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THE CONDITION FOR APPLICATION OF THE CROCCO INTEGRAL IN THE MATHEMATICAL DESCRIPTION OF A LASER WELDING PLASMA PLUME

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Based on the theoretical approach developed in the article by G.A. Turichin et al. (High Temperature. 2006. Vol. 44. No. 5. Pp. 647–655), the characteristics of the plasma plume forming in the keyhole laser welding have been investigated. A condition corresponding to the existence of the Crocco integral was defined, making it possible to simplify the system of gas dynamics equations and obtain analytical solutions for a plasma plume in the form of a classical submerged jet. These solutions were analyzed for a wide range of metal vapor velocities and temperatures at the keyhole top. For some of the selected values, an agreement with the numerical calculations of other authors was found.

Keywords: gas dynamics equations, plasma plume, laser welding, shear layer, submerged jet

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УСЛОВИЕ ПРИМЕНЕНИЯ ИНТЕГРАЛА КРОККО ПРИ МАТЕМАТИЧЕСКОМ ОПИСАНИИ ПЛАЗМЕННОГО ФАКЕЛА ПРИ ЛАЗЕРНОЙ СВАРКЕ

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На основе теоретического подхода, разработанного в статье Туричина Г.А. и др. (ТВТ, 2006. Т. 44, № 5. С. 655–663), исследуются характеристики плазменного факела, образующегося при лазерной сварке с глубоким проплавлением. Находится условие, которое отвечает существованию интеграла Крокко и позволяет упростить систему газодинамических уравнений, дающую аналитические решения для плазменного факела в виде классической затопленной струи. Полученные решения анализируются для широкого интервала значений скорости и температуры паров металла на вылете из парогазового канала. Для некоторых выбранных значений найдено согласие с численными расчетами других авторов.

Ключевые слова: газодинамические уравнения, плазменный факел, лазерная сварка, слой смешения, затопленная струя

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Introduction

The essence of keyhole laser welding is that a laser beam directed at a metal workpiece causes the welded metal to heat and evaporate with a vapor-gas channel (the so-called keyhole) forming. The interaction between the vapor pressure of the material, hydrostatic forces of the surrounding liquid metal and surface tension forces prevent the keyhole from closing [1]. A jet of metal vapors escapes from the keyhole (KH) into the shielding gas atmosphere, propels it and mixes with it, attenuating at a certain height due to viscosity. The interaction of the laser beam with the evaporated metal can lead to partial ionization of these vapors, generating plasma above the surface treated, i.e., a plasma plume and plasma inside the KH [2 – 4]. The plasma plume serves as a convenient medium for transferring energy from laser radiation to the workpiece. However, it also affects the welding process, due to absorption and scattering of laser radiation, as well as heating of the laser head.

The interactions between the plasma plume, laser beam and shielding gas paint a complex picture of gas flow and heat/mass transfer [5 – 7]. When the absorbed energy becomes comparable to the energy losses due to conduction, convection and radiation of the plasma plume, the plume becomes capable of maintaining a stationary shape with respect to the laser beam. The spatial distributions of the gas temperature and particle concentration in the plasma plume are always non-uniform. The presence of density gradients within the plasma plume leads to refraction and defocusing of the laser beam, reducing the power density of laser radiation and thus affecting the laser welding process.

Combustion of the plasma plume during welding can be accompanied by different complex processes depending on the specific type of material processing. For example, condensed-phase clusters can evolve in the plasma plume during hybrid laser-arc welding of metallic materials due to expansion and cooling of metal vapors, reducing the mean energy of electrons in the plasma and affecting the course of kinetic processes [8, 9].

As new methods need to be developed for controlling the processes occurring during keypoint laser welding in order to improve the quality of material processing and obtain welds with the required properties, studies into the distributions of temperature, density and concentration of particles in a plasma plume are of paramount importance.

Turichin et al. [5] considered the characteristics of a plasma plume appearing during welding in a helium-iron atmosphere using a solid-state neodymium-doped yttrium aluminum garnet laser (Nd:YAG). The absorption of laser radiation by the plume in this particular case is typically very insignificant. The authors then used a system of gas-dynamic equations to solve the problem on a jet of hot iron vapors ejected into an atmosphere of cold helium acting as the shielding gas [10 – 12]. The influence of insignificant volumetric heat release due to absorption of laser radiation by the plume was taken into account by introducing an appropriate source into the energy balance equation. The properties of the source were determined from the solution of the kinetic equation for the energy spectrum of plasma electrons [13].

Analytical solutions of the system of gas-dynamic equations corresponding to the problem of a laminar submerged jet [10] were obtained in [5] taking the approximation of an axially symmetric boundary layer and neglecting the heat source. Several assumptions were made to obtain these solutions, allowing, in particular, to introduce the Crocco integral, but the authors did not investigate the condition for its existence in detail.

In view of the above, one of the main goals of this study was to identify the domain where the values of the observed quantities can be described by analytical solutions obtained in [5]. An additional task was to analyze the characteristics of the plasma plume in a wide range of velocities and temperatures of metal vapors ejected from the keyhole, comparing the results with the data of numerical calculations obtained by other authors [6, 7].

Theoretical description of a plasma plume

We consider the stationary case, i.e., neglect all the transient processes associated with the initial instants when the laser is switched on and the plume starts to form. Assuming that welding is performed by a Nd: YAG laser in a helium atmosphere, we will also neglect the heat source (similar to [5]), associated with the absorption of laser radiation by the plume.

Fig. 1 schematically shows the plasma plume, which is a metal vapor (MV) jet escaping from the keyhole (KH) into an atmosphere of shielding helium gas (HG). The vapors advance the shielding gas, mixing with it, and the jet attenuates at some distance from the surface due to viscosity. The axes of the Cartesian coordinate system are located so that the axis z is directed along the keyhole axis, which can be represented as a cylinder with a small depth Δz.

Let us denote the density of the mixture consisting of metal vapors with shielding gas as ρ, the mass fraction of metal atoms in the mixture as C. Then the densities of metal vapors ρ_V and shielding gas ρ_g are expressed as

$$\begin{cases} \rho = \rho_V + \rho_g, \\ \rho_V = C\rho,\ \rho_g = (1-C)\rho. \end{cases} \tag{1}$$

Some of the properties that mainly determine the properties of a stabilized plasma plume occur in the layer where metal vapors are mixed with shielding gas atoms. We consider the edge of the mixing layer, where the concentration of heavy metal atoms in the mixture is low, i.e., the region where the heavy gas diffuses in the light one. In this case, the properties of the plume can be described in the approximation of the boundary layer. We assume that the boundary layer is laminar (see the estimate of the Reynolds number below, in the description of the results).

Let us take two length scales: longitudinal $\sim z$ (along the direction in which metal vapors are ejected) and transverse $\sim r = \sqrt{x^2 + y^2}$ (across the keyhole axis). Similar to theory of the boundary layer, the transverse scale is assumed to be much smaller than the longitudinal one.

The complete system of gas dynamic equations for the given binary mixture, written in the approximation of an axially symmetric boundary layer, has the form [10, 11]:

$$\begin{cases} \rho\left\{ v_r \dfrac{\partial v_z}{\partial r} + v_z \dfrac{\partial v_z}{\partial z} \right\} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\mu \dfrac{\partial v_z}{\partial r} \right) - \dfrac{\partial p}{\partial z}, \\ \rho\left\{ v_r \dfrac{\partial C}{\partial r} + v_z \dfrac{\partial C}{\partial z} \right\} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\rho D_{gV} \dfrac{\partial C}{\partial r} \right), \\ \rho\left\{ v_r \dfrac{\partial h}{\partial r} + v_z \dfrac{\partial h}{\partial z} \right\} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r \dfrac{\kappa}{C_p} \dfrac{\partial h}{\partial r} \right) + v_z \dfrac{\partial p}{\partial z} + \mu\left(\dfrac{\partial v_z}{\partial r} \right)^2 + q, \\ \dfrac{\partial}{\partial r}(r\rho v_r) + \dfrac{\partial}{\partial z}(r\rho v_z) = 0, \\ \dfrac{\partial p}{\partial r} = 0, \end{cases} \tag{2}$$

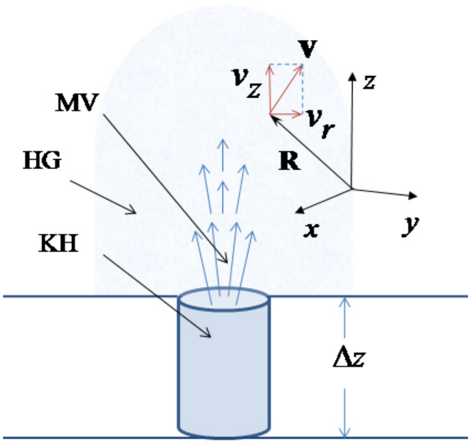

Fig. 1. Schematic of plasma plume and velocity field $\mathbf{v}(\mathbf{R})$, where $\mathbf{R} = (r, \varphi, z)$: metal vapors MV; shielding helium gas HG; keyhole KH, its depth Δz. A laser beam (not shown in the figure) falls along the normal to the surface of the metal workpiece

where v_r, v_z are the cylindrical components of the velocity field $\mathbf{v}$; μ is the dynamic viscosity; D_{gV} is the interdiffusion coefficient; κ is the thermal conductivity; C_p is the heat capacity of the mixture, $C_p = (1-C)\cdot C_{p_g} + C\cdot C_{p_V}$ (C_{p_g}, C_{p_V} are the specific heat capacities of helium and metal vapors); $h = C_p T$ is the enthalpy written in terms of temperature T; p is the pressure.

The source q (which we are going to neglect below) is the volumetric heat release related to the absorption of laser radiation energy by the mixture.

System of equations (2) is written in the cylindrical coordinate system (r, φ, z) with the axis z along the keyhole top (see Fig. 1). Assuming axial symmetry for the problem implies that the velocity field $\mathbf{v}$ is independent of the angle φ and that the rotational component v_φ of the velocity is equal to zero, i.e.,

$$\mathbf{v}(r,z) = \mathbf{e}_r v_r(r,z) + \mathbf{e}_z v_z(r,z).$$

The first equation in system (2) is obtained from the main dynamic equation (Navier – Stokes equations), the second and third are derived from the convective diffusion equation and the general energy balance equation, respectively. The fourth equation in system (2) is the continuity equation, and the fifth equation is the well-known condition of constant pressure across the boundary layer.

System (2) differs from a similar system of equations given in [5] by a quadratic term ~ $(\partial v_z/\partial r)^2$, characterizing the action of viscous forces in the equation for enthalpy. The authors of [5] assumed in advance that this term was small and omitted it. On the other hand, we preserved this term for the sake of generality; we are going to explain at the end of the section why it can be neglected. Below we make a natural assumption about the constant pressure [10], in the same manner as it was done in [5], i.e., we take $\partial p/\partial z = 0$ in system (2).

We use the Mendeleev – Clapeyron equation as an equation of state closing system (2):

$$p = \left(\frac{C}{M_V} + \frac{1-C}{M_g}\right)\rho R T, \tag{3}$$

where M_i is the molar mass for the i^{th} component of the mixture, R is the universal gas constant.

System (2), (3) can be solved by finding the transfer coefficients μ, D_{gV}, κ. These coefficients contain the free path lengths λ_g, λ_V (for helium and metal, respectively), which can be estimated as follows based on molecular kinetic theory [14, 15]:

$$\begin{cases} \lambda_g = \dfrac{1}{n_g\sigma_{gg}\sqrt{2} + n_V\sigma_{gV}\sqrt{1+m_g/m_V}}, \\ \lambda_V = \dfrac{1}{n_V\sigma_{VV}\sqrt{2} + n_g\sigma_{Vg}\sqrt{1+m_V/m_g}}, \end{cases} \tag{4}$$

where m_g, m_V are the masses of helium and metal atoms, respectively; n_g, n_V are the corresponding concentrations; σ_{gg}, σ_{gV}, σ_{Vg}, σ_{VV} are the effective cross sections of gas – gas, gas – metal, metal – gas and metal – metal interactions.

The mass of a helium atom is $m_g \approx 4$ amu; we took the mass of an iron atom $m_V \approx 56$ amu as the mass of the metal atom; concentrations $n_g = \rho_g/m_g$, $n_V = \rho_V/m_V$.

The effective cross sections are expressed as

$$\sigma_{gg} = 4\pi r_g^2, \sigma_{gV} = \pi\left(r_g + r_V\right)^2, \sigma_{VV} = 4\pi r_V^2,$$

where r_g, r_V are the effective radii of the atoms.

The Van der Waals radii were chosen as these radii [16, 17]: $r_g \approx 140$ pm, $r_V \approx 210$ pm.

Therefore,

$$\sigma_{gg} \approx 2.46\cdot 10^5 \text{ (pm)}^2,\ \sigma_{gV} \approx 3.85\cdot 10^5 \text{ (pm)}^2,\ \sigma_{VV} \approx 5.54\cdot 10^5 \text{ (pm)}^2 \quad (5)$$

and

$$\lambda_g \approx \gamma\lambda_V, \quad (6)$$

where the value of γ lies in the range from 2 to 4. The number 2 corresponds to the $C \to 1$ limit, and 4 corresponds to the $C \to 0$ or $C \to 0.5$ limit.

The expressions for the transfer coefficients have the following form [14, 15]:

$$\begin{cases} \nu = \dfrac{1}{3}\left[C\lambda_V \langle v_V \rangle + (1-C)\lambda_g \langle v_g \rangle\right], \\ D_{gV} = \dfrac{1}{3}\left[(1-C)\lambda_V \langle v_V \rangle + C\lambda_g \langle v_g \rangle\right], \\ \alpha = \dfrac{C_v}{3C_p}\left[C\lambda_V \langle v_V \rangle + (1-C)\lambda_g \langle v_g \rangle\right], \end{cases} \quad (7)$$

where $\nu = \mu/\rho$ is the kinematic viscosity; $\alpha = \kappa/(\rho C_p)$ is the thermal diffusivity coefficient; $\langle v_V \rangle, \langle v_g \rangle$ are the mean thermal velocities of iron and helium atoms, respectively.

The mass fraction C of iron atoms in the mixture should be less than 0.6 in the region of the mixing layer under consideration, with predominant diffusion of heavy gas in light one; this value corresponds to the approximate equality $n_g \approx 10\, n_V$. Let us consider a particular case $C \approx 0.5$. The reason why we have chosen it will become clear later on.

Then, taking into account relation (6), as well as the ratio of thermal velocities $\langle v_g \rangle / \langle v_V \rangle = \sqrt{m_V/m_g} \approx 4,$ we can rewrite the expressions for the transfer coefficients as follows:

$$\begin{cases} \nu \approx \nu_\infty \dfrac{\rho_\infty}{\rho}\left(\dfrac{h}{h_\infty}\right)^{1/2}(1-C), \\ D_{gV} \approx \nu_\infty \dfrac{\rho_\infty}{\rho}\left(\dfrac{h}{h_\infty}\right)^{1/2} C,\ \alpha \approx \alpha_\infty \dfrac{\rho_\infty}{\rho}\left(\dfrac{h}{h_\infty}\right)^{1/2}(1-C), \end{cases} \quad (8)$$

where

$$\nu_\infty = \frac{4(kT_\infty)^{3/2}}{3p\sigma_{gg}(\pi m_g)^{1/2}},\ \alpha_\infty = \frac{3\nu_\infty}{5},\ h_\infty = C_p T_\infty;$$

T_∞, ρ_∞ are the temperature and density at infinity, i.e., the parameters of the shielding gas away from the plume; k is the Boltzmann constant.

Let us rewrite system (2) in view of the remarks made and the transfer coefficients (8) obtained:

$$\begin{cases}\left\{v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}\right\}=\nu_\infty\frac{\rho_\infty}{\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(r(1-C)\left(\frac{h}{h_\infty}\right)^{1/2}\frac{\partial v_z}{\partial r}\right),\\ \left\{v_r\frac{\partial C}{\partial r}+v_z\frac{\partial C}{\partial z}\right\}=\nu_\infty\frac{\rho_\infty}{\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(rC\left(\frac{h}{h_\infty}\right)^{1/2}\frac{\partial C}{\partial r}\right),\\ \left\{v_r\frac{\partial h}{\partial r}+v_z\frac{\partial h}{\partial z}\right\}=\alpha_\infty\frac{\rho_\infty}{\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(r(1-C)\left(\frac{h}{h_\infty}\right)^{1/2}\frac{\partial h}{\partial r}\right)+\\ +\nu_\infty\frac{\rho_\infty}{\rho}(1-C)\left(\frac{h}{h_\infty}\right)^{1/2}\left(\frac{\partial v_z}{\partial r}\right)^2,\\ \frac{\partial}{\partial r}(r\rho v_r)+\frac{\partial}{\partial z}(r\rho v_z)=0.\end{cases} \quad (9)$$

The first two equations are almost identical with respect to two different variables for values of C close to 0.5, and the boundary conditions of the solutions are similar up to a constant value: $C \to 0$, $v_z \to 0$ at $r \to \infty$, and C, v_z tend to some constant values depending on z at $r \to 0$.

Thus, the Crocco integral can be introduced for $C \approx 0.5$:

$$C = av_z + f$$

(a and f are constants), allowing to exclude the equation for C from system (9).

Notably, the refined condition for using the Crocco integral, $C \approx 0.5$, can be regarded as the main result of this section.

Because it was not discussed in [5] whether the Crocco integral could be used in system (2), it was impossible to find the domain where the analytical solutions obtained would give a more accurate result. For example, the value of ν differs from D_{gV} by about ten times for $C << 1$, making it impossible to introduce the Crocco integral that linearly relates the quantities C and v_z.

Let us solve the system of three equations with respect to the unknown fields v_r, v_z and h in the same way as it was done in [5]. Let us write down the main steps of the solution.

Step 1. Transition to Dorodnitsyn variables x, y [10]:

$$\begin{cases} ydy = \dfrac{\rho}{\rho_\infty} rdr, \\ x = z. \end{cases} \tag{10}$$

Step 2. Assumption of a weak dependence of the density ρ on its argument r, i.e., $\partial\rho/\partial r \approx 0$ [5]. The basis for this important assumption is that the escaping metal vapors are heavy but hot, while the surrounding helium gas is, on the contrary, light but cold.

Then,

$$y^2 = \frac{1}{\rho_\infty}\left(r^2\rho - \int_0^r r^2 \frac{\partial\rho}{\partial r} dr \right) \approx \frac{\rho}{\rho_\infty} r^2, \; \frac{\partial y}{\partial r} \approx \left(\frac{\rho}{\rho_\infty}\right)^{1/2}. \tag{11}$$

Step 3. Introducing new velocities U, $\tilde{V}$ dependent on x, y, and their stream function ψ:

$$\begin{cases} V = v_r, \; U = v_z, \\ \tilde{V} = \left(V\dfrac{\partial y}{\partial r} + U\dfrac{\partial y}{\partial z} \right); \end{cases} \quad \begin{cases} U = \dfrac{1}{y}\dfrac{\partial\psi}{\partial y}, \\ \tilde{V} = -\dfrac{1}{y}\dfrac{\partial\psi}{\partial x}. \end{cases} \tag{12}$$

Step 4. Linearization of equations, corresponding to the following substitutions with respect to system (9):

$$(1-C)\left(\frac{h}{h_\infty}\right)^{1/2} \to \frac{1}{2}\left(\frac{h_0}{h_\infty}\right)^{1/2}, \; C\left(\frac{h}{h_\infty}\right)^{1/2} \to \frac{1}{2}\left(\frac{h_0}{h_\infty}\right)^{1/2}, \tag{13}$$

where h_0 is the characteristic value of the enthalpy in the given region.

Step 5. Introducing the self-similar variable $\xi = y\big/\left(x\tilde{\nu}_\infty^{1/2}\right)$ and writing the stream function and enthalpy in terms of the new variables ξ, x:

$$\psi(\xi, x) = \tilde{\nu}_\infty x \cdot c(\xi), \; h = \frac{\tilde{\nu}_\infty}{x} d(\xi), \tag{14}$$

where

$$\tilde{\nu}_\infty = \frac{\nu_\infty}{2}\left(\frac{h_0}{h_\infty}\right)^{1/2}, \; \tilde{\alpha}_\infty = \frac{\alpha_\infty}{2}\left(\frac{h_0}{h_\infty}\right)^{1/2}.$$

Step 6. Transition to differential equations for functions $c(\xi)$, $d(\xi)$:

$$\begin{cases} \xi^2 c''' + \xi(c-1)c'' + \xi(c')^2 - (c-1)c' = 0, \\ \left(\tilde{\alpha}_\infty \xi \cdot d'' + (\tilde{\alpha}_\infty + \tilde{\nu}_\infty c)d' + \tilde{\nu}_\infty c' \cdot d\right)\xi^3 + \dfrac{1}{x}(\xi c'' - c')^2 = 0. \end{cases} \tag{15}$$

The solution of the first equation for the function $c(\xi)$ is known, corresponding to the solution for a laminar submerged jet of incompressible gas [10]. The second equation for the function $d(\xi)$ defines the enthalpy and contains a term proportional to $1/x$. Omitting this term assuming that it is small ($x = z$ is a large scale) is equivalent to omitting the term $\sim (\partial v_z/\partial r)^2$ in the original equations (9). Notice that the equation for $d(\xi)$ can be solved exactly and its solution is expressed in quadratures. Here, following [5], we do not take the influence of this term into account, considering it a correction of higher order $\sim 1/x^2$.

Let us write down the final solutions for the field of velocities, enthalpy, and the fraction of metal vapors in the mixture:

$$U = \frac{2b^2}{x}\cdot\frac{1}{\left(1+\frac{1}{4}b^2\xi^2\right)^2},\quad \tilde{V} = \frac{\tilde{\nu}_\infty^{1/2} b}{x}\cdot\frac{b\xi\left(1-\frac{1}{4}b^2\xi^2\right)}{\left(1+\frac{1}{4}b^2\xi^2\right)^2},$$
$$h = \frac{\tilde{\nu}_\infty}{x}\frac{b_1}{\left(1+\frac{1}{4}b^2\xi^2\right)^{\frac{2\tilde{\nu}_\infty}{\tilde{\alpha}_\infty}}},\quad C = aU. \tag{16}$$

Here the constants a, b_1, b are found from the boundary conditions for the function $c(\xi)$ and from the conditions for preserving the corresponding total fluxes. The explicit expressions for these constants take the form

$$a = \frac{I_0}{J_0},\quad b_1 = \frac{C_p H_0}{8\pi\tilde{\nu}_\infty^2}\left(1+2\frac{\nu_\infty}{\alpha_\infty}\right),\quad b = \left(\frac{3J_0}{16\pi\tilde{\nu}_\infty\rho_\infty}\right)^{1/2}, \tag{17}$$

where J_0, I_0, C_p, H_0 are the total fluxes (along the jet) of mixture momentum, metal vapor mass, and enthalpy, respectively.

We should emphasize that the described procedure for solving system (9) is borrowed from [5]; therefore, solutions (16) completely coincide in their structure with the solutions obtained in [5].

However, the slight difference is in the constants in $\tilde{\alpha}_\infty$, $\tilde{\nu}_\infty$ (14) (see Eq. (11) in [5]), which are half the size of the similar constants given in [5]. The reason for this difference is that we refined the condition for using the Crocco integral above.

To write the quantities U, $\tilde{V}$, h, C found in terms of the 'physical coordinates' r, z, we should first make a transition from the coordinates x, ξ to coordinates x, y, and then perform the inverse Dorodnitsyn transformation. The dependence of the variable y on ρ should be taken into account.

The solutions found are diverging at $z \to 0$, which is typical for the self-similar case. The origin along the jet axis was shifted deep into the keyhole by Δz in this example to eliminate the divergence.

Results and discussion

We analyze solutions (16) considering the following problem statement. The KH has a shape close to a straight cylinder with a small depth Δz of the order of several millimeters (see Fig. 1). The latter was estimated provided that $h(r = 0, z = \Delta z) = h_m$, where $h_m = C_p T_m$ is the maximum enthalpy corresponding to the temperature in the center of the keyhole top. The KH radius r_k corresponds to the radius of the laser spot on the surface of the workpiece and is approximately equal to $r_k \approx 1.5\cdot 10^{-4}$ m for the given case [6]. The temperature of the side walls and the bottom is considered constant and equal to the boiling point T_b of the welded material, which is iron in our case. The maximum velocity of metal vapors U_m is reached in the center of the keyhole top, where, accordingly, the mixture temperature reaches its maximum T_m. Welding is performed in a shielding helium gas under normal conditions, i.e., at a temperature of 298 K and a pressure of 101.325 kPa.

A pair of boundary values (T_m, U_m) was set using the calculated data from [6], presenting numerical studies of keyhole laser welding of an iron workpiece. Let us give the values for these pairs of quantities used below:

$$13800\text{ K, }360\text{ m/s; }14100\text{ K, }690\text{ m/s;}$$

$$14400\text{ K, }920\text{ m/s; }14500\text{ K, }1120\text{ m/s.}$$

The quantities H_0, I_0, J_0 included in solutions (16) were estimated from the above values of temperature, velocity, pressure, and radius of the keyhole.

The table lists the boundary values of various characteristics of the iron-helium mixture used in the calculations.

Table

Boundary values for characteristics of iron-helium mixture

T_b	3200 K
T_∞	298 K
C_{pV} at T_b	480.8 J/(kg·K)
C_{pg} at T_∞	5193 J/(kg·K)
μ_∞ at T_∞	$1.99 \cdot 10^{-5}$ Pa·s
κ_∞ at T_∞	0.157 W/(m·K)
ρ_∞ at T_∞	0.164 kg/m^3

Setting explicit values for temperature, velocity and pressure, we can now estimate the Reynolds number Re for our problem. Let us choose U_m as the characteristic velocity, and estimate the characteristic linear dimensions by the KH diameter $2r_k$. We calculate the dynamic viscosity of metal vapors using the Enskog equation [15]:

$$\mu_V = 1{,}016 \frac{5}{16(2r_V)^2} \left(\frac{m_V k T_m}{\pi} \right)^{1/2},$$

then, taking into account the equation of state, we find the kinematic viscosity $\nu_V = \mu_V / \rho_V \approx$ $\approx 3 \cdot 10^{-3}$ m^2/s. Therefore,

$$\mathrm{Re} = \frac{2 r_k U_m}{\nu_V} \approx 20.$$

The obtained Reynolds number is less than the typical value at which a steady laminar jet loses its stability [18, 19].

Let us now move on to the results. Fig. 2 shows the radial dependences of the temperature and mass fraction of iron atoms in the mixture at a distance of 5 mm from the surface of the workpiece,

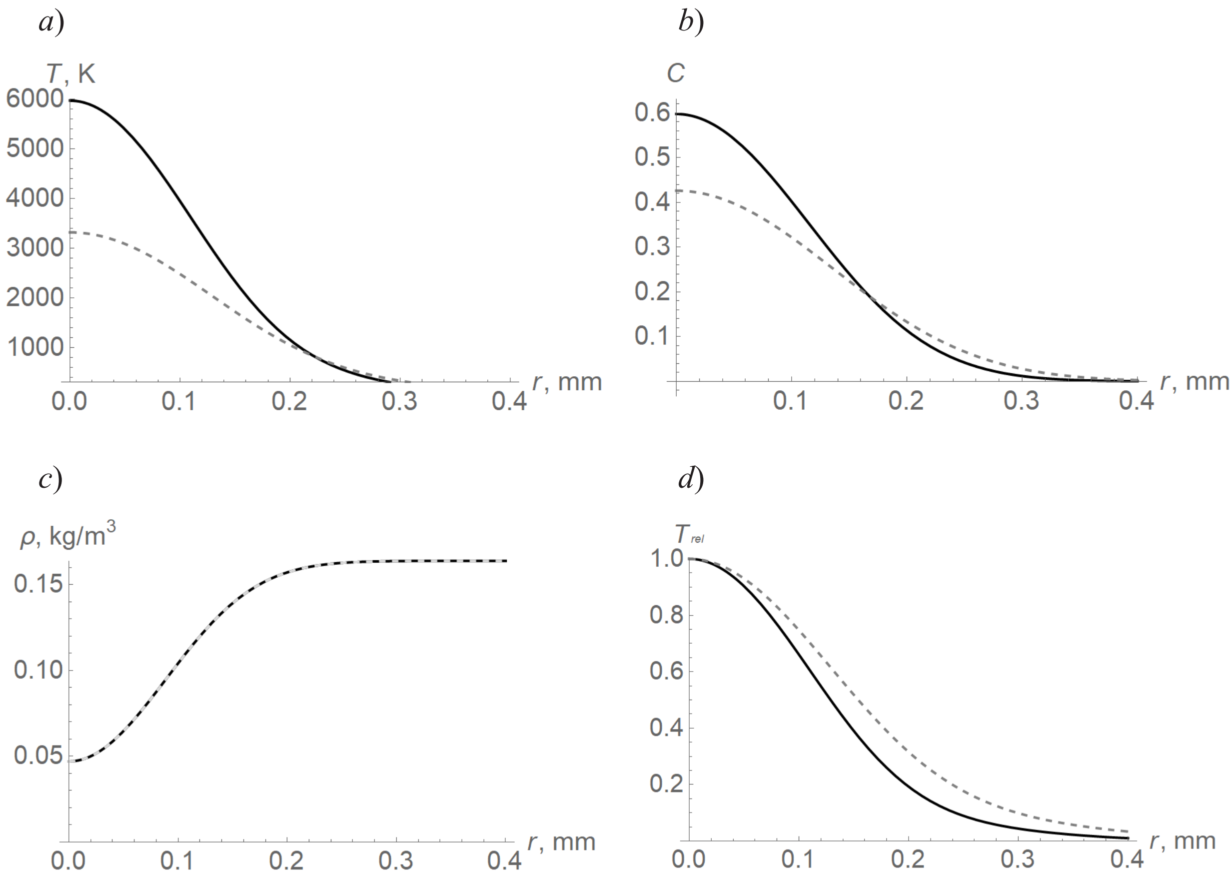

Fig. 2. Radial distributions for jet temperature (*a*) and mass fraction of iron atoms in the jet (*b*) at a distance of 5 mm from the workpiece surface; radial distributions of mixture density at KH top (*c*) and radial distributions normalized to the maximum temperature T_{rel} (*d*).
Fig. 2,*d* corresponds to Fig. 2,*a* but each curve is normalized to its maximum.
The values shown are α_∞, ν_∞ (solid lines) and $2\alpha_\infty$, $2\nu_\infty$ (dashed lines), see Table

as well as the density of the mixture at the KH top. The dependences are constructed by Eqs. (16). The maximum velocity and the corresponding temperature at the KH top are equal to 1120 m/s and 14500 K, respectively.

Analyzing the graphs in Fig. 2, we can conclude that the transfer processes decide the final width of the submerged jet. The characteristic radius of jet attenuation can be estimated by equating the temperature of the mixture to the temperature of the shielding gas; it turns out that this radius is approximately equal to 0.3 – 0.4 mm.

The dashed lines in Fig. 2 correspond to the curves obtained following [5], i.e., the quantities α_∞, ν_∞ included in Eqs. (16) turn out to be doubled. As expected in this case, the absolute value of the temperature decreases due to the viscosity preventing the jet from propagating, while the width increases due to thermal conductivity (see Fig. 2,*d*). The maximum temperature is approximately halved. A similar dependence is observed for the quantity C, however, the maximum value decreases by only 1.5 times, which is associated with the upward motion of cooled iron atoms.

The submerged jet of iron vapor in the shielding helium gas attenuates at a certain distance from the treated surface. The attenuation height depends on many factors, including the velocity at which the vapor is ejected from the keyhole.

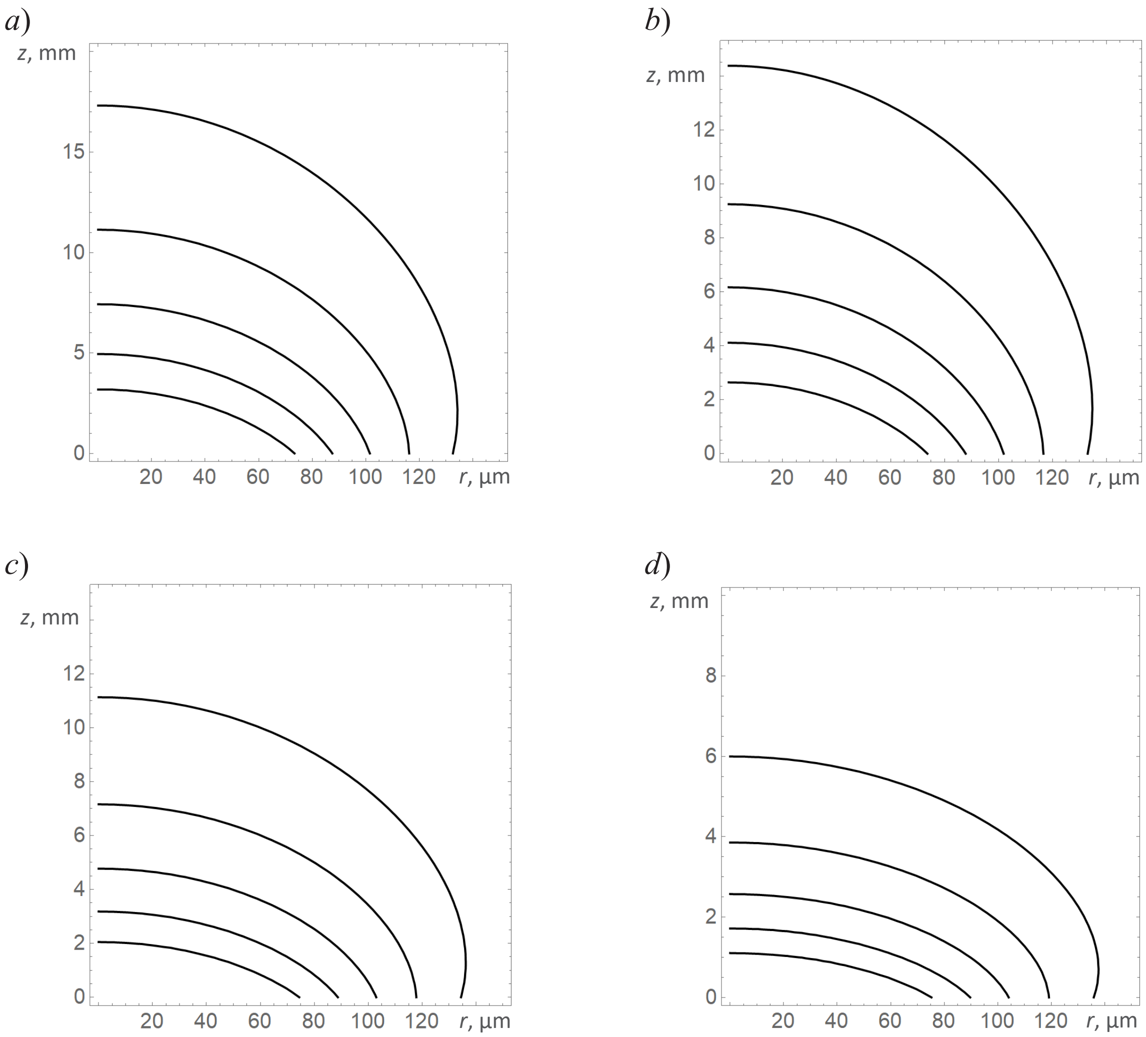

Fig. 3. Family of isolines $C(r, z) = C_0$ for different pairs of parameter values (T_m, K and U_m, m/s): 14500 and 1120 (*a*), 14400 and 920 (*b*), 14100 and 690 (*c*), 13800 and 360 (*d*); the values taken are C_0 = 0.3; 0.4; 0.5; 0.6 and 0.7

We use Eqs. (16) to analyze the behavior of the plasma plume with varying boundary values of maximum velocity U_m and temperature T_m.

Fig. 3 shows a family of isolines for the same mass fractions of iron atoms in the mixture for different pairs (T_m, *K* and U_m, m/s):

$$C(r, z) = C_0 \ (C_0 = 0.3 - 0.7 \text{ with a step } 0.1).$$

The graphs show an approximately linear relationship between the maximum velocity U_m and the penetration depth of iron atoms into the shielding gas. For example, the isoline point with $C_0 = 0.5$ at $r = 0$ for $U_m = 1120$ m/s is located at a height of $z \approx 7.5$ mm, and the height of the same point decreases by three times for a velocity three times less (U_m = 360 m/s): $z \approx 2.5$. This linear dependence is an obvious consequence from introducing the Crocco integral in our problem. A similar picture is observed for the isotherms of the gas mixture $T(r, z) = T_0$, K: $T_0 = (1 - 14)\cdot 10^3$ K at a step of 10^3 K (see Fig. 4).

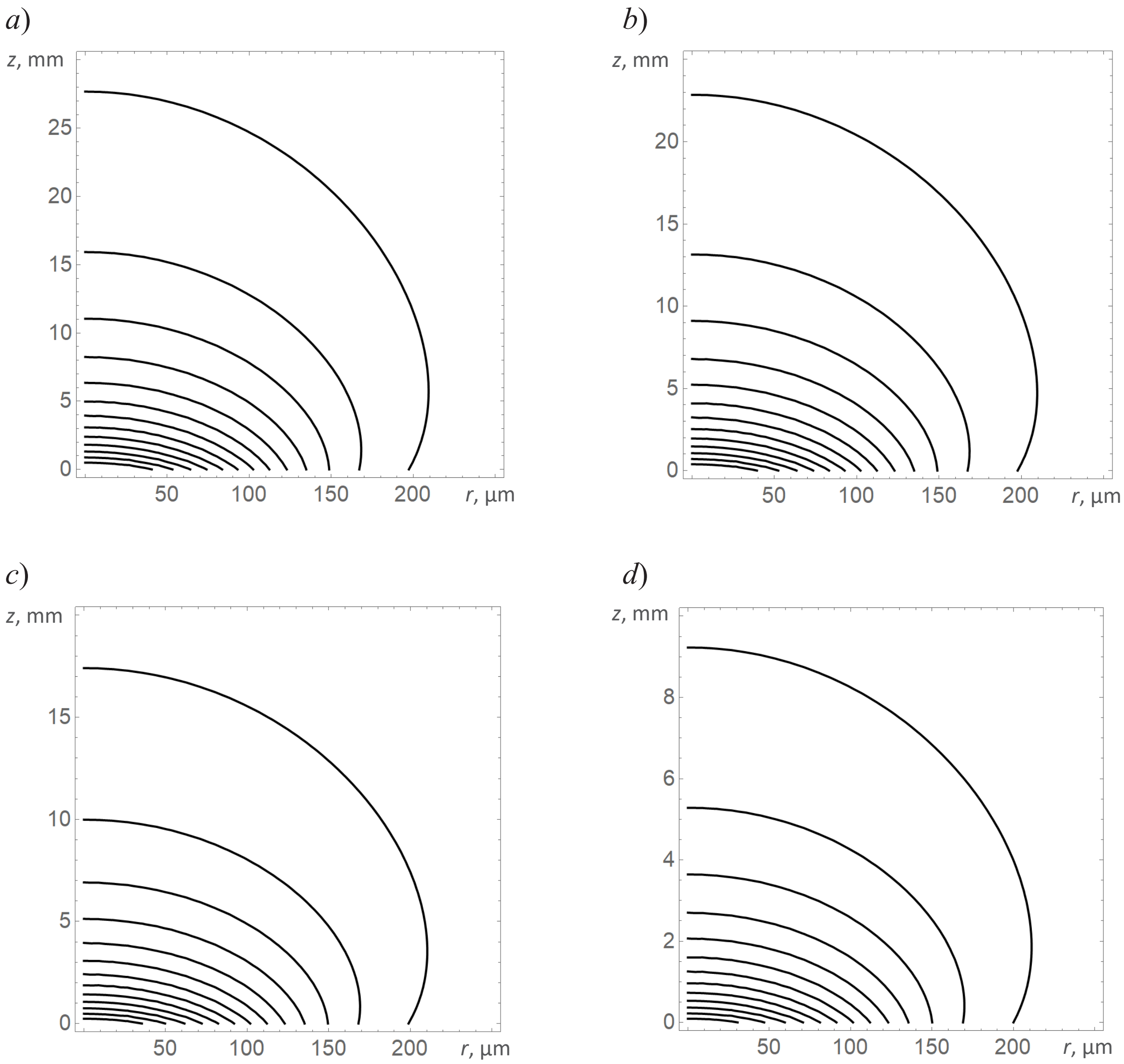

Fig. 4. Family of gas mixture isotherms $T(r, z) = T_0$
for the same pairs of parameter values (T_m, K and U_m, m/s) as in Fig. 3.
The values taken are T_0, K: 1000, 2000, 3000, etc.

The results we obtained were compared with the numerical data from [6]. considering the process of laser keyhole welding of an iron workpiece in shielding argon gas. A system of gas-dynamic equations was solved taking into account the heat source associated with absorption of laser radiation. Although a slightly different model is considered in [6], some patterns obtained in it are described by solutions (16): the penetration depth of iron atoms into the shielding gas along the direction in which iron vapor is ejected is linearly related to the maximum velocity U_m (of course, with the corresponding temperature T_m); a similar linear dependence was observed for the temperature of the gas mixture.

Conclusion

We used the analytical solutions of the system of gas-dynamic equations written in the boundary-layer approximation to analyze the characteristics of a plasma plume generated under keyhole welding of an iron workpiece with a neodymium-doped (Nd:YAG) laser in shielding helium gas.

Analysis of the transfer coefficients included in the system of gas-dynamic equations made it possible to find the condition for using the Crocco integral. This condition corresponds to the mass fractions C of metal vapors in the mixture close to 0.5. The kinematic viscosity for such values becomes practically equal to the diffusion coefficient, and the equation for the velocity field v_z is written in the same form as the diffusion equation (see system (9)), allowing to introduce the Crocco integral and consequently solve a system with a smaller number of equations. The solutions (16) obtained in this case for the field of velocities, enthalpy and mass fraction of metal vapors in the mixture completely coincide in their structure with the solutions obtained in [5]. There is, however, a slight difference in that the transfer coefficients for the shielding gas are redefined. For example, it is necessary to substitute α_∞, $\nu_\infty \rightarrow 2\alpha_\infty$, $2\nu_\infty$ in the final solutions (16) to make a transition from solutions (16) presented here to the corresponding solutions in [5], producing in particular noticeable changes in the radial dependences of the mixture temperature and mass fraction of metal vapors in the mixture. The reason why this difference between the solutions appears is that the Crocco integral was introduced in [5] without rigorous analysis of the conditions under which it can be used.

The dependences of the mass fraction of iron atoms in the gas mixture and the mixture temperature have been considered for various boundary values of velocity and temperature of the vapors at the keyhole top. We have found a linear relationship (natural, due to the Crocco integral introduced) between the maximum velocity of iron vapor ejected from the keyhole and the penetration depth of iron atoms into the shielding gas (a similar dependence was found for the mixture temperature). The same linear relationship between these quantities was observed in [6], where the system of gas-dynamic equations was solved numerically for a similar problem.

To conclude, we should note that the estimate of the Reynolds number in our problem yielded $\mathrm{Re} \approx 20$, which is less than the typical value at which a laminar stationary jet becomes unstable [18, 19]. The Reynolds number grows larger for higher velocities of metal vapors U_m, and the assumption that the jet in the mixture is laminar and steady may become inapplicable. In this case, the problem will have to be solved relying on the theory of a turbulent boundary layer [20].

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