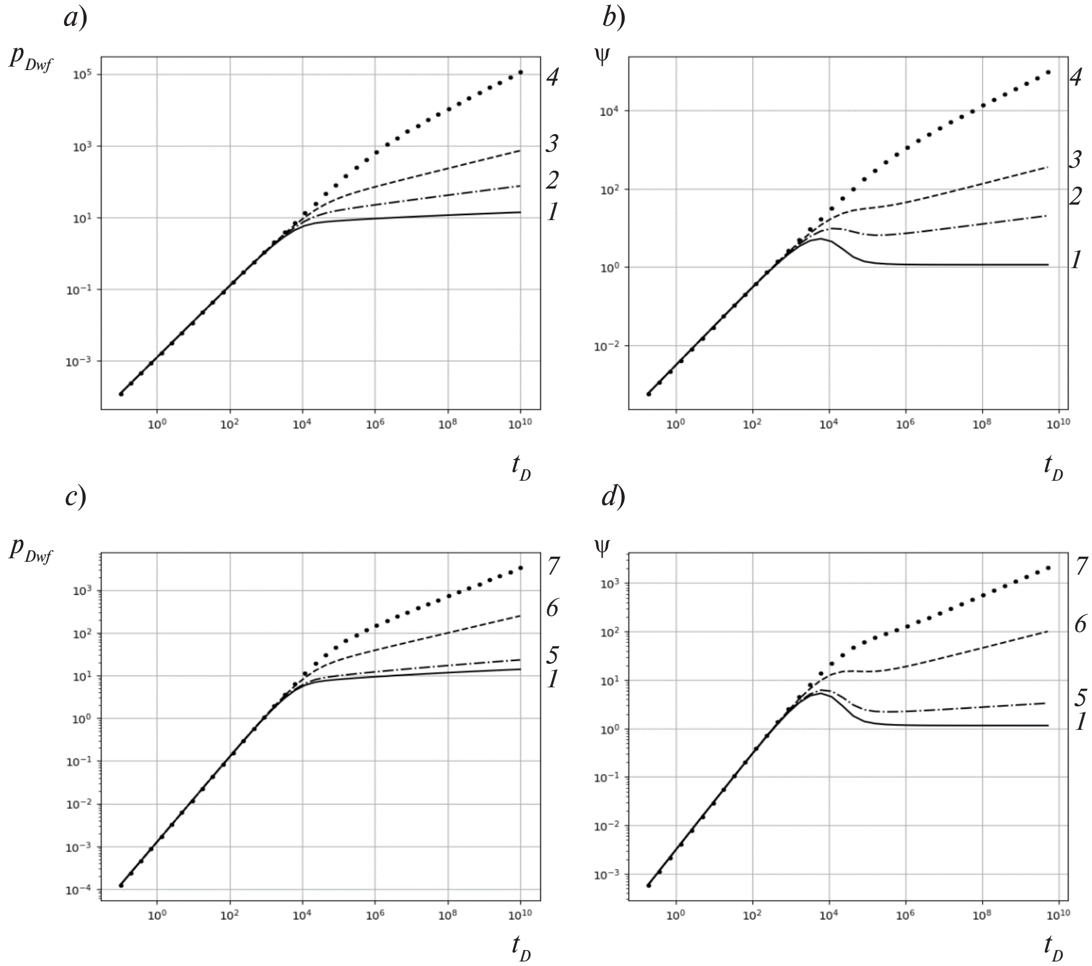


Fig. 7. Dependences of dimensionless pressure (a, c) and its derivative $\psi = d(p_{Dwf}(t_D))/d(\ln t_D)$ (b, d) on dimensionless time t_D, satisfying the diffusivity equation, for different values of the parameters D_m and θ: $D_m = 2.00$ (curves *1*, *5*–*7*), 1.75 (*2*), 1.50 (*3*), 1.00 (*4*); $\theta = 0.0$ (*1*–*4*), 0.1 (*5*), 0.5 (*6*), 1.0 (*7*)

used to obtain the expression for dimensionless pressure:

$$p_D\left(r_D, t_D\right) = \frac{r_D^{\theta+2-D_m}}{\Gamma\left(\dfrac{D}{\theta+2}\right)(\theta+2)} \int_y^{\infty} z^{\frac{D}{\theta+2}-2} e^{-z} dz, \tag{37}$$

which is actually reduced to solving (11) for $D_m = 2$ and $\theta = 0$.

The skin factor and wellbore storage effect in the model can be taken into account by analogy with the conventional diffusivity equation (6), by introducing other boundary conditions and solving the equation by numerical methods or using the Laplace transform [3, 42, 43].

Numerical solutions of the generalized diffusivity equation (35) for different values of the two parameters D_m and θ are plotted in Fig. 7.

Similar dependencies $p_{Dwf}(t_D)$ and $d(p_{Dwf}(t_D))/d(\ln t_D)$ were also obtained in [44], but without accounting for the wellbore storage effect and the skin factor. These dependences were verified in [44] using numerical modeling of fluid flow to the well along an artificial system of fractures with fractional values of D_m and θ; however, the analysis did not include the physical aspects of fracture growth in the rock.

The dimensionless bottom-hole pressures and derivatives can be obtained in asymptotic form for large values of time t_D ($y \ll 1$) and fractal dimension $D_m \neq 2$ [43]:

$$p_D\left(1,t_D\right)=p_{D_{wf}}\left(t_D\right)\approx\frac{\left(\theta+2\right)^{2(1-\delta)}}{\Gamma\left(\delta\right)\left(\theta+2-D_m\right)}t_D^{\;(1-\delta)},\tag{38}$$

which can be then used to obtain the formula for the pressure derivative for this asymptote:

$$\frac{\partial p_D}{\partial\left(ln\left(t_D\right)\right)}\left(1,t_D\right)=\frac{dp_{D_{wf}}}{d\left(ln\left(t_D\right)\right)}\left(t_D\right)\approx\left(1-\delta\right)p_{D_{wf}}\left(t_D\right),\tag{39}$$

where $\delta = D_m/(\theta + 2)$.

Analyzing Eqs. (38), (39), we can draw several conclusions.

Firstly, accounting for the fractal dimension of a naturally fractured system D_m and the anomalous diffusion coefficient θ indeed allows describing the power-law dependence of pressure on time observed in [45–48]. Fractal (fractional) dimension and the anomalous flow (anomalous diffusion) effect have little effect on fluid filtration during the period when the wellbore storage effect is observed (at the initial instants of time) but make a significant contribution at a later stage, i.e., when radial flow would appear in a standard model (see Fig. 2).

Secondly, two characteristics can act as indicators of fractal fluid flow to the well.

The first one is that both well test curves have the same slope in double logarithmic coordinates, so that its arctangent is equal to $(1 - \delta)$ (see Fig. 7).

The second one is that the difference between the values of the well test curves at later times (when the wellbore storage effect and the skin factor are no longer noticeable) is constant and equal to $(1 - \delta)$.

Unlike plane-radial flow, the dependences obtained from well tests (see Fig. 7) and estimates of h and μ does not allow to unambiguously estimate the permeability value k_w on the well wall. Indeed, in contrast to expression (15), the asymptote of the bottom-hole pressure derivative in logarithmic coordinates does not have a horizontal section, and its slope depends on two unknown parameters: D_m and θ. Moreover, the relationship between these parameters is determined only for a certain class of bodies [1].

For this reason, Ref. [1] proposes to use the model (see Fig. 7) for assessing the reservoir's fracturing type rather than permeability.

The presence of fractal fracturing can point to anomalous fluid flow to the well in such a reservoir. In this case, well test simulation of fluid filtration should involve the approaches described in [17], since it would be incorrect to use traditional models of continuum mechanics and dual-porosity models, yielding overestimated forecasts for fluid flow at the well.

The slope of the asymptote $d(p_{Dwf}(t_D))/d(\ln t_D))$ in logarithmic coordinates is traditionally interpreted in engineering practice as the one-dimensional flow to the main fracture that the well has intersected. If a similar asymptote slope is observed in well tests in several wells, it should be assumed that a main fracture has evolved in each of the wells, which requires additional verification, since it can lead to an incorrect forecast of the pressure or fluid flow rate in these wells.

The model of fluid flow to a fractal fracture (see Eq. (35)) is based on a more reasonable assumption: if there are associated fractures in the reservoir, then the main fluid flow to the wells penetrating the reservoir occurs out along this complex (fractal) system of natural fractures, ultimately affecting the dynamics of well test results.

The fractal model additionally allows to interpret the well test results when the graph $d(p_{Dwf}(t_D))/d(\ln t_D))$ in logarithmic coordinates is *V*-shaped. This is interpreted in engineering practice as a dual-porosity mode [44], which may turn out to be incorrect if the graph does not reach a plateau later on (see Fig. 4).

Such a *V*-shape may be due to the presence of fractal fractures or anomalous diffusion (see Fig. 7) together with the wellbore storage effect. A *V*-shaped dip appears on the curve at the point in time when the wellbore storage effect weakens, causing this feature to appear on the plot.

Besides, the fractal model can be modified to interpret the well test data from two or more wells. For example, the fractal model in [45, 46] made it possible to approximate the data from cross-hole tests for pressure interference with a smaller error using the example of two fields in Indonesia.

Conclusion

The paper considers the models of oil flow to a vertical well in naturally fractured reservoirs, generalizing it to the case of anomalous fluid flow to the well along a fractal system of fractures. Analyzing these models, we discovered evidence from well test studies that anomalous flow to the well is established. We propose to use the model of fluid flow along a fractal fracture system for interpreting the data obtained in such tests.

This model is based on scaling laws for fractal structures, describing the mechanics of fluid flow in a fractal medium via the generalized diffusivity equation, which makes it possible to interpret the well test data for which the dependence of bottom-hole differential pressure on time follows a power law. Applying the model should prevent potential misinterpretations of well test data in the scenarios when models of linear flow along a single fracture or double porosity are used in engineering practice.

The fractal model of flow can be validated in the future both in field tests with real core samples and in numerical simulation. In particular, this can be resolved by means of the Smoothed Particle Hydrodynamics method (SPH) or finite-volume modeling of the Navier–Stokes equations for a geometry approximately simulating a fractal system of fractures.

It would be of interest to further advance the model for artificially generated hydraulic fracturing, other processes when fractal channels of fluid flow can evolve near the well, and filtration modes involving both fractures and pores.

Furthermore, it seems intriguing to draw an analogy between the model and the process of anomalous diffusion traditionally described by equations containing fractional derivatives. This description allows simulating the state of the system depending on its previous history.

Determining the parameters D_m and θ which characterizing the fractal properties of the system also remains an open question. Estimating these parameters will allow extracting valuable data on the properties of the reservoir from well tests without waiting for radial flow to evolve.

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