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ABOUT THE PROPER TIME AND THE MASS OF THE UNIVERSE

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For a closed universe, a modification of the quantum gravity where the dynamics is reduced to the motion in the orbit of a general covariance groups has been proposed. To connect these motion parameters, namely, proper time and spatial shifts, to observations, classical equations of motion were introduced into the quantum theory as additional conditions. The equations account for differential conservation laws for additional dynamical variables, which form the spatial density of distribution and motion of the universe's proper mass in the representation of Arnovitt, Deser and Misner (ADM). This made it possible to determine the average values of the parameters of proper time and spatial shifts in the evolutionary history of the universe. In order to preserve the homogeneity and isotropy of space, the proper mass of the universe should next be set equal to zero. Nonzero values of its proper mass (mass spectrum) were allowed in the operator canonical representation of the quantum gravity, which was also introduced instead of the ADM representation.

Keywords: universe, proper time, proper mass, quantization, Hermitian operator, Dirac spinor

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О СОБСТВЕННОМ ВРЕМЕНИ И МАССЕ ВСЕЛЕННОЙ

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Для случая замкнутой Вселенной предложена модификация квантовой теории гравитации, в которой динамика сводится к движению по орбите групп общей ковариантности. Чтобы связать с наблюдениями параметры этого движения, а именно собственное время и пространственные сдвиги, в качестве дополнительных условий в квантовую теорию вводятся классические уравнения движения указанных параметров. Эти уравнения отражают дифференциальные законы сохранения дополнительных динамических переменных, которые в представлении Арновитта, Дезера и Мизнера (АДМ) образуют пространственную плотность распределения и движения собственной массы Вселенной. Определены средние значения параметров собственного времени и пространственных сдвигов в истории эволюции Вселенной. Инвариантное определение собственной массы (спектра масс) сформулировано в операторном каноническом представлении теории гравитации, которое также вводится вместо представления АДМ.

Ключевые слова: Вселенная, собственное время, собственная масса, квантование, Эрмитов оператор, спинор Дирака

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Introduction

For a closed universecase, the following Wheeler – DeWitt equation (a system of wave equations) is the heart of quantum gravity:

$$\hat{H}^{\mu}\Psi = 0. \quad (1)$$

According to this equation, the wave function of the universe Ψ does not depend from any external parameter of time. Albeit adopting this concept, we, nonetheless, assume that time is required for interpretation and description of the observation results, and thus it should be introduced in the quantum cosmology as well. This calls for a modification of the canonical procedure for quantizing the theory.

This paper proposes a variant of such a modification in case of a closed universe, which allows introducing the time parameter (parameters). The modification is simultaneously a variant of the quasiclassical approximation and presents no changes into the dynamical content of the theory at the classical level.

As a model for our constructions, let us consider the mechanics of a relativistic particle. In relativistic mechanics, mass particle Hamiltonian m is proportional to the Hamiltonian constraint, i.e.

$$h = NH,\, H \equiv p^2 - m^2c^2, \quad (2)$$

which expresses the known condition imposed on a particle's four-momentum.

We use a simplified notation for a four-vector square $p^2 = p^{\mu}p_{\mu}$. Let us base our constructions on the formal symmetry of relativistic mechanics: the reparametrization-invariance of action which has geometrical meaning of a particle's world line in Minkowski space. The arbitrary function N of the parameter τ on the world line provides this invariance. This symmetry is the simplest analog of the general covariance principle in Einstein's theory of gravity.

The most universal tool of quantizing covariant theories is Batalin – Fradkin – Vilkovisky formalism (BFV-formalism) which prescribes the way to constructing Becchi – Rouet – Stora – Tyutin propagator (BRST-propagator) [1, 2]. In the simplest case of a relativistic particle, this formalism gives a simple result as well: functional-integral representation of the Green's function for the Klein – Gordon equation [3], which contains an additional integral over particles proper time within [0, ∞). This representation was first proposed by V.A. Fock [4] and J. Schwinger [5]. The problem of interpretation of this covariant quantum theory consists in the fact that proper time here is not a parameter of the evolution, while the Green's function itself has no dynamical meaning. We obtain the same result for homogenous models of the universe in the quantum theory.

For the proper time in the quantum theory to obtain the meaning of an evolution parameter, we need additional constructions allowing proper time integration in the propagator. The authors of Ref. [6] propose a modification of the original theory for the homogenous anisotropic model of the universe which allows removing the proper time integral without changing its dynamical content at the classical level.

Let us conditionally divide the modification into two stages. At the first one, the proper time is introduced into the initial action of the classical theory as a new dynamical variable using a relation

$$N = \dot{s}. \quad (3)$$

At the second stage, we add Euler – Lagrange (EL) to the initial action for the new dynamical variable as an additional condition. It is equivalent to adding to the initial action its variation generated by an infinitesimal shift of the proper time:

$$\delta s = -\varepsilon. \tag{4}$$

In the simplest homogenous model of the universe [6], this EL equation is reduced to the law of conservation of Hamiltonian constraint which in the original theory was a consequence of equations of physical dynamical variables motion. Such a modification obviously does not change dynamical content at the classical level, but in this case, it already leads to some additions.

Consider the consequences of the theory under consideration on the example of relativistic mechanics. As an initial system, let us take a massless particle ($m = 0$ in relation (2)) with Lagrangian

$$L = \frac{1}{2}\frac{\dot{x}^2}{N}. \tag{5}$$

After two stages of modification move on to the Lagrange function:

$$\hat{L} = \frac{1}{2}\frac{\dot{x}^2}{\dot{s}}\left(1 + \frac{\dot{\varepsilon}}{\dot{s}}\right), \tag{6}$$

where the infinitesimal shift of the proper time ε should also be considered as an independent dynamical variable.

Pass on to the canonical form of the modified theory. Its Hamiltonian equals zero as Lagrangian (6) is a homogenous function of velocities of the first order, while the constraint equation takes the form

$$p_s = P_\varepsilon - \sqrt{2P_\varepsilon}\sqrt{p^2}, \tag{7}$$

where

$$P_\varepsilon = \frac{1}{2}\frac{\dot{x}^2}{\dot{s}^2} \tag{8}$$

is a canonical momentum conjugated to ε.

Another canonical equation of motion is the conservation law for the additional dynamical variable $\dot{P} = 0.$ Bearing in mind Eq. (7), this means the conservation law of the initial theory constraint. However, the particle's mass can be not equal to zero now, if we set

$$P_\varepsilon = 2m^2c^2. \tag{9}$$

Thus, the proposed modification leads to appearance of the additional parameter P_ε in the theory, which in this case serves as the constant of motion. This could also be understood as an expansion of the admissible initial values of the particle's velocity due to its "gained" mass. Note that the initial constraint (2) is an invariant of the Lorentz transformations acting in Minkowski space and phase space of the relativistic particle. The proper mass obtained in this construction is an invariant as well.

There is another significant result of the modification: a square root of the 4-momentum square p_μ in Eq. (7). In the quantum theory, it is a source of an additional conditional in the form of δ, a function in the functional integral which defines the proper time on the world line of the particle as integral Eq. (8):

$$s = \int \frac{\sqrt{dx_\mu dx^\mu}}{\sqrt{2P_\varepsilon}}. \tag{10}$$

This exact equation allows eliminating proper time integration in the propagator for the relativistic particle. A generalization of this modification of the covariant quantum theory for a case of a system with two Hamiltonian constraints and two proper time parameters is considered in paper [7].

In this paper, the authors proposed a modification of the gravity theory in the general case of an inhomogeneous universe. It is based on the representation of the action obtained by Arnovitt, Deser and Misner (ADM) [8, 9] using a (3 + 1)-split of 4D-metric. A part of elements of this metric (N, N_i), which are called successor and shift functions, play a role of Lagrange multipliers in the canonical representation of the ADM action. The modification of the theory in this representation leads to occurrence of additional dynamical variables, which form scalar and vector densities with respect to transformations of space coordinates. Similar to the relativistic particle, they can be called density and flux density distribution of the proper mass of the universe. Assumption of non-zero values of these variables in the ADM representation means a breach of homogeneity and isotropy of the space. Generalization of the theory be means of introducing invariant non-zero values of the additional dynamical variables is possible in the operator representation of gravitational constraints [10], which is also considered in this paper.

In the operator representation, additional variables are invariants of $3D$-diffeomorphisms and form a spectrum of proper mode masses of the universe.

Proper time and mass of the universe in the ADM representation

The action of the gravity theory in ADM representation obtained by means of (3 + 1)-split of $4D$-metric has the form of [9]:

$$I_{\text{ADM}} = \int N dt \int_{\Sigma} \sqrt{g} d^3 x \times \times \left[R + \text{Tr}K^2 - (\text{Tr}K)^2 \right], \quad (11)$$

where

$$K_{ik} = \frac{1}{2N}\left[\nabla_i N_k + \nabla_k N_i - \dot{g}_{ik}\right] \quad (12)$$

is the tensor of the hypersurface extrinsic curvature of the constant time $\sum$.

To simplify, we exclude the action of matter fields from action (11). The addition of the matter does not change the main conclusions of the article. Here, N, N_i are successor and shift functions which are the elements of the $4D$-metric. Time derivatives of these functions are absent in the ADM action, so they play the role of Lagrange multipliers in the canonical representation of the action.

EL equations for N, N_i are essentially classic constraint equations expressed via time derivatives from $3D$-metric:

$$\frac{\delta I_{\text{ADM}}}{\delta N} = H = = \sqrt{g}\left[R + (\text{Tr}K)^2 - \text{Tr}K^2 \right] = 0, \quad (13)$$

$$\frac{\delta I_{\text{ADM}}}{\delta N_i} = H^i = = -2\nabla_k \left[\sqrt{g}\left(g^{ik} K - K^{ik} \right) \right] = 0. \quad (14)$$

The Hamiltonian in the case of the closed universe has the form of a linear combination

$$h_{\text{ADM}} = \int d^3 x N_\mu \Pi^\mu, \quad (15)$$

where Π^μ are ADM constraints expressed via canonical momenta

$$\pi^{ik} = \sqrt{g}\left(g^{ik} K - K^{ik} \right), \quad (16)$$

conjugated to $3D$-metric elements.

Momentum quadratic Hamiltonian constraints are canonical generators of shifts normal to the space cross-section $\sum$, while the linear momentum constraints serve as canonical generators of $3D$-space diffeomorphisms.

Here, we have no need for explicit form of these constraints in the ADM representation. Although, let us note that they form scalar and vector densities with respect to transformations of space coordinates on $\sum$. Further, we follow general notations [11] sufficient for any covariant theories. Summation over repeated indices implies integration if the range of the possible index values forms continuum.

In the gravity theory, the variation range of the Latin index is as follows:

$$\alpha = (\mu, x); \ \mu = 0; \ i, x \in \Sigma.$$

In these general notations, the infinitesimal shifts of the proper (multipoint) time are united with the infinitesimal space shifts on the hypersurface by means of a united symbol ε_α, so that the infinitesimal variations of the canonical variables generated by these shifts are written in the form

$$\delta q_\alpha = \varepsilon_b \{ q_\alpha, \varphi_b \}, \quad \delta p_\alpha = \varepsilon_b \{ p_\alpha, \varphi_b \}. \quad (17)$$

The constraints form closed algebra with respect to Poisson brackets, i.e.

$$\{\varphi_\alpha, \varphi_b\} = C_{abd}\varphi_d, \tag{18}$$

and we will not need its structural functions in the explicit form here as well.

Transformations (17) in this case act as symmetry transformations of the theory only in so far as the action written in the canonical form

$$I = \int dt L(q_i, \dot{q}_i, \lambda_\alpha) = \\ = \int dt \left[p_i \dot{q}_i - \lambda_\alpha \varphi_\alpha \right] \tag{19}$$

is an invariant of the transformations with an additional Lagrange multipliers transformation:

$$\delta\lambda_\alpha = \dot{\varepsilon}_\alpha - C_{bd\alpha}\lambda_b \varepsilon_d. \tag{20}$$

Take Eq. (20) as the basis of our further constructs. Let us consider them as functional differential equations in the form

$$\frac{\delta\lambda_\alpha(t)}{\delta s_b(t')} = \\ = \delta_{\alpha b}\frac{d}{dt}\delta(t-t') - C_{db\alpha}\lambda_b(t)(t-t') \tag{21}$$

with respect to λ_α, and here, we assume them to be functionals of the proper time s_α parameters.

Solution to these equations at additional initial conditions $\lambda_\alpha[0] = 0$ has the following form:

$$\lambda_\alpha = \dot{s}_b \Lambda_{b\alpha}. \tag{22}$$

It can be obtained using iterations in the form of functional Taylor series, in which with the accuracy of up to second order of smallness with respect to s_α

$$\Lambda_{b\alpha}. = \delta_{b\alpha} - \frac{1}{2!}C_{bd\alpha}s_d + \\ + \frac{1}{3!}C_{b'd'\alpha'}C_{bdb'}s'_d s_d - \ldots. \tag{23}$$

We will also need a variation of Lagrange multipliers (22) at the infinitesimal shift of the proper time ε_α parameters:

$$\delta\lambda_\alpha = \dot{\varepsilon}_b \Lambda_{b\alpha} + \dot{s}_b \frac{\partial \Lambda_{b\alpha}}{\partial s_d}\varepsilon_d. \tag{24}$$

Eqs. (22) and (24) are a generalization of Eqs. (3) and (4) for the general case of an inhomogeneous universe. Using an analogy with relativistic mechanics, let us write Lagrange function of the modified gravity theory at once in the general case:

$$\tilde{L}(q, \dot{q}, s, \dot{s}, \varepsilon, \dot{\varepsilon}) = L(q, \dot{q}, \lambda(s, \dot{s})) + \\ + \frac{\partial L(q, \dot{q}, \lambda(s, \dot{s}))}{\partial \lambda_d}\delta\lambda_d(s, \dot{s}, \varepsilon, \dot{\varepsilon}). \tag{25}$$

The peculiarities connected with the time problem and possible solutions to it in the modified theory reveal themselves after a transition to the canonical form of the action (25). Modified Lagrange function (25) is a homogenous first order function of all generalized velocities. For this reason, the Hamiltonian of the modified theory equals zero.

Taking this into account, we deviate here from the standard formalism of covariant dynamics quantization in terms of the external time parameter [1, 2]. Such a description remains possible in the island universe model, the energy of which is equal to zero, and the time is measured in hours at infinity [12]. Instead, in a closed universe, we can talk about symmetry transformations or motion of general covariance groups in the orbit generated by constraints, while the parameters of this motion form the proper (multipoint) time s_α.

In the modified theory, this intrinsic dynamics consists in the equations that determine the canonical momenta conjugated to the proper time:

$$p_{s_\alpha} = \frac{\partial \tilde{L}}{\partial \dot{s}_\alpha} = -\tilde{h}_\alpha. \tag{26}$$

These equations will play a role of constraints in the modified theory after the transition to its canonical form. For this purpose, we should exclude all velocities from the right-hand side of Eq. (26) by expressing them through the corresponding canonical momenta.

In the quantum theory, the constraints of Eq. (26) transform into a system of self-consistent wave equations of Schrödinger equation type

$$i\hbar\frac{\partial\Psi}{\partial s_\alpha}=\hat{\tilde{h}}\Psi \tag{27}$$

for the universe wave function Ψ.

The general covariance principle demands excluding the dependence of the universe wave function on the additional dynamical variables s_α.

Obtain the invariant propagator by additional integration of solution to system (27) along the whole orbit of the general covariance groups with a simple measure:

$$\tilde{K}=\int\prod_\alpha ds_\alpha\Psi\left(s_\alpha,...\right). \tag{28}$$

We will not develop the modified quantum theory here, but focus on those aspects of the canonical form of the classical theory, which lead to eliminating the integrals in propagator (28). One of such aspects is excluding the infinitesimal shift velocities of the proper time $\dot{\varepsilon}_\alpha$ as a result of which there are square roots in Eq. (26) which are similar to those contained in constraints Eq. (7). To exclude them, there are equations determining the corresponding canonical conjugated momenta in the form

$$\begin{gathered}P_{\varepsilon_\alpha}=\frac{\partial L\left(q,\dot{q},\lambda\left(s,\dot{s}\right)\right)}{\partial\lambda_d}\Lambda_{ad}=\\ =\Lambda_{ad}\varphi_d,\end{gathered} \tag{29}$$

which we will further require in explicit form.

Eqs. (29) allow obtaining a generalization of simplest Eq. (10) for the proper time of the relativistic particle in a form of a system of equations for the proper (multipoint) time of the universe.

Let us write off this system in explicit form keeping in mind our agreement on the condensed Latin index:

$$\begin{gathered}\frac{\sqrt{N^2\left(x\right)\left[\mathrm{Tr}K^2\left(x\right)-\left(\mathrm{Tr}K\left(x\right)\right)^2\right]}}{\sqrt{P_{\varepsilon_\alpha}\Lambda^{-1}_{\alpha 0}\left(x\right)+R\left(x\right)}}=\\ =N\left(x\right),\end{gathered} \tag{30}$$

$$\begin{gathered}\sqrt{g\left(x\right)}P_{\varepsilon_\alpha}\Lambda^{-1}_{\alpha i}\left(x\right)=\\ =-2\nabla_k\left[\sqrt{g\left(x\right)}\left(g^{ik}\left(x\right)K\left(x\right)-K^{ik}\left(x\right)\right)\right].\end{gathered} \tag{31}$$

Here, successor and shift functions are determined by Eqs. (22). Both sides of Eqs. (30) and (31) are homogenous functions of the first and zero degrees of velocities, respectively. The integrals of these equations define the proper time of the universe as a trajectory function in its configuration space. This time also acts as a functional of the additional dynamical variables $P_{\varepsilon\alpha}$, therefore, Eqs. (30), (31) should be solved together with their motion equations. They are obtained as EL equations for infinitesimal shifts of the proper time in the modified time and have the form

$$\frac{d}{dt}P_{\varepsilon_\alpha}+P_q\Lambda^{-1}_{qd}\dot{s}_b\frac{\partial\Lambda_{bd}}{\partial s_\alpha}=0. \tag{32}$$

According to expressions (29), the additional dynamical variables $P_{\varepsilon\alpha}$, as well as the constraints in the ADM representation, form spatial densities. Permission of their non zero values violates the covariance of the modified theory with respect to 3D-diffeomorphisms. There is no breach of the covariance, if we assume the additional dynamical variables to be identically equal to zero. Moreover, the result of the modification in the form of the system of Eqs. (30), (31) defining the proper time of the universe in the ADM representation is maintained. The invariant definition of the proper time and mass of the universe can be achieved by using 3D-invariant representation of the gravitational constraints.

Operator representation of gravitational constraints

The 3D-invariant representation of the gravitational constraints is based on the operator equality

$$H=D^2+\frac{1}{2}\Delta=0, \tag{33}$$

which is equivalent to a complete set of gravitational constraints in the ADM-representation [9]. Here, D is the Dirac 3D-operator, while Δ is Laplace – Beltrami 3D-operator in Dirac bispinor space at the compact space cross-section $\sum$ with the given scalar product

$$\left(\psi_1, \psi_2\right) = \int \sqrt{g} d^3 x \psi_1^+ \psi_2. \quad (34)$$

Both operators Δ and D are Hermitian with respect to product (34), their coefficients are the essence of the function of canonical variables of the gravitational field (g_{ik}, π^{ik}).

To introduce a new canonical representation, let us consider the fact the proper functions of the Hermitian operator H form a complete set in Dirac bispinor space, while the necessary and sufficient conditions of its equality to zero is the equality to zero of all of its proper values defined by a secular equation.

$$H\psi_\alpha = h_\alpha \psi_\alpha. \quad (35)$$

In its turn, this means that the proper values of H form closed algebra with respect to Poisson brackets with the previous canonical variables:

$$\left\{h_\alpha, h_\beta\right\} = \tilde{C}_{\alpha\beta\gamma} h_\gamma. \quad (36)$$

Operator Eq. (33) allows us to transform the Hamiltonian of the gravity theory from the initial representation in the form of integral (15) of the ADM local constraints distribution at the space cross-section $\sum$ into a linear combination of mode Hamiltonians:

$$h_H = \lambda_\alpha h_\alpha \left(g, \pi\right). \quad (37)$$

Here, modes are understood as proper states of the operator H.

Secular Eq. (35) can be represented in a matrix form based on spectral decomposition for each Hermitian and elliptical operator in equality (33).

Write the secular equation for the Dirac operator square:

$$D^2 \psi_n = d_n^2 \psi_n. \quad (38)$$

Assuming the set of proper functions ψ_n is orthonormal, we will seek a solution to Eq. (35) in a form of decomposition

$$\psi_\alpha = \sum_n c_{\alpha n} \psi_n, \quad (39)$$

for the coefficients of which we obtain a system of equations

$$\sum_n \Delta_{mn} c_{\alpha n} = \left(h_\alpha - d_m^2\right) c_{\alpha m} \Delta_{mn} = \\ = \left(\psi_m, \Delta\psi_n\right). \quad (40)$$

Now, let us write the secular equation for the Hermitian matrix Δ_{mn} in the form:

$$\sum_n \Delta_{mn} f_n^p = \delta_p f_m^p \quad (41)$$

and look for a solution to system (40) as a decomposition

$$c_{\alpha n} = \sum_p a_{\alpha p} f_n^p. \quad (42)$$

Assuming again the set of vector-sequences f_n^p orthonormal with respect to a common Hermitian scalar product in the space of sequences, for the coefficients of this decomposition and the proper values of operator H of interest, we obtain:

$$\sum_m d_m^2 f_m^{+p'} f_m^p a_{\alpha p} = \left(h_\alpha + \delta_{p'}\right) a_{\alpha p'}. \quad (43)$$

In this form, the system of equations defining mode Hamiltonians of the universe h_α, can be useful, in particular, to formulate finite-dimensional approximations. Thus, for a homogenous universe, obviously, we have the sole mode with Hamiltonian

$$h = d_1^2 - \delta_1, \quad (44)$$

which coincides with the Hamiltonian of the homogenous anisotropic universe considered in paper [6].

Mode Hamiltonians of h_α are invariants of $3D$-transformations of the coordinates in the space cross-section. Consequently, all variables appearing in the constructs of the previous chapter are also invariants. Mode parameters of the proper time and the proper mass spectrum are invariant as well from direct analogy of relativistic mechanics. This allows us to consider the evolution of the universe in the modified quantum theory without violating the general covariance principle even at non-zero values of the proper mass. For the wave function of the universe origin, we postulate the conditional principle of minimum space energy defined by the functional

$$W = \frac{\left\langle \Psi_0 \left| d_1^2 \right| \Psi_0 \right\rangle}{\left\langle \Psi_0 \middle| \Psi_0 \right\rangle}. \qquad (45)$$

Equal-zero Hamiltonian of the universe (37) here serves as an additional condition, while the wave function Ψ_0 and Lagrange multipliers λ_n are the variational parameters. To calculate this energy defined by the elliptical operator d_1^2, we take its minimal proper value. Further, we solve the system of wave equations (27) (written now in the operator representation). The propagator (28) obtained in this fashion has additional dependence on the spectrum of invariant mode mass. This, in turn, allows us to define observable mode parameters of the proper time as average values of the corresponding observables:

$$\left\langle \hat{\varepsilon}_\alpha \right\rangle_\Psi = \left\langle \frac{\hbar}{i} \frac{\delta}{\delta P_\alpha} \right\rangle_\Psi. \qquad (46)$$

These parameters of time or the corresponding spatial scale, obviously, can be associated with the hierarchy of the spatial structures emerging in the process of the universe evolution. After calculating mean values (46), in the frame of the original theory, the mode mass should be set equal to zero. However, the general covariance principle now does not exclude the non-zero values of these parameters as well. The presence of absence of the proper mass of the universe is a question of observations and their interpretation we leave open here.

The new canonical representation of the gravity theory allows us to modify original Wheeler – DeWitt form (1) as well. The system of local (for each point of the space) wave equations superimposed on the physical state of the universe should now be replaced by non-local mode conditions. In case of strict adherence to the conventional formulation of the quantum constraints, the operator representation leads to the following system of wave equations for the physical state of the universe Ψ:

$$\hat{h}_\alpha \Psi = 0. \qquad (47)$$

Nonetheless, direct form of operator representation (33) as a self-consistent definition of the modes themselves with the wave equation for the universe wave function in the frame of the functional differential equation seems more natural:

$$\hat{H}\psi_\alpha \Psi_\alpha = 0. \qquad (48)$$

In this formulation of the quantum cosmology, the solutions should be grouped in sequences with increasing mode index α, which by that takes the meaning of a quantum parameter of the proper time in this sequence of the physical states of the universe Ψ_α.

Conclusion

The absence of the traditional view of time in the quantum cosmology is one of the consequences of the general covariance principle, which excludes any external numbering of the universe structure. This means that the universe evolution should be defined in the intrinsic terms. In fact, the structure of the covariance group itself, after additional constructs, defines the intrinsic dynamics of the universe. The constructs proposed in this paper are based on the structure of the general covariance group in the canonical ADM-representation obtained by means of (3+1)-split of the time-space geometry. The proper time and the spatial shifts as the natural parameters of the symmetry transformations are introduced in the initial action as independent dynamical variables. In this case, the dynamics of the closed universe is reduced to the motion of the general covariance group in the orbit. In quantum theory, such a motion is described by a system of wave equations of Schrödinger type. However, the general covariance principle demands independence of the wave function on the parameters of this motion: symmetry transformation. The independence is achieved by means of averaging the wave function on the symmetry group orbit. The task of the second stage of modification consists in eliminating additional averaging on the orbit using a correlation of the intrinsic dynamics with the classical motion integrals. In the original theory, these integrals play a role of constraints, i.e. become zero due to the general covariance principle. In the modified theory, these values can differ from zero and become additional dynamical variables. Their motion is described by EL equations for the parameters of the general covariance transformations. Introducing additional dynamical variables associated with the motion integrals in quantum theory is a variant of the quasiclassical approximation. In this case, the approximation requires no substantiation by corresponding estimates. There is only one requirement left, which is the compliance with the observations. "Exact" quantum theory in the absence of the time parameter has no connection to the observations.

The additional dynamical parameters P_ε act as observable ones in the modified theory. In the ADM-representation, they form space distribution of the universe proper mass, as well as the space-time shifts canonically subjugated to them. For the latter, average (along the whole history of the universe) values can be determined in the original theory as well, where the universe proper mass should be taken as equal to zero. Non-zero proper mass of the universe (mass spectrum) is admissible in the operator canonical representation of the gravity theory for a closed universe. In this representation, the mass spectrum is associated with the hierarchy of the spatial structures emerging in the process of the universe evolution. The sequence of formation of the spatial structures of various scale itself may serve as a material basis for the definition of the proper time of the universe.

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