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AN ANALYSIS OF CORRECTIONS TO THE PROPAGATION CONSTANTS OF A MULTIMODE PARABOLIC OPTICAL FIBER UNDER BENDING

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The goal of our work was to study a circularly bent, weakly guiding, multimode optical fiber with a parabolic refractive index profile. With this in mind, the second-order corrections to propagation constants of longitudinally perturbed arbitrary dielectric waveguide's modes were found using the perturbation theory. Based on that general result, a simple analytic equation describing the corrections to the propagation constants of the modes in the bent parabolic optical fiber was derived. It was shown that the increments of squares of mode propagation constants were the same for all modes. Moreover, the increments of mode propagation constants' differences in the bent fiber were proportional to those in the straight fiber. The proportionality coefficient was independent of the mode number. The obtained results are of high importance for development of optical fiber sensors, in which fiber bending is possible.

Keywords: fiber, curvature, graded index, bent waveguide, perturbation analysis, propagation constant

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АНАЛИЗ ПОПРАВОК К ПОСТОЯННЫМ РАСПРОСТРАНЕНИЯ В ИЗОГНУТОМ МНОГОМОДОВОМ ПАРАБОЛИЧЕСКОМ ОПТИЧЕСКОМ ВОЛОКНЕ

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Работа посвящена исследованию равномерно изогнутого слабонаправляющего оптического многомодового волокна с параболическим профилем показателя преломления. В рамках формализма малых возмущений записана формула для поправок второго порядка малости к постоянным распространения мод равномерно возмущенного диэлектрического волновода. На этой основе получено простое аналитическое выражение для поправок к постоянным распространения мод изогнутого параболического многомодового волокна. Показано, что поправки к квадратам постоянных распространения мод одинаковы для всех мод. При этом поправки к разности постоянных распространения мод в изогнутом волокне пропорциональны разности постоянных распространения мод прямолинейного волокна с коэффициентом, не зависящим от номеров мод. Результат особенно важен для анализа интерферометрических оптоволоконных датчиков на основе изгиба чувствительного волокна.

Ключевые слова: оптическое волокно, градиентный профиль, изогнутый волновод, метод возмущений, постоянная распространения

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Introduction

The fiber in any extended fiber optic paths has some degree of bending. This is undesirable in some systems, for instance, leading to optical losses in communication devices. Conversely, the operating principles of other types of systems are actually based on bending, for example, in optical modulators [1], multiplexers [2], splitters [3]; bending can also be measured using fiber optic sensors. In particular, fiber optic sensors allow tracking the state of buildings, structures and mechanical assemblies [4 – 6], monitoring human health [7 – 9] by monitoring the bending of the fiber; such sensors are used to fabricate medical devices [10], equipment in robotics [11], and for other purposes.

In view of the above, it is essential to develop an array of convenient analytical tools making it possible to calculate the effect of fiber bending on the parameters of light propagation in it. Recent studies have focused closely on analytical [12, 13] and numerical [14, 15] calculations of optical losses in bent fibers. However, the change in phase progression or propagation constants (PC) of the waveguide modes of optical fiber under bending is an equally important factor. For example, this change plays a decisive role in interferometric fiber optic sensors, where the interference signal directly depends on the difference in propagation constants (PCD) of the interfering modes [16 – 26]. Even though this issue is crucial for fiber optic bending sensors, scarce attention has been given in the literature to calculation of the corrections to mode PC under fiber bending, in particular, for the most common fiber with a parabolic refractive index. The well-known studies calculating the changes in the phase progression or propagation constants of waveguide modes under fiber bending [27 – 33] have failed to yield simple expressions for the corrections to the PC and PCD of the modes in a parabolic fiber. A general expression was introduced in [34] for the changes in the mode PC for the case of uniform fiber perturbation, obtained from a system of differential equations for coupled modes. However, this expression does not allow formulating an explicit analytical expression for the case of a parabolic fiber, which would be convenient for computational estimates. Appendix 1 of this study contains this expression and its brief analysis.

The goal of this paper consists in analyzing the propagation constants and their difference for the case of a weakly guiding multimode optical fiber with a parabolic refractive index profile, uniformly bent in a circle.

In view of the goal set, we first derived a general formula for calculating the corrections to the PC of the modes of a dielectric waveguide uniformly perturbed along the axis (the method of small perturbations was used); the case of a parabolic optical fiber was analyzed in detail based on this formula.

Calculation of corrections to the PC of the m^{th} mode of a perturbed dielectric waveguide

To apply the method of small perturbations to solving the problem, we rely on the approach used in monograph [35] for obtaining a first-order correction to the PC of the modes of a perturbed waveguide. We are going to establish below that the first-order correction vanishes in the case of uniform bending of the fiber. For this reason, we determined a correction for the second-order mode PC to account for the bending.

Based on this approach, let us formulate the statement of a specific problem. Suppose an unperturbed dielectric waveguide has a relative permittivity profile

$$\varepsilon_0(r,\varphi) = n_0^2(r,\varphi),$$

where n_0 is the refractive index profile; r, φ are the coordinates in a cylindrical coordinate system where the axis z coincides with the fiber axis.

If perturbation that is uniform along the axis is introduced, the relative permittivity profile of the perturbed waveguide can be written in the form

$$\varepsilon(r,\varphi)=\varepsilon_0(r,\varphi)+\Delta\varepsilon(r,\varphi). \tag{1}$$

Let the unperturbed modes have the form

$$\mathbf{E}'_m=\mathbf{E}_m(r,\varphi)\exp\left[i(\omega t-\beta_m z)\right],\quad m=0,1,2,\ldots, \tag{2}$$

where the transverse mode functions $\mathbf{E}_m(r,\varphi)$ satisfy the unperturbed wave equation

$$\left[\nabla_t^2+k_0^2\varepsilon_0\right]\cdot\mathbf{E}_m(r,\varphi)=\beta_m{}^2\mathbf{E}_m(r,\varphi). \tag{3}$$

Here we omitted the term $\nabla(\nabla\cdot\mathbf{E}'_m)$, which is justified for linearly polarized modes of optical fiber [36].

These modes are mutually orthogonal and satisfy the following orthogonality condition:

$$\int\mathbf{E}^*_m\cdot\mathbf{E}_l\,dS=N_m\delta_{ml}. \tag{4}$$

Consider the influence from the perturbation of relative permittivity $\Delta\varepsilon(r,\varphi)$, which is small compared with the magnitude of $\varepsilon(r,\varphi)$. Let us assume that such a small perturbation only causes small changes in mode functions and propagation constants.

Let the mode functions change by $\delta\mathbf{E}_m{}^{(1)}$, and the squares of the propagation constants by $\delta\beta_m^{2\,(1)}$. In this case, the wave equation takes the form

$$\left[\nabla_t^2+k_0^2\varepsilon_0+k_0^2\Delta\varepsilon\right]\cdot\left(\mathbf{E}_m+\delta\mathbf{E}_m{}^{(1)}\right)=\left(\beta_m{}^2+\delta\beta_m^{2\,(1)}\right)\cdot\left(\mathbf{E}_m+\delta\mathbf{E}_m{}^{(1)}\right). \tag{5}$$

If we neglect the second-order terms $\Delta\varepsilon\cdot\delta\mathbf{E}_m{}^{(1)}$ and $\delta\beta_m^{2\,(1)}\cdot\delta\mathbf{E}_m{}^{(1)}$ and use relation (3), Eq. (5) can be written in a simpler form:

$$\left[\nabla_t^2+k_0^2\varepsilon_0\right]\cdot\delta\mathbf{E}_m{}^{(1)}+k_0^2\Delta\varepsilon\cdot\mathbf{E}_m=\beta_m{}^2\cdot\delta\mathbf{E}_m{}^{(1)}+\delta\beta_m^{2\,(1)}\cdot\mathbf{E}_m. \tag{6}$$

To solve this equation, we expand $\delta\mathbf{E}_m{}^{(1)}$ in terms of the unperturbed mode functions:

$$\delta\mathbf{E}_m{}^{(1)}=\sum a_{ml}\mathbf{E}_l, \tag{7}$$

where a_{ml} are constant coefficients.

Substituting expression (7) for $\delta\mathbf{E}_m{}^{(1)}$ into Eq. (6) and using (3), we obtain the following relation:

$$\sum_l a_{ml}\left(\beta_l{}^2-\beta_m{}^2\right)\cdot\mathbf{E}_l=\left(\delta\beta_m^{2\,(1)}-k_0^2\Delta\varepsilon\right)\cdot\mathbf{E}_m. \tag{8}$$

If relation (8) is scalarly multiplied by the complex conjugate quantity $\mathbf{E}^*_m$ and this product is integrated over the entire transverse plane, then the expression on the left-hand side will be equal to zero by virtue of orthogonality relation (4). Thus, we have the following expression:

$$\int\mathbf{E}^*_m\cdot\left(\delta\beta_m^{2\,(1)}-k_0^2\Delta\varepsilon\right)\cdot\mathbf{E}_m dS=0. \tag{9}$$

Since $\delta\beta_m^{2\,(1)}$ is a constant, it follows from expression (9) that

$$\delta\beta_m^{2\,(1)}=\frac{k_0^2\int\mathbf{E}^*_m\cdot\Delta\varepsilon\cdot\mathbf{E}_m dS}{N_m}. \tag{10}$$

This expression gives the first-order correction to the square of the propagation constant β_m^2. Since the correction to the PC square is small, we can write the actual correction to the PC [35]:

$$\delta\beta_m=\delta\beta_m^2/2\beta_m.$$

As a result, we obtain the following expression:

$$\delta\beta_m^{(1)}=\frac{k_0^2\int\mathbf{E}^*_m\cdot\Delta\varepsilon\cdot\mathbf{E}_m dS}{2\beta_m N_m}. \tag{11}$$

To calculate the coefficients aml $a_{ml}(m\neq l)$, we scalarly multiply the left and right-hand sides of Eq. (8) by $\mathbf{E}^*_l$, $(m\neq l)$ and integrate over the entire transverse plane. Then the following expression can be written:

$$a_{ml} = \frac{k_0^2}{\left(\beta_m^2 - \beta_l^2\right) N_l} \int \mathbf{E}_m \cdot \Delta\varepsilon \cdot \mathbf{E}_l^* dS,$$
$$l \neq m. \tag{12}$$

Thus, we obtained (with an accuracy up to notations) expressions for the first-order correction to the PC and coefficients $a_{ml} (m \neq l)$, similar to those given in monograph [35], except for the normalization parameter N_m introduced.

Similarly, we derive the second-order corrections to the mode PC. For this purpose, we consider the wave equation in the form

$$\begin{gathered}\left[\nabla_t^2 + k_0^2 \varepsilon_0 + k_0^2 \Delta\varepsilon\right] \times \\ \times \left(\mathbf{E}_m + \delta\mathbf{E}_m^{(1)} + \delta\mathbf{E}_m^{(2)}\right) = \\ = \left(\beta_m^2 + \delta\beta_m^{2\,(1)} + \delta\beta_m^{2\,(2)}\right) \times \\ \times \left(\mathbf{E}_m + \delta\mathbf{E}_m^{(1)} + \delta\mathbf{E}_m^{(2)}\right),\end{gathered} \tag{13}$$

where $\delta\mathbf{E}_m^{(2)}, \delta\beta_m^{2\,(2)}$ are the second-order corrections to the mode function and the PC square, respectively.

If we use relation (3) and further assumptions, such as neglecting the third and fourth-order terms and taking (6) as an initial approximation, Eq. (13) can be reduced to the following form:

$$\begin{gathered}\left[\nabla_t^2 + k_0^2 \varepsilon_0\right] \cdot \delta\mathbf{E}_m^{(2)} + k_0^2 \Delta\varepsilon \delta\mathbf{E}_m^{(1)} = \\ \beta_m^2 \delta\mathbf{E}_m^{(2)} + \delta\beta_m^{2\,(1)} \delta\mathbf{E}_m^{(1)} + \delta\beta_m^{2\,(2)} \mathbf{E}_m.\end{gathered} \tag{14}$$

To solve this equation, we expand $\delta\mathbf{E}_m^{(2)}$ in terms of the unperturbed mode functions:

$$\delta\mathbf{E}_m^{(2)} = \sum_l b_{ml} \mathbf{E}_l, \tag{15}$$

where b_{ml} are constant coefficients.

Substituting expressions (7) for $\delta\mathbf{E}_m^{(1)}$ and (15) for $\delta\mathbf{E}_m^{(2)}$ into Eq. (14), and using (3), we obtain the following relation:

$$\begin{gathered}\sum_l b_{ml} \left(\beta_l^2 - \beta_m^2\right) \cdot \mathbf{E}_l + \\ + k_0^2 \Delta\varepsilon \sum_l a_{ml} \mathbf{E}_l = \\ = \delta\beta_m^{2\,(1)} \sum_l a_{ml} \mathbf{E}_l + \delta\beta_m^{2\,(2)} \cdot \mathbf{E}_m.\end{gathered} \tag{16}$$

If this ratio is scalarly multiplied by $\mathbf{E}_m^*$ and integrated over the entire transverse plane, then the first term on the left-hand side and all terms of the sum, except for the term with $l = m$, are equal to zero by virtue of orthogonality relation (4).

If we also use expression (10), then we can write the following ratio:

$$\begin{gathered}k_0^2 \sum_l a_{ml} \int \mathbf{E}_m^* \cdot \Delta\varepsilon \cdot \mathbf{E}_l dS = \\ = k_0^2 a_{mm} \int \mathbf{E}_m^* \cdot \Delta\varepsilon \cdot \mathbf{E}_m dS + \\ + \delta\beta_m^{2\,(2)} N_m.\end{gathered} \tag{17}$$

Then, keeping in mind that the first term on the right-hand side of this expression is mutually canceled out with the summation term $l = m$ in its left-hand side, and using expression (12), the second-order correction to the PC square of the m^{th} mode can be written in the following form in the case when the waveguide is perturbed uniformly along the axis:

$$\delta\beta_m^{2\,(2)} = \sum_{l \neq m} \left(\beta_l^2 - \beta_m^2\right) a_{ml} a_{lm}, \tag{18}$$

or

$$\begin{gathered}\delta\beta_m^{2\,(2)} = \sum_{l \neq m} \frac{k_0^4}{\left(\beta_m^2 - \beta_l^2\right) N_m N_l} \times \\ \times \left| \int \mathbf{E}_m \cdot \Delta\varepsilon \cdot \mathbf{E}_l^* dS \right|^2.\end{gathered} \tag{19}$$

Given that $\delta\beta_m^{2\,(2)}$ is the correction to the PC square, and the actual correction to the PC is

$$\delta\beta_m^{(2)} = \delta\beta_m^{2\,(2)} / 2\beta_m,$$

we obtain the following expression:

$$\begin{gathered}\delta\beta_m^{(2)} = \sum_{l \neq m} \frac{k_0^4}{2\beta_m \left(\beta_m^2 - \beta_l^2\right) N_m N_l} \times \\ \times \left| \int \mathbf{E}_m \cdot \Delta\varepsilon \cdot \mathbf{E}_l^* dS \right|^2.\end{gathered} \tag{20}$$

Expressions (19) and (20) are the desired expressions for the second-order correction to the PC of the optical fiber mode.

Notably, expressions (19) and (20) hold true for any dielectric waveguides where the term $\nabla\left(\nabla\cdot\mathbf{E}'_m\right)$ can be neglected in formulating the wave equation.

Appendix 2 of this paper compares Eq. (20) with the formula obtained by the approach used in [34].

Case of uniform bending of optical fiber with a parabolic refractive index profile

Expressions (11) and (20) describe the general form of corrections to the mode PC under perturbation of the dielectric waveguide. Let us consider their specific form for the case of a uniformly bent parabolic fiber.

The relative permittivity profile of parabolic optical fiber is described by the following expression:

$$\varepsilon(r)=n^2(r)= \\ =n_1^2\begin{cases}1-2\Delta\cdot\left(r^2/a^2\right), & r<a,\\ 1-2\Delta, & r>a,\end{cases} \tag{21}$$

where a is the radius of the core; n_1 is the maximum value of the refractive index in the cross section; Δ is the relative difference of refractive indices,

$$\Delta=\left(n_1^2-n_2^2\right)/2n_1^2$$

(n_2 is the refractive index in the cladding region).

The eigenmodes are described as linearly polarized LP_{lp}-modes within the model of weakly guiding fiber with an unlimited parabolic profile [37]. The scalar component of the mode has the form

$$E'_{lp}=E_{lp}(r,\varphi)\exp\left[j\left(\omega t-\beta_{lp}z\right)\right], \tag{22}$$

where $E_{lp}(r, \varphi)$ are the scalar transverse mode functions, l is the azimuthal order of the mode, and the index p is the radial order of the mode.

Transverse mode functions are given by the following expressions:

$$E_{lp}(r,\varphi)=\Psi(r)\cdot\begin{pmatrix}\cos l\varphi\\ \sin l\varphi\end{pmatrix}, \tag{23}$$

$$\Psi(r)=\left(\sqrt{2}\frac{r}{w}\right)^l L_q^{(l)}\left(2\frac{r^2}{w^2}\right)\exp\left[-\frac{r^2}{w^2}\right]. \tag{24}$$

Here $L_q^{(l)}$ are the generalized Laguerre polynomials of order l and degree q ($q = p - 1$); they have the form

$$L_q^{(l)}(u)=\sum_{\nu=0}^{q}\begin{pmatrix}q+l\\ q-\nu\end{pmatrix}\frac{(-u)^\nu}{\nu!}. \tag{25}$$

The parameter w gives the boundaries where the field exists in the radial direction and is defined by the following expression:

$$w=\sqrt{\frac{2a}{n_1k_0\sqrt{2\cdot\Delta}}}. \tag{26}$$

The propagation constant β_{lp} of the LP_{lp}-mode, taking into account expression (26) and the data from monograph [37], is found by the expression

$$\beta_{lp}=n_1k_0\sqrt{1-\frac{4}{w^2}(2p+l-1)}. \tag{27}$$

To calculate the corrections to the PC of the LP_{lp}-mode, we use the normalized mode functions:

$$E_{lpN}=\frac{E_{lp}}{\sqrt{\int E_{lp}E_{lp}dS}}. \tag{28}$$

In this case, the normalization parameter Nlp introduced in condition (4) is equal to unity. Then relations (10), (11), (19), and (20) can be rewritten in the following form:

$$\delta\beta_{lp}^{2\,(1)}=k_0^2\int E_{lpN}\cdot\Delta\varepsilon\cdot E_{\eta\mu N}dS, \tag{29}$$

$$\delta\beta_{lp}^{(1)}=\frac{k_0^2}{2\beta_{lp}}\int E_{lpN}\cdot\Delta\varepsilon\cdot E_{\eta\mu N}dS, \tag{30}$$

$$\delta\beta_{lp}^{2\,(2)} = \sum_{lp\neq\eta\mu} \frac{k_0^4}{\left(\beta_{lp}^2 - \beta_{\eta\mu}^2\right)} \times \\ \times\left(\int E_{lpN}\cdot\Delta\varepsilon\cdot E_{\eta\mu N} dS\right)^2, \quad (31)$$

$$\delta\beta_{lp}^{(2)} = \sum_{lp\neq\eta\mu} \frac{k_0^4}{2\beta_{lp}\left(\beta_{lp}^2 - \beta_{\eta\mu}^2\right)} \times \\ \times\left(\int E_{lpN}\cdot\Delta\varepsilon\cdot E_{\eta\mu N} dS\right)^2. \quad (32)$$

The following common approach can be applied to analyze the bending of a fiber with the radius R (see Figure) [37]: instead of a fiber uniformly bent around the circumference of the fiber, we introduce an equivalent straight fiber with a relative dielectric permittivity profile $\varepsilon(r, \varphi)$ of the form (1). In this case, $\varepsilon_0(r, \varphi)$ is the relative permittivity profile of an unbent straight fiber, and the perturbation $\Delta\varepsilon(r, \varphi)$ is described by the expression

$$\Delta\varepsilon = \frac{2n_1^2 r\cos\varphi}{R}. \quad (33)$$

In this case, the distributions of the transverse fields coincide for the bent and equivalent straight fibers [37].

Within the framework of the model for the equivalent straight fiber, the first-order correction to the PC of the LP_{lp}-mode for bent fiber can be obtained by substituting expression (33) into expression (30). However, it is easy to prove that it is equal to zero due to the antisymmetric nature of the perturbation. Therefore, it is insufficient to confine the consideration with only the first correction in the bending case.

Substituting expression (33) into expression (31), we obtain a second-order correction to the PC square of the LP_{lp}-mode:

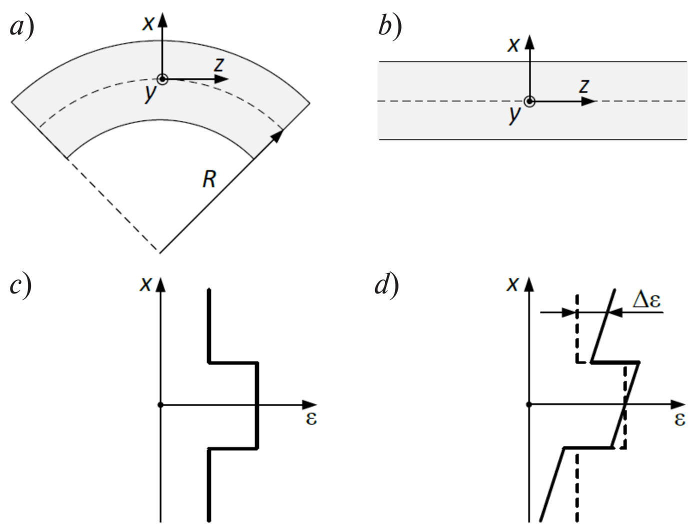
$$\delta\beta_{lp}^{2\,(2)} = \sum_{lp\neq\eta\mu} \frac{4k_0^4 n_1^4}{\left(\beta_{lp}^2 - \beta_{\eta\mu}^2\right)R^2} I_{lp,\eta\mu}^2; \quad (34)$$

here we have introduced the following notation for the integral characterizing the relationship between the LP_{lp}-mode and the $LP_{\eta\mu}$-mode:

$$I_{lp,\eta\mu} = \int\int E_{lpN}\cdot r^2\cos\varphi\cdot E_{\eta\mu N} dr d\varphi. \quad (35)$$

This integral can be calculated explicitly for all combinations of LP_{lp}-modes [37]. The following integrals (35) are nonzero in this case:

1. Integral characterizing the relationship of the LP_{lp}-mode with the $LP_{l+1,p}$-mode, i.e.,



Schematic diagrams for bending analysis for fiber with the radius R: *a*, *b* are the bent and equivalent straight fiber; *c*, *d* are the profiles of relative permittivity for fibers *a* and *b*, respectively

$$I_{lp,(l+1)p}=\frac{w}{2}\sqrt{\frac{p+l}{\sigma_l}};\qquad(36)$$

2. Integral characterizing the relationship of the LP_{lp}-mode with the $LP_{l-1,p+1}$-mode, i.e.,

$$I_{lp,(l-1)(p+1)}=\frac{w}{2}\sqrt{\frac{p}{\sigma_{l-1}}};\qquad(37)$$

3. Integral characterizing the relationship of the LP_{lp}-mode with the $LP_{l-1,p}$-mode, i.e.,

$$I_{lp,(l-1)p}=\frac{w}{2}\sqrt{\frac{p+l-1}{\sigma_{l-1}}};\qquad(38)$$

4. Integral characterizing the relationship of the LP_{lp}-mode with the $LP_{l+1,p-1}$-mode, i.e.,

$$I_{lp,(l+1)(p-1)}=\frac{w}{2}\sqrt{\frac{p-1}{\sigma_l}},\qquad(39)$$

at the same time, $\sigma_l = 1$ if $l = 0$, and $\sigma_l = 2$ if $l \neq 0$.

The remaining modes turned out to be unrelated to the LP_{lp}-mode. We should note that all coupled modes differ by unity in their composite mode number $m = 2p + l - 1$.

Thus, the absolute value of the difference between the PC squares of the coupled modes does not depend on the values of l and p and follows the expression

$$\left|\beta_{lp}^2-\beta_{\eta\mu}^2\right|=\frac{4}{w^2}.\qquad(40)$$

Therefore, the factor in front of the integral in expression (34) can be simplified and taken outside the sum sign (accounting for the plus or minus signs). Then expression (34) can be rewritten as follows:

$$\delta\beta_{lp}^{2\,(2)}=\frac{k_0^4n_1^4w^2}{R^2}\times$$
$$\times\left(I_{lp,(l+1)p}^2+I_{lp,(l-1)(p+1)}^2-I_{lp,(l-1)p}^2-I_{lp,(l+1)(p-1)}^2\right).\qquad(41)$$

By enumerating all possible combinations of the indices l and p, it is easy to prove that the expression in brackets in relation (41) is equal to $w^2/4$ regardless of the values of the indices and the orientation of the mode function LP_{lp} ($\cos(l\varphi)$ or $\sin(l\varphi)$).

Thus, the desired increment of the propagation constant square of the LP_{lp}-mode in a parabolic optical fiber takes on a relatively simple form:

$$\delta\beta_{lp}^2=\delta\beta_{lp}^{2\,(2)}=\frac{k_0^4n_1^4w^4}{4R^2}=\frac{k_0^2n_1^2a^2}{2\Delta\cdot R^2}\qquad(42)$$

and, importantly, does not depend on the mode number.

The actual increment of the PC $\delta\beta_{lp} = \delta\beta_{lp}^2/2\beta_{lp}$ is written as

$$\delta\beta_{lp}=\frac{k_0^2n_1^2a^2}{4\beta_{lp}\cdot\Delta\cdot R^2}.\qquad(43)$$

Now we can proceed to calculating the difference

$$\Delta\beta_{lp,\eta\mu}^b=\beta_{lp}^b-\beta_{\eta\mu}^b$$

that is the difference in the mode propagation constants under fiber bending, which is important for calculations of interferometric sensors:

$$\Delta\beta_{lp,\eta\mu}^b=$$
$$=\Delta\beta_{lp,\eta\mu}^s-\frac{k_0^2n_1^2a^2}{4\cdot\Delta\cdot\beta_{lp}^s\beta_{\eta\mu}^sR^2}\Delta\beta_{lp,\eta\mu}^s,\qquad(44)$$

where $\Delta\beta_{lp,\eta\mu}^b$, $\Delta\beta_{lp,\eta\mu}^s$ are the mode PCD of bent and straight fibers, respectively.

The resulting expression can be simplified taking into account that $\beta_{lp}^s\beta_{\eta\mu}^s \approx k_0^2n_1^2$:

$$\Delta\beta_{lp,\eta\mu}^b=\Delta\beta_{lp,\eta\mu}^s\left(1-\frac{a^2}{4\Delta\cdot R^2}\right).\qquad(45)$$

Thus, the change in mode PCD in a bent parabolic optical fiber is proportional to the mode PCD in a straight fiber $\Delta\beta_{lp,\eta\mu}^s$ with the proportionality coefficient $a^2/(4\Delta)$ independent of mode numbers. It is important for assessing the efficiency of fiber optic sensors that detect bending to obtain an estimate of the sensitivity of the change in mode PCD to the inverse square of the bending radius, which, according to expression (45), has the form

$$\frac{\partial\left(\Delta\beta^{b}_{lp,\eta\mu}\right)}{\partial\left(\dfrac{1}{R^2}\right)}=\frac{a^2}{4\Delta}\Delta\beta^{s}_{lp,\eta\mu}. \qquad (46)$$

It should be borne in mind that both the theoretical formulas (22) – (27) for the mode functions and the mode PC of a parabolic fiber, and the formulas (42) – (46) that we derived for the corrections to the PC and PCD of the mode were obtained in the approximation of an unlimited parabolic profile of the fiber's refractive index. For this reason, strictly speaking, they cannot be used to describe the behavior of modes close to cutoff, whose field appears to largely extend beyond the fiber core.

The elasto-optical effect should be taken into account in the formulation of expression (33) for the perturbed profile of relative permittivity to use the expressions obtained for analysis of real fibers. The following approach can be used for approximate account of this effect: the real curvature radius of the bend is replaced with an effective one, for example, R_{eff} = 1.27 R for glass fiber [30].

Conclusion

To analyze the influence of bending of a parabolic multimode fiber on the propagation constants of the modes and their differences, we have obtained analytical expressions for second-order corrections using the method of small perturbations. We have established that the increments to the squares of mode propagation constants are the same for all modes. Furthermore, we have confirmed that the change in the difference between the mode propagation constants in a bent fiber is proportional to the difference in the mode propagation constants in a straight fiber with a proportionality factor that does not depend on the mode numbers. In this case, the magnitude of the changes depends on the ratio between the radius of the fiber core and the relative difference between the refractive indices of the core and the cladding. The result obtained has major importance for developing interferometric fiber optic sensors with fiber bending detection, as well as in analysis of phase effects in fiber optic systems with multimode fibers. The generalized expression derived for the second-order correction (20) to the mode propagation constant presents interest not only for analysis of optical fibers but also arbitrary dielectric waveguides with uniform perturbation.

Appendix 1

Known estimate of the correction to the PC of the mth mode of optical fiber with a parabolic profile under uniform perturbation

The influence of fiber bending on the PC of fiber modes is analyzed in [34] but since the first-order correction vanishes in the perturbation method, the authors suggested using, instead of the perturbation method, a more specific approach to analyzing the system of differential equations of coupled modes.

As a result, the following estimate for the PC increment was obtained in [34]:

$$\delta\beta_m=-\sum_l\frac{\kappa_{ml}\kappa_{lm}}{\beta_m-\beta_l}, \qquad (A1)$$

where $\delta\beta_m$ is the correction to the PC of an m^{th} mode due to fiber perturbation leading to mode coupling; β_m is the PC of an m^{th} mode without perturbation; κ_{mn} is the coupling coefficient between the m^{th} and n^{th} mode.

The expression for the coupling coefficients can be found, for example, in [36, 37]:

$$\kappa_{ml}=\frac{k_0^2}{2\beta_m}\int\int E_{mN}\left(r,\varphi\right)\cdot\Delta\varepsilon\left(r,\varphi\right)\times \\ \times E_{lN}\left(r,\varphi\right)\cdot rdrd\varphi, \qquad (A2)$$

where k_0 is the wavenumber; $\Delta\varepsilon(r, \varphi)$ is the perturbation of the profile of the relative permittivity of the fiber, in particular, under bending; $E_{mN}(r, \varphi)$ is the normalized mode function; r, φ are the coordinates in a cylindrical coordinate system where the z axis coincides with the fiber axis. In this case, the mode functions satisfy the normalization condition:

$$\iint E_{mN}^2 r dr d\varphi = 1. \tag{A3}$$

If we substitute the expression (A2) to (A1), we can obtain an estimate for the increment of the mode PC:

$$\delta\beta_m = \sum_{l \neq m} \frac{k_0^4}{4\beta_m\beta_l(\beta_m - \beta_l)} \times \\ \times \left(\iint E_{mN} \cdot \Delta\varepsilon \cdot E_{lN} r dr d\varphi \right)^2 . \tag{A4}$$

In the case of an optical parabolic fiber in the formalism LP_{lp}, the mode PC can be found using expression (27). The integrals in estimate (A4) for the given fiber are also known (see Eqs. (35) – (39)). However, the expression obtained based on (A4) still remains cumbersome, complicating analysis of the physical meaning.

Appendix 2

Comparison of expressions for the PC increment under fiber bending

Let us compare expression (20) that we obtained to expression (A4). Notice that estimate (20) differs from estimate (A4) in the denominator of the factor in front of the integral. The denominator in expression (20) can be written in the following form:

$$2\beta_m(\beta_m^2 - \beta_l^2) = 2\beta_m(\beta_m + \beta_l)(\beta_m - \beta_l).$$

For a weakly guiding optical fiber, when the relative difference in refractive indices tends to zero, $\beta_m \approx \beta_l$, therefore the following relation holds true:

$$2\beta_m(\beta_m + \beta_l)(\beta_m - \beta_l) \approx 4\beta_m\beta_l(\beta_m - \beta_l)$$

and the formulas converge asymptotically.

However, Eq. (20) seems to be more appropriate for use for three reasons:

firstly, it was obtained by method of small perturbations generally accepted for such analysis, yielding simple and understandable approximations; on the contrary, a number of approximations made in [34] do not allow clearly assessing the order of smallness of the correction to the PC and the conditions for the applicability of the obtained estimate;

secondly, we obtained a simple and convenient expression for the PC increments under bending of a parabolic fiber from Eq. (20), while expression (A4) is a complex sum where the PC has to be written out for all modes associated with the calculated one;

thirdly, the expression that we obtained allowed drawing important physical conclusions about the increment of the mode PC under bending (they are discussed in Conclusion), while it is difficult to draw such conclusions based on expression (A4).

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