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NUMERICAL AND ANALYTICAL STUDY OF GAS SUSPENSION EXPANSION IN A CLOSED SHOCK TUBE

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Abstract: The expansion problem of ideal gas and incompressible spherical monodisperse particles mixture in a closed shock tube has been posed and solved. An asymptotically accurate solution of the initial (self-similar) stage of expansion of the gas suspension with a fine powder fraction was obtained. For the nonequilibrium case of larger particles, the problem was solved numerically by the hybrid large-particle method of the second-order approximation in space and time. The double-speed splitting effects of the flow stratification and the interface both into the gas-phase contact and the porosity jump as well as the interaction of the reflected shock wave with them were revealed. For the late stage of expansion, the dynamics of the gas suspension layer was established to be similar to a nonlinear oscillatory dissipative system with attenuation and a drift of the split contact interfaces depending on the size of dispersed inclusions.

Keywords: gas suspension expansion, closed shock tube, self-similar solution, oscillatory dissipative system, hybrid large-particle method

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ЧИСЛЕННОЕ И АНАЛИТИЧЕСКОЕ ИССЛЕДОВАНИЕ РАЗЛЕТА ГАЗОВЗВЕСИ В ЗАКРЫТОЙ УДАРНОЙ ТРУБЕ

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Аннотация. Поставлена и решена задача разлета смеси идеального газа и несжимаемых сферических монодисперсных частиц в закрытой ударной трубе. Получено асимптотически точное решение начальной (автомодельной) стадии разлета газовзвеси с мелкодисперсной фракцией. Для неравновесного случая частиц бульшего размера задача решена численно гибридным методом крупных частиц второго порядка аппроксимации по пространству и времени. Выявлены двухскоростные эффекты расслоения течения и границы раздела сред на контакт в газовой фазе и скачок пористости, а также взаимодействия с ними отраженной ударной волны. Для поздней стадии разлета установлено, что динамика слоя газовзвеси подобна нелинейной колебательной диссипативной системе с затуханием и дрейфом расщепленных контактных границ, зависящих от размеров дисперсных включений.

Ключевые слова: разлет газовзвеси, закрытая ударная труба, автомодельное решение, колебательная диссипативная система, гибридный метод крупных частиц

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Introduction

The phenomenon of dusty gas expanding in a confined space is interesting from the standpoint of applied problems on explosion protection of production facilities, as well as on powder and chemical technologies for pulse mixing of reagents.

Monographs [1, 2] are dedicated to fundamental challenges and approaches to modeling the dynamics of heterogeneous media. Expansion of gas suspension in a shock tube was considered in studies on the evolution of shock waves in ambient gas [3, 4], the parameters of discharged powder layer or gas suspension [5, 6], the interaction of a shock wave with an obstacle screened by a porous layer [7], the structures loaded by the flow of a two-phase medium [8, 9], the effect that the properties of the mixture components have on the expanding gas suspension [10, 11].

Different aspects of oscillatory processes in gas suspensions and aerosols are discussed in monographs [12, 13]. Experimental studies in [14, 15] deal with the behavior of gas suspension in a nonlinear wave field evolving in a tube, where longitudinal oscillations are excited by piston displacement following the harmonic law (in a resonator). Numerical simulation of gas oscillations with dispersed inclusions in the resonator was carried out using the finite-volume scheme in the Fluent CFD package [16] and the McCormack finite-difference scheme [17]. The results of physical experiments on nonlinear oscillations of aerosol in acoustic resonators were compared in [18] with numerical calculations by the McCormack method. The author attributes the discrepancy between the experimental data and the simulation results to the contribution from numerical viscosity of the method used.

Our study was aimed at analysis of the initial stage in the expansion of gas suspension in a closed shock tube based on accurate self-similar solutions for a two-phase equilibrium medium, comparing these solutions with the results obtained by the hybrid large-particle method; furthermore, we carried out numerical simulation for a nonlinear oscillatory process at a late stage taking into account the nonequilibrium velocity and temperature of gas and particles.

Mathematical model and calculation procedure

The expansion dynamics in a gas suspension is described within the framework of a two-fluid model [1] of a calorically perfect gas and solid incompressible particles as formulated in [19]:

$$\begin{gathered}
\frac{\partial \rho_i}{\partial t} + \frac{\partial \rho_i v_i}{\partial x} = 0,\ \frac{\partial \rho_1 v_1}{\partial t} + \frac{\partial \rho_1 v_1^2}{\partial x} + \alpha_1 \frac{\partial p}{\partial x} = -F_\mu, \\
\frac{\partial \rho_2 v_2}{\partial t} + \frac{\partial \rho_2 v_2^2}{\partial x} + \alpha_2 \frac{\partial p}{\partial x} = F_\mu,\ \frac{\partial \rho_2 e_2}{\partial t} + \frac{\partial \rho_2 e_2 v_2}{\partial x} = Q_T, \\
\frac{\partial}{\partial t}\left(\rho_1 E_1 + \rho_2 E_2\right) + \frac{\partial}{\partial x}\left(\rho_1 E_1 v_1 + \rho_2 E_2 v_2 + p\left(\alpha_1 v_1 + \alpha_2 v_2\right)\right) = 0, \\
\rho_i = \rho_i^\circ \alpha_i\ (i = 1,\ 2),\ \rho = \rho_1 + \rho_2,\ \alpha_1 + \alpha_2 = 1,\ E_i = e_i + \mathrm{v}_i^2/2.
\end{gathered} \tag{1}$$

The subscripts 1 and 2 here and below correspond to the parameters of the carrier and dispersed phases, respectively, the superscript ○ to the true densities. The quantities α_i, ρ_i, kg/m^3, v_i, m/s, E_i, e_i, J/kg, p, N/m^2, ρ, kg/m^3 refer to the volume fraction, reduced density, velocity, total and internal energy per unit mass of the ith phase, gas pressure, and density of a mixture of gas and particles; F_μ, N/m^3, and Q_T, W/m^3, are, respectively, the viscous component of the phase interaction force and the heat flow rate between gas and particles per unit volume; x, m, t, s, are the coordinate and the time.

The closing relations of system (1) are the equations of state for an ideal, calorically perfect gas and incompressible solid particles:

$$p = (\gamma_1 - 1)\rho_1^\circ e_1,\ e_1 = c_v T_1,\ e_2 = c_2 T_2,\ \{\gamma_1, c_v, c_2, \rho_2^\circ\} \equiv \text{const},$$

where T_1, T_2, K, is the temperature of the carrier phase and particles; γ_1, c_v, J/(kg·K), are the adiabatic exponent and specific heat capacity of the gas at constant volume; c_2, J/(kg·K), is the specific heat capacity of the particles.

The interaction force F_μ and the heat flow rate Q_T between the phases are found from empirical relationships [1, 5]:

$$\mathbf{F}_\mu = (3/8)(\alpha_2 / r) C_\mu\left(\text{Re}_{12}\right)\rho_1\left(\mathbf{v}_1 - \mathbf{v}_2\right)\left|\mathbf{v}_1 - \mathbf{v}_2\right|,$$

$$C_\mu^{(1)} = \frac{24}{\text{Re}_{12}} + \frac{4.4}{\text{Re}_{12}^{1/2}} + 0.42\ \left(\alpha_2 < 0.08\right),$$

$$Q_T = (3/2)(\alpha_2 / r^2)\lambda_1 \text{Nu}_1 (T_1 - T_2),$$

$$\text{Nu}_1 = \begin{cases} 2 + 0.106\text{Re}_{12}\text{Pr}_1^{1/3} & (\text{Re}_{12} \le 200), \\ 2.274 + 0.6\text{Re}_{12}^{0.67}\text{Pr}_1^{1/3} & (\text{Re}_{12} > 200), \end{cases}$$

$$\text{Re}_{12} = 2r\rho_1^\circ\left|\mathbf{v}_1 - \mathbf{v}_2\right| / \mu_1,\ \text{Pr}_1 = c_v \gamma_1 \mu_1 / \lambda_1.$$

where Re_{12}, Nu_1, Pr_1 are the Reynolds, Nusselt and Prandtl numbers; C_μ, μ_1, Pa·s, r, m, are the interfacial friction coefficient, dynamic viscosity and particle radius.

The hybrid large-particle method with second-order of approximation in space and time was used to numerically simulate the expansion of the gas suspension [20]. The numerical solution was regularized by two techniques: using Christensen-type artificial viscosity and nonlinear correction of flows with limiters. The applicability of the method was previously verified for this class of problems using the Shu–Osher test problem on nonlinear acoustics [21].

Problem statement

At the initial moment of time, a shock tube with the length $X = 1$ m with closed ends is divided into two parts (Fig. 1,*b*). A high-pressure chamber L filled with air at $p_L = 10^6$ Pa, mixed with monodisperse spherical incompressible particles with a material density $\rho_2^\circ = 2500$ kg/m^3, was located to the left of the membrane M (at $0 < x \le x_M = 0.5$ m). The dispersed phase occupies a volume fraction $\alpha_{2L} = 0.001$.

The region R ($x_M < x < x_w$) with low-pressure dust-free air ($p_R = 10^5$ Pa) is located to the right of the membrane M. Gas and dispersed medium are in a state of thermodynamic equilibrium ($T_{1L,R} = T_{2L} = 293$ K) and at rest ($v_{1L,R}$, $v_{2L,R} = 0$). The boundary conditions on the left (at $x = 0$) and on the right (at $x = x_w$) are given in the no-flow formulation, where the gas and particle velocities on the end walls of the tube equal zero. The membrane M is instantly removed at time $t = 0$. It is required to determine the movement of the medium at $t > 0$. The problem was solved numerically by the hybrid large-particle method with a Courant number of 0.4 on a grid containing 400 cells.

Problem solution and discussion of the results

First, consider the solution to the problem posed in a short time interval: the structure of the solution is shown in the x–t diagram (Fig. 1,*a*). After the membrane is removed, the discontinuity in the parameters decays at the point x_M, separating the initial states of the medium to the right (*0*) and to the left (*1*) of the discontinuity. This produces a centered rarefaction wave r, a constant-flow region *2* before the contact discontinuity c_0, and a cocurrent flow region *3* behind the shock wave s_0. As the wave front s_0 reaches the right wall of the tube at the instant in time t_w, a reflected shock wave s_w is generated, moving in the opposite direction. When this wave interacts with the interface between the media c_0, another discontinuity decay occurs at the point x_c: with the shock waves reflected (s_{c1}) and transmitted into the gas suspension layer (s_{c2}).

Following the main relations of the Riemann problem in a two-phase equilibrium medium [22, 23], we construct an asymptotically accurate solution (at $d \to 0$) for the initial stage in the expansion of a gas suspension in a closed shock tube. The solution is found assuming the velocity and temperature equilibrium as a sequence of discontinuity decays at points x_M, x_w, x_c. The values of pressure p_j, density ρ_j, velocity v_j of the mixture and volume fraction α_{1j} of gas are complemented with subscripts $j = 0, 1, ..., 6$, corresponding to the numbers of the regions (see Fig. 1,*a*).

Discontinuity decay at the point x_M. The pressure in the regions with constant p_2 and cocurrent flow behind the shock wave $p_3 = p_2$ is determined for the given parameters of an arbitrary discontinuity in the regions *0* and *1* as a solution to the equation

$$\frac{2a_1\alpha_{11}}{\gamma_1^*-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma_1^*-1}{2\gamma_1^*}}\right]=\left(p_2-p_0\right)\sqrt{\frac{\chi_0-1}{\rho_0 p_2+p_0}}.$$

The following notations are used here and below: $\chi_j = (\gamma_j^* + 2\alpha_{1j} - 1)/(\gamma_j^* - 1)$, $\kappa_j = (\gamma_j^* + 1)/(\gamma_j^* - 1)$ are auxiliary functions; $\gamma_j^* = 1 + (\zeta_{1j}R_1)/(\zeta_{1j}c_v + \zeta_{2j}c_2)$ is the polytropic index of the mixture depending on the mass concentrations of the phases $\zeta_{ij} = \rho_{ij}/\rho_j$; R_1 is the gas constant; $a_j = \sqrt{\gamma_j^* p_j/(\rho_j\alpha_{1j})}$ is the equilbrium speed of sound in a two-phase medium.

Given the obtained pressure value and the invariable polytropic index of the mixture along the trajectories $\gamma_3^* = \gamma_0^*$ and $\gamma_2^* = \gamma_1^*$, other parameters of the gas suspension are calculated:

$$v_3 = v_2 = \left(p_3 - p_0\right)\sqrt{\frac{\chi_0-1}{\rho_0\left(\kappa_0 p_3 + p_0\right)}},$$

$$\rho_3 = \rho_0\frac{\kappa_0 p_3 + p_0}{\chi_0 p_0 + \dfrac{\gamma_0^* - 2\alpha_{10} + 1}{\gamma_0^* - 1}p_3},\quad \alpha_{13} = 1 - \frac{\rho_3}{\rho_0}\left(1-\alpha_{10}\right),$$

$$\rho_2 = \rho_1\left[\alpha_{21} + \alpha_{11}\left(1 - \frac{\gamma_1^*-1}{2\alpha_{11}}\frac{v_2}{a_1}\right)^{-\frac{2}{\gamma_1^*-1}}\right]^{-1},\quad \alpha_{12} = 1 - \frac{\rho_2}{\rho_1}\left(1-\alpha_{11}\right),$$

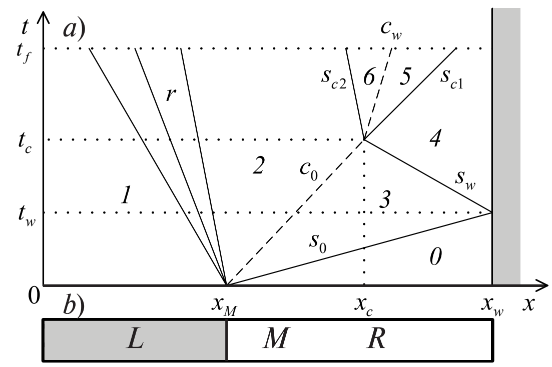

Fig. 1. Structure of the solution for the initial stage in the expansion of gas suspension: *x–t* diagram (*a*), shock tube (*b*)

The regions considered are numbered; *r* is the centered rarefaction wave; s_0 is the primary shock wave, and s_w, s_{c1}, s_{c2} are the shock waves evolving behind it; c_0, c_w are the interfaces between the media before and after refracting the shock wave reflected from the wall, respectively; *L* is the high-pressure chamber, *M* are the membranes, *R* is the low-pressure region with dust-free air.

$$\mathrm{M}_0=\sqrt{\frac{\alpha_{10}}{\alpha_{13}}\left(\frac{\gamma_3^*+2\alpha_{13}-1}{2\gamma_3^*}\frac{p_3}{p_0}+\frac{\gamma_3^*-2\alpha_{13}+1}{2\gamma_3^*}\right)},\ D_0=\mathrm{M}_0a_0,$$

where M_0 and D_0 are the Mach number of the shock wave s_0 (see Fig. 1,*a*) and its velocity, respectively.

The centered rarefaction wave *r* in the region *1–2* (see Fig. 1,*a*) has a self-similar solution depending on the variable $\xi = (x - x_M)/a_1\alpha_{11}t$:

$$\left[\frac{(1-\alpha_{11})\alpha_1(\xi)}{(1-\alpha_1(\xi))\alpha_{11}}\right]^{\omega}=\frac{\alpha_1(\xi)+\omega}{\alpha_1(\xi)(1-\omega\xi)},\ \omega=\frac{\gamma_1^*-1}{2},$$

$$\mathrm{M}_c(\xi)=\frac{2\alpha_{11}}{\gamma_1^*+2\alpha_1(\xi)-1}(\alpha_1(\xi)\xi+1),\ \mathrm{M}_c(\xi)=\frac{v(\xi)}{a_1},$$

$$\rho(\xi)=\rho_1\frac{\alpha_1(\xi)}{\alpha_{11}}\left(1-\frac{\gamma_1^*-1}{2\alpha_1(\xi)}\mathrm{M}_c(\xi)\right)^{\frac{1}{\omega}},\ p(\xi)=p_1\left(1-\frac{\gamma_1^*-1}{2\alpha_{11}}\mathrm{M}_c(\xi)\right)^{\frac{\gamma_1^*}{\omega}},$$

Reflection from the wall at point x_w. The parameters characterizing the processes occurring behind the reflected shock wave s_w at $\gamma^*_4 = \gamma^*_3$ are calculated from the system of equations:

$$2v_3=2(p_4-p_3)\sqrt{\frac{\chi_3-1}{\rho_3(\kappa_3p_4+p_3)}},$$

$$v_4=v_3-(p_4-p_3)\sqrt{\frac{\chi_3-1}{\rho_3(\kappa_3p_4+p_3)}}$$

$$\rho_4=\rho_3\frac{\kappa_3p_4+p_3}{\chi_3p_3+\dfrac{\gamma_3^*-2\alpha_{13}+1}{\gamma_3^*-1}p_4},\ \alpha_{14}=1-\frac{\rho_4}{\rho_3}(1-\alpha_{13}),$$

$$\mathrm{M}_w=\sqrt{\frac{\alpha_{13}}{\alpha_{14}}\left(\frac{\gamma_4^*+2\alpha_{14}-1}{2\gamma_4^*}\frac{p_4}{p_3}+\frac{\gamma_4^*-2\alpha_{14}+1}{2\gamma_4^*}\right)},\ D_w=v_3-\mathrm{M}_wa_3,$$

where M_w and D_w are the Mach number of the shock wave s_w reflected from the wall (see Fig. 1,*a*) and its velocity, respectively.

Discontinuity decay at the contact boundary at point x_c. Interaction with the contact discontinuity of the shock wave reflected from the wall produces a configuration with two shock waves characterized by the parameters at $\gamma^*_5 = \gamma^*_4$ and $\gamma^*_6 = \gamma^*_2$:

$$v_2-(p_6-p_2)\sqrt{\frac{\chi_2-1}{\rho_2(\kappa_2p_6+p_2)}}=(p_6-p_4)\sqrt{\frac{\chi_4-1}{\rho_4(\kappa_4p_6+p_4)}},$$

$$v_6=v_5=v_2-(p_6-p_2)\sqrt{\frac{\chi_2-1}{\rho_2(\kappa_2p_6+p_2)}},\ \rho_6=\rho_2\frac{\kappa_2p_6+p_2}{\chi_2p_2+\dfrac{\gamma_2^*-2\alpha_{12}+1}{\gamma_2^*-1}p_6},$$

$$\alpha_{16} = 1 - \frac{\rho_6}{\rho_2}(1 - \alpha_{12}).$$

$$M_2 = \sqrt{\frac{\alpha_{12}}{\alpha_{16}}\left(\frac{\gamma_6^* + 2\alpha_{16} - 1}{2\gamma_6^*}\frac{p_6}{p_2} + \frac{\gamma_6^* - 2\alpha_{16} + 1}{2\gamma_6^*}\right)},\ D_4 = M_4 a_4,$$

$$\rho_5 = \rho_4 \frac{\kappa_4 p_5 + p_4}{\chi_4 p_4 + \dfrac{\gamma_4^* - 2\alpha_{14} + 1}{\gamma_4^* - 1} p_5},\ \alpha_{15} = 1 - \frac{\rho_5}{\rho_4}(1 - \alpha_{14}),$$

$$M_4 = \sqrt{\frac{\alpha_{14}}{\alpha_{15}}\left(\frac{\gamma_5^* + 2\alpha_{15} - 1}{2\gamma_5^*}\frac{p_5}{p_4} + \frac{\gamma_5^* - 2\alpha_{15} + 1}{2\gamma_5^*}\right)},\ D_4 = M_4 a_4,$$

where M_2 and D_2, M_4 and D_4 are the Mach numbers and velocities of the shock waves s_{c2} and s_{c2}, respectively, formed after decay at the contact interface (see Fig. 1,*a*).

The instants when the shock wave is reflected from the wall t_w and refracted at the contact discontinuity t_c, as well as the coordinate of refraction for the shock wave reflected from the wall x_c are found from the expressions:

$$t_w = \frac{x_w - x_0}{D_0},\ t_c = \frac{x_w - x_0 - D_w t_w}{v_2 - D_w},\ x_c = x_0 + v_2 t_c.$$

The solution to the problem on the expansion of gas suspension at an early stage, in a closed shock tube at successive instants as relative density profiles of the mixture along the dimensionless coordinate $x' = x/X$, is shown in Fig. 2, *a*–*d* for particle size $d = 0.1$ μm and in Fig. 2, *e*–*h* for $d = 20$ μm. The dashes correspond to the calculation results in both cases for the 1/400 grid, and the solid lines (see Fig. 2) are the exact solutions of the equilibrium mixture given above.

The flow in a two-phase medium with a coarse fraction is nonequilibrium, and analytical solutions are unknown. For this reason, Fig. 2, *e*–*h* shows the calculation results for a detailed 1/4000 grid (solid curves) for comparison. Notice that the hybrid large-particle method used effectively resolves the flow details.

In contrast to almost homogeneous flow, stratification of gas flow and dispersed phase (Fig. 2, *e*–*h*) is observed for larger particles in the shock tube with a gas suspension containing a fine powder fraction (see Fig. 2, *a*–*d*). Consequent splitting of the initial interface between the media occurs, producing a contact discontinuity in gas c_0 and a particle suspension interface c'_0. The shock wave s_w reflected from the wall is subsequently refracted first at the gas contact c_w (Fig. 2,*g*), and then at the porosity jump c'_w (Fig. 2,*h*).

A self-similar solution to a two-phase equilibrium medium exists for a limited time interval. Multiple interactions of shock waves and rarefaction waves with the channel walls and interface boundaries occur in the shock tube at the given late stage in the expansion of the gas suspension considered numerically. Fig. 3 presents data for a mixture with different particle sizes, namely $d = 0.1$, 4.0, 10 and 20 μm, showing the dimensionless trajectories of the contact in the gas phase *1* ($x'_1 = x_1/X$) and the particle suspension boundaries *2* ($x'_2 = x_2/X$) with their common initial position at point x_M.

As evident from the figures, mixture expansion in a confined space can be interpreted as a dissipative oscillatory system. The initial interface between the media for a fine powder fraction makes oscillatory movements as a single contact (Fig. 3, *a*). The porosity jump oscillates with increasing diameter of the particles, shifting towards the right wall of the tube; furthermore, more intense attenuation of the oscillations is observed due to interfacial friction and heat transfer (Fig. 3, *b*–*d*).

Conclusion

We have considered the initial stage in the expansion of a mixture of gas and particles in a closed shock tube, when discontinuity decay occurs, the shock wave is reflected from the wall and refracted at the interface between the media. An asymptotically accurate solution was obtained for the case of fine particles. The hybrid large-particle method was used to numerically obtain the effects related to stratification of two-phase flow, with two contact surfaces appearing (the interface between suspension and contact in the gas phase), interacting with the reflected shock wave. The resolution and accuracy of the method are confirmed by comparison with self-similar solutions. We have established that the gas suspension layer behaves like an oscillatory dissipative system with the decay depending on the size of dispersed particles in a long-term period.

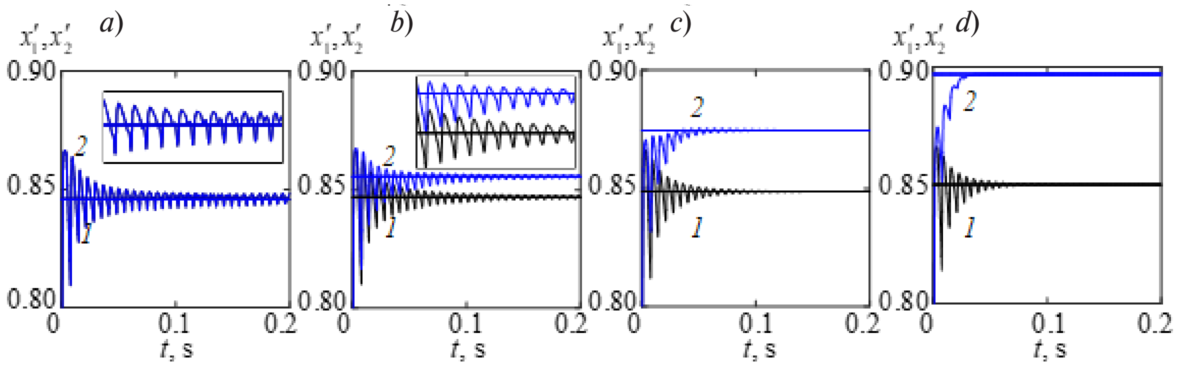

Fig. 3. Trajectories of contact motion in the gas phase *1* (x'_1, solid curves) and particle suspension interfaces *2* (x'_2, bold blue lines) for a mixture with different particle sizes d, μm: 0.1 (*a*), 4.0 (*b*), 10 (*c*) and 20 (*d*); horizontal lines are asymptotes of contact trajectories

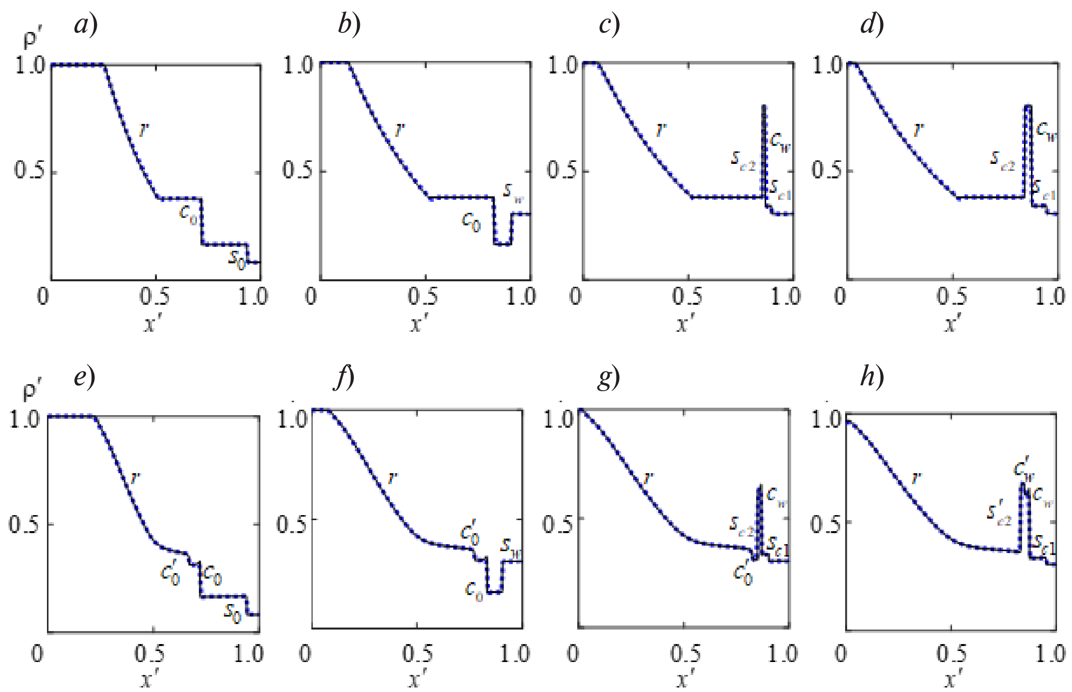

Fig. 2. Relative density distribution for the gas and particle mixture; d = 0.1 μm (*a*–*d*) and d = 20 μm (*e*–*h*) at instants, ms: 0.8 (*a*,*e*), 1.2 (*b*,*f*), 1.4 (*c*,*g*), 1.5 (*d*,*h*).
The notations correspond to those given in Fig. 1,*a*, however, c_0, c'_0, c_w, c'_w in the second row are contact discontinuities in the gas phase (without primes) and porosity jumps (with primes) before (subscript 0) and after (subscript w) refraction of the shock wave reflected from the wall; s_{c2}, s'_{c2} is the refracted shock wave at gas contact (without a prime) and at porosity jump (with a prime)

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