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MATHEMATICAL MODELING OF INFORMATION CONFRONTATION

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The article continues our studies in the previously constructed mathematical model of dissemination of new information in the society. The model is a system of four ordinary differential equations with quadratic nonlinearity in the right parts. Two fundamental domains have been taken in the parameter space of the model and they may be of interest in application. In some sense, these domains provide two diametrically opposite and essentially different scenarios of new information dissemination. In every case, the global properties of the phase pattern of the constructed dynamic system were investigated using qualitative methods of the theory of differential equations. Both conceptual and geometric interpretations of the obtained results were given.

Keywords: differential equation, stationary solution of system, invariant set, asymptotic stability

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ИНФОРМАЦИОННОГО ПРОТИВОБОРСТВА

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В статье излагается продолжение исследований построенной ранее базовой математической модели распространения в обществе новой информации. Данная модель представляет собой автономную систему четырех обыкновенных дифференциальных уравнений с квадратичной нелинейностью в правых частях. В пространстве параметров системы выделены две важные области, представляющие интерес для приложений. В определенном смысле в этих областях реализуются два диаметрально противоположных и принципиально разных сценария распространения новой информации в обществе. С помощью качественных методов теории дифференциальных уравнений в каждом случае изучены глобальные свойства фазового портрета построенной динамической системы. Даны содержательная и графическая интерпретации полученных результатов.

Ключевые слова: дифференциальное уравнение, стационарное решение системы, инвариантное множество, асимптотическая устойчивость

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Introduction

In paper [1], we presented a constructed mathematical model of dissemination of new information in the society:

$$\frac{dN}{dt} = \beta N - \gamma AN,$$

$$\frac{dC}{dt} = \alpha AN - \mu\left(C - C_*\right), \qquad (1)$$

$$\frac{dA}{dt} = \rho C - \eta\gamma AN - \lambda A,$$

$$\frac{di}{dt} = \sigma N - \omega i.$$

While constructing, it was considered that the main factors for the dissemination of new information were the following values, depending on the time t:

$N(t)$ (after "News") is a quantitative characteristic of information related to news which corresponds to promotion of new views in the information space;

$C(t)$ (after "Censorship") is a number of censor bodies with a certain resource base which aim to maintain the previously established concepts;

$A(t)$ (after "Alternative view") is a quantitative characteristic of information flow (probably initiated by censor bodies) which opposes the dissemination of a new concept in the information space;

$i(t)$ (after "index") is a parameter of a share of population favorably disposed towards to new ideas emerging in mass media at the time of occurrence:

$$i = 1 - \frac{I^*}{I},$$

where I, %, corresponds to complete acceptance of conventional statements before the beginning of observations; I^*, %, is a respective characteristic of acceptance of conventional statements with new views disseminating in mass media.

Parameters $\beta \geq 0$, $\gamma \geq 0$ respectively shows the capacity of new information dissemination via mass media and probabilistic characteristic of the effect neutralized by means of stating the opposite opinion. In turn, coefficient $\alpha \geq 0$ shows the intensity of the reaction to the confrontation of opposite views, parameter $\mu > 0$ is inversely related to the time of operation of additionally formed bodies (let us assume that a society always has some special resource available in the number of C_* in order to protect the previous concept).

Average velocity of news generated by one body of information C is characterized by parameter $\rho \geq 0$, while $\eta \geq 0$ is the amount of information A aimed at neutralizing the impact of messages N. Coefficient $\lambda > 0$ is inversely related to the time the information A is forgotten.

Parameters $\sigma > 0$, $\omega \geq 0$ characterize respectively the acceptance rate of the new idea and return to the former concept due to inertia of mentality.

Undoubtedly, the proposed mathematical model does not account for all the subtle features and details while describing the process of new information dissemination via mass media in the society. However, this generalized form of the model allows binding main factors dedicated to this action into a system and deeper understanding of the process of information confrontation.

Previously, using the methods described in papers [2–5], we showed that system (1) has characteristics that allow studying global properties of its solutions: uniqueness, continuous dependence on parameters, their unlimited extendibility. In addition, for this system, we proved invariance of the set

$$R_+^4 = \\ = \left\{\left(N, C, A, i\right) \in R^4 : N \geq 0,\, C \geq 0,\, A \geq 0,\, i \geq 0\right\}.$$

We also found two stationary solutions which have rather clear interpretation [1]:

$$X_{1st} = \left(N_{1st}, C_{1st}, A_{1st}, i_{1st}\right) = \\ = \left(0, C_*, \rho C_*/\lambda, 0\right),$$

$$X_{2st} = \left(N_{2st}, C_{2st}, A_{2st}, i_{2st}\right),$$

where

$$N_{2st} = \frac{\mu(\lambda\beta - \gamma\rho C_*)}{\beta(\alpha\rho - \mu\eta\gamma)},$$
$$C_{2st} = \frac{\alpha\lambda\beta - \eta\mu\gamma^2 C_*}{\gamma(\alpha\rho - \mu\eta\gamma)},$$
$$A_{2st} = \frac{\beta}{\gamma},$$
$$i_{2st} = \frac{\sigma\mu(\lambda\beta - \gamma\rho C_*)}{\omega\beta(\alpha\rho - \mu\eta\gamma)}.$$

In the space of system parameters, we indicated two domains in which $X_{ist} \in R_+^4$, $i = 1.2$, but with properties significantly different from each other:

$$\Omega_1 : \begin{cases} \gamma\rho C_* > \lambda\beta \\ \mu\eta\gamma > \alpha\rho \end{cases}, \quad \Omega_2 : \begin{cases} \gamma\rho C_* < \lambda\beta \\ \mu\eta\gamma < \alpha\rho \end{cases}.$$

By means of using qualitative methods of studying differential equations, we studied global properties of the phase pattern of the constructed dynamic system. This allowed us to identify several possible scenarios of dissemination of new information in the society.

This paper investigates the solution properties of system (1) in parameter domains

$$\Lambda_1 : \begin{cases} \gamma\rho C_* < \lambda\beta \\ \mu\eta\gamma > \alpha\rho \end{cases},$$
$$\Lambda_2 : \begin{cases} \gamma\rho C_* > \lambda\beta \\ \mu\eta\gamma < \alpha\rho \end{cases},$$

which may be of interest in application.

In these domains, set R_+^4 contains only one stationary solution:

$$X_{1st} = (N_{1st}, C_{1st}, A_{1st}, i_{1st}) = = (0, C_*, \rho C_*/\lambda, 0).$$

Before the study, one comment should be made. Since in system (1), a variable $i(t)$ is present only in the last equation, a further investigation may be conducted for a system of lesser dimension which is expedient to present in a form more suitable for the studies:

$$\frac{dC}{dt} = \alpha AN - \mu(C - C_*),$$
$$\frac{dA}{dt} = \rho C - (\lambda + \eta\gamma N)A, \qquad (2)$$
$$\frac{dN}{dt} = (\beta - \gamma A)N.$$

The obtained results can be then easily extended to the variable $i(t)$.

It is easy to show that the set

$$R_+^3 = = \{(C, A, N) \in R^3 : C \geq 0, A \geq 0, N \geq 0\}$$

for this set is invariant and contains only one stationary solution

$$X_{st} = (C_{st}, A_{st}, N_{st}) = = (C_*, \rho C_*/\lambda, 0).$$

Note that in Λ_1 this stationary solution in unstable, while in Λ_2 it is stable.

The conceptual meaning of the stationary solution X_{st} was formulated in paper [1] as a system of views predominant in a society, for the support of which administrative resource in the number of C_* employs a sufficient (from the point of view of this resource) amount of information $\rho C_*/\lambda$ in mass media.

Due to autonomy of system (2), initial conditions can be written as follows:

$$C(0) = C_0 \geq 0,$$
$$A(0) = A_0 \geq 0, N(0) = N_0 \geq 0. \qquad (3)$$

Analysis if models (2), (3) in the parameter domain

A reduced two-dimensional system of differential equations

$$\frac{dA}{dt} = \rho C - (\lambda + \eta\gamma N) A,$$
$$\frac{dN}{dt} = (\beta - \gamma A) N, \tag{4}$$

obtained from system (2) at $\alpha = 0$ and $C(t) = C_*$ when $t \geq 0$ gives a better understanding of the behavior of solutions to three-dimensional system (2), (3) in the Λ_1 domain.

We should explain that system (4) describes a situation when the bodies of information protection due to various reasons do not react to targeted media "injections" of data that contradict the accepted opinions of the society.

System (4) in the parameter domain Λ_1 in the invariant set

$$R_+^2 = \{(A, N) \in R^2 : A \geq 0, N \geq 0\}$$

has only one stationary solution:

$$X_{st} = (A_{st}, N_{st}) = (\rho C_*/\lambda, 0),$$

which is a saddle.

Known techniques of qualitative analysis of two-dimensional differential equations systems [6] allow constructing a phase pattern and investigating the behavior of system (4) trajectories (Fig. 1). As seen from the Figure, all trajectories of system (4) with initial conditions

$$A(0) = A_0 \geq 0, N(0) = N_0 > 0$$

when $t \to +\infty$ have the same behavior:

$$A(t) \to 0, N(t) \to +\infty.$$

We show that in

$$R_+^3 = \{(C, A, N) \in R^3 : C \geq 0, A \geq 0, N \geq 0\}$$

system (2), (3) has a qualitatively similar phase pattern.

Let

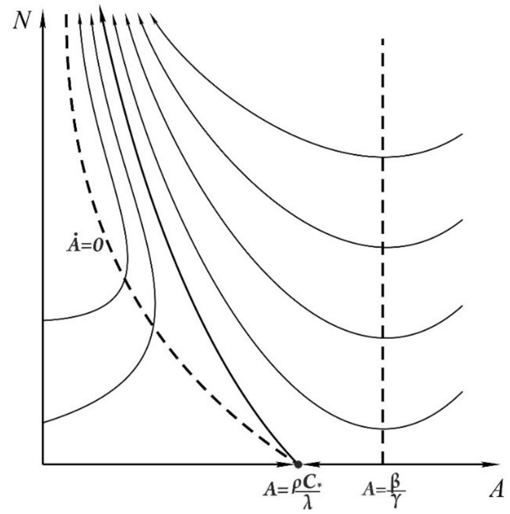

Fig. 1. Phase pattern of system (4) in the R_+^2 set

$$R^+ = \{(C, A, N) \in R_+^3 : N > 0\},$$
$$\partial R^+ = \{(C, A, N) \in R_+^3 : N = 0\}.$$

For an arbitrary solution of system (2), (3), specifically for

$$X(t) = (C(t), A(t), N(t)),$$

let us denote $\Lambda^+(X)$ as a ω-bounding set of this solution [7].

Lemma. *For all trajectories of system(2), originating in the R^+ set, $\Lambda^+ \cap \partial R^+$ is an empty set.*

P r o o f. The set ∂R^+ is invariant due to system (2), (3). Indeed, if

$$X_0 = (C_0, A_0, N_0) \in \partial R^+,$$

then system (2) is determined by linear equations

$$\frac{dC}{dt} = -\mu(C - C_*),$$
$$\frac{dA}{dt} = \rho C - \lambda A, \tag{5}$$
$$N(t, X_0) \equiv 0,$$

For which a special X_{st} point has global, uniform, asymptotic stability in ∂R^+.

Assume that the $\Lambda^+ \cap \partial R^+$ set is not empty. Then there is such a trajectory $X(t, X_0)$ of system (2), (3), that from $X(t, X_0) \in R^+$ it follows that $X(t, X_0) \to \partial R^+$. Based on the theorem of continuous dependence of system (2), (3) solutions on the initial data [8], $X(t, X_0) \to X_{st}$ when $t \to +\infty$. Which is impossible, as X_{st} is an unstable stationary solution to system (2), (3).

Thus, the lemma is proved.

Theorem 1. *All trajectories of system (2), originating in the R^+ set, are not bounded.*

P r o o f. Assume the converse. Let there be such a trajectory $X(t, X_0)$ which is bounded when

$$X_0 = \left(N(0), C(0), A(0), i(0)\right) \in R^+$$

Introduce Lyapunov's function into consideration

$$V(X,t) = \gamma AN - \beta N - \gamma \int_0^t \dot{A}N\, d\tau,$$

and find its derivative due to system (1):

$$\dot{V}(X,t) = \gamma \dot{N}A + \gamma \dot{A}N - \\ - \beta \dot{N} - \gamma \dot{A}N = \dot{N}(\gamma A - \beta) = \\ = -(\beta - \gamma A)^2 N \le 0.$$

This derivative is locally Lipschitzian along X and continuous. The $V(X, t)$ function itself is bounded from below. Let us show this:

The boundedness from below of the difference $\gamma AN - \beta N$ is obvious given the boundedness of the $X(t, X_0)$ trajectory. The third summand of the presented function is also bounded from below. Indeed, it is positive in a set, where $\dot{A} < 0$. In a set, where $\dot{A} > 0$, it can be estimated as follows:

$$-\gamma \int_0^t \dot{A}N\, d\tau \ge -\gamma \int_0^t \dot{A}N_{\max} d\tau = \\ = -\gamma N_{\max}\left[A(t) - A(0)\right] \ge \\ \ge -\gamma N_{\max} A_{\max} + \gamma N_{\max} A(0).$$

The second derivative $\ddot{V}(X,t)$, due to system (1), is obviously bounded from below as well. Consequently (see [7], statement VIII.4.7),

$$\dot{V}(X,t) \to 0 \text{ at } t \to +\infty.$$

This fact ensures that the trajectory is approaching to its ω-bounding set

$$\Lambda^+ \subset M = \\ = \left\{ (N, C, A, i) \in R_+^4 : A = \frac{\beta}{\gamma} \vee N = 0 \right\}.$$

Since system (1) is autonomous, this set is invariant due to the system.

Consider an Λ^+ set.

In the R^+ set, a plane $A = \beta/\gamma$ does not contain sets invariant due to system (2). Therefore, the trajectory of system (1) cannot approach this plane when $t \to +\infty$. The plane $N = 0$, according to lemma, can also not contain the points of ω-bounding set. Consequently, the system trajectory cannot approach the $N = 0$ plane when $t \to +\infty$ as well. We arrived at a contradiction.

This proves the theorem.

Let us select two subsets in the R^+ set (Fig. 2):

$$H_1 = \left\{ (C, A, N) \in R^+ : A \le \frac{\beta}{\gamma} \right\}, \\ H_2 = \left\{ (C, A, N) \in R^+ : A > \frac{\beta}{\gamma} \right\}. \quad (6)$$

In the H_1 subset, consider such surfaces, on which values $C(t)$, $A(t)$, $N(t)$ respectively equal zero:

$$N = \frac{\mu(C - C_*)}{\alpha A}, \quad (7)$$

$$N = \frac{\rho C}{\eta\gamma A} - \frac{\lambda}{\eta\gamma}, \quad (8)$$

$$A = \frac{\beta}{\gamma}. \quad (9)$$

Let us estimate the mutual arrangement of surfaces (7) and (8) by prior determination of their intersection with the plane $N = 0$ (see Fig. 2): $C = C_*$ and $C = \lambda A/\rho$ for Eqs. (7) and (8) respectively.

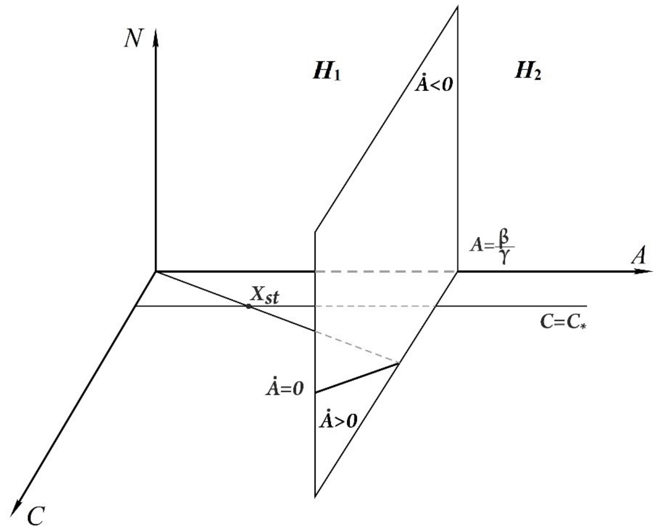

Fig. 2. Two selected subsets (6) in the R^+ set

At the intersection of these lines, there is a stationary solution

$$X_{st}=\left(C_{st},A_{st},N_{st}\right)=\left(C_*,\rho C_*/\lambda,0\right).$$

At any intersection with a plane $A=\tilde{A}$, surfaces (7) and (8) respectively have the form:

$$N=\frac{\mu C}{\alpha\tilde{A}}-\frac{\mu C_*}{\alpha\tilde{A}}, \quad (10)$$

$$N=\frac{\rho C}{\eta\gamma\tilde{A}}-\frac{\lambda}{\eta\gamma}. \quad (11)$$

For the Λ_1 domain parameters, the coefficient of C in the expression for line (10) turns out to be greater than the respective coefficient for line (11), as it results from inequality $\mu\eta\gamma > \alpha\rho$ after dividing by $\alpha\eta\gamma\tilde{A}$ that

$$\frac{\mu}{\alpha\tilde{A}}>\frac{\rho}{\eta\gamma\tilde{A}}.$$

Note here, that a diagram of the considered problem (Fig. 3) allows graphical estimation of the mutual arrangement of surfaces (7), (8) in the H_1 subset. This representation will be used to prove the following theorem.

Theorem 2. Assume that in the R^+ set of the parameter domain H_1,

$$X\left(t,X_0\right)=\left(C\left(t,X_0\right),A\left(t,X_0\right),N\left(t,X_0\right)\right)$$

is the solution of system (2), (3). Then, at t $\to +\infty$*, a component of this solution* $A(t) \to 0$*, while* $N(t) \to +\infty$.

P r o o f. Let us prove the theorem in three steps.

Step 1. Let us show that any solution $X\left(t\right)=\left(C\left(t\right),A\left(t\right),N\left(t\right)\right)$ from the H_2 subset of the R^+ set after a finite time falls into the H_1 subset. Indeed, if $A > \beta/\gamma$, it follows from the third equation of system (2) that $\dot{N}\left(t\right)<0$. Then, still in H_2, the $X(t)$ solution in finite time falls in a rather close neighborhood of plane $N = 0$. However, in this plane, all solutions of system (2) at $t \to +\infty$ approach the stationary solution of X_{st}, for which

$$A_{st} = \rho C_*/\lambda < \beta/\gamma$$

in Λ_1. Therefore, the theorem of continuous dependence on the initial data [8] guarantees that any solution to system (2) out of the H_2 subset falls into the H_1 subset in finite time.

Step 2. It is clear that solutions of $X(t)$ fall from H_2 into H_1, where $\dot{N}\left(t\right)>0$, through a part of plane (9) $A = \beta/\gamma$, in which $\dot{A}<0$ (see Fig. 2). Now, let us show that from the part of the H_1 set, where $\dot{A}<0$, at $t \to +\infty$, the component

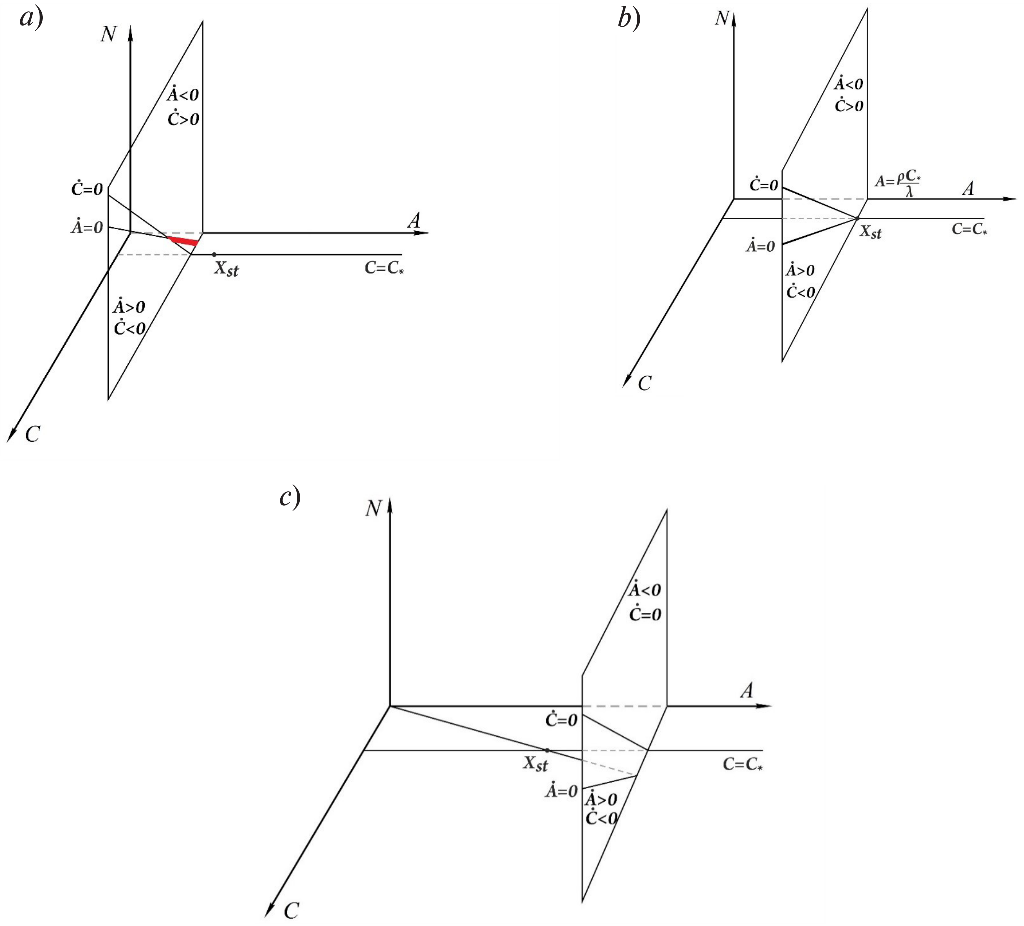

Fig. 3. Surface (7), (8) for various values of A:
$0 < A < A_{st}$ (*a*), $A = A_{st}$ (*b*), $A_{st} < A < \beta/\gamma$ (*c*)

$A(t) \to 0$, and $N(t) \to +\infty$.

Let us introduce two functions: $V_1(X) = \dot{A}$ and $V_2(X) = \dot{C}$. Using the relations (7), (8), it is easy to verify that in the H_1 subset on the surface $V_1(X) = 0$, when

$$A_{st} < A \leq \beta/\gamma$$

$V_2(X) < 0$ (see Fig. 3,*b*,*c*). Therefore, due to system (2),

$$\dot{V}_1(X)\Big|_{X:V_1(X)=0} =$$

$$= \dot{A} = \rho\dot{C} - \eta\gamma\dot{N}A < 0.$$

Consequently, the system solution from the surface $\dot{A} = 0$ falls into the domain, where $\dot{A} < 0$. Only for a few values $0 < A < A_{st}$, indicated in Fig. 3,*a* (red line segment), the solution may fall from the surface $\dot{A} = 0$ into a domain, where $\dot{A} > 0$. When it starts to go up, the $A(t)$ component can grow up to value $A = A_{st}$ only under the condition that $\dot{C} < 0$ at $A = A_{st}$ (Fig. 3,*b*). For this, we need to cross the surface $V_2(X) = \dot{C} = 0$. But on this surface due to system (2)

$$\dot{V}_2(X) = \ddot{C} = \alpha\dot{A}N + \alpha A\dot{N} > 0.$$

Therefore, as $\dot{N} > 0$ in the H_1 subset, then $A(t)$ through surface (8) again falls into the domain, where $\dot{A} < 0$, and consequently starts to drop. Thus, when $t \to +\infty$ the component $A(t) \to 0$, while $N(t) \to +\infty$.

Step 3. Solution $X(t, X_0)$, starting in the H_1 part, where $\dot{A} > 0$ and $\dot{C} < 0$, falls either through the surface $A = \beta/\gamma$ into the H_2 subset or into the surface $\dot{A} = 0$, where $V_1(X) = 0$. Although, in the first case, as it was shown in the previous steps,

$A(t) \to 0$, and $N(t) \to +\infty$ at $t \to +\infty$. In the second case, due to system (2), we have

$$\dot{V}_1(X)\Big|_{X:V_1(X)=0} = \\ = \ddot{A} = \rho\dot{C} - \eta\gamma\dot{N}A < 0.$$

Thus, we find ourselves in the part of the H_1 domain, in which $\dot{A} < 0$, where due to $\dot{N} > 0$, $A(t) \to 0$ again, and $N(t) \to +\infty$ at $t \to +\infty$. Which is what we set out to prove.

This proves Theorem 2.

Interpretation. With this ratio of parameters of system (2), the results of the mathematical studies allow us to make a conclusion on the preparedness of the society to accept new ideas and opinions. Any appearance of views and opinions in mass media that do not coincide with the traditional ones will find support of the society members. In this case, the recipients replace the previously dominating concept completely.

Analysis of models (2), (3) in the parameter domain Λ_2

It is easy to show (see paper [1], statement 2) that in the parameter domain Λ_2, a stationary solution

$$X_{st} = (C_{st}, A_{st}, N_{st}) = \\ = (C_*, \rho C_*/\lambda, 0)$$

of system (2), (3) has asymptotic stability. Let us study its domain of attraction.

Note that the right part of the equation for $C(t)$ of system (2) ensures the trajectory under the initial conditions falls from the R_+^3 set into a subset, where $C(t) \ge C_*$, which is invariant. Therefore, we will study only this part of R_+^3. In addition, from the equation for $A(t)$ of system (2) we can see that plane $A = \beta/\lambda$ divides this subspace into two sets (Fig. 4):

$$R_+^N = \{(C, A, N) \in R_+^3 : C \ge C_*, \dot{N} \ge 0\},$$

$$R_-^N = \{(C, A, N) \in R_+^3 : C \ge C_*, \dot{N} \le 0\},$$

in which X_{st} is located.

Theorem 3. *Assume that for systems (2), (3) in the parameter domain* Λ_2*, a condition*

$$\beta\eta\gamma + \mu\eta\gamma < \rho\alpha. \quad (12)$$

is satisfied. Then the entirety of the space

$$R_+^3 = \\ = \{(C, A, N) \in R^3 : C \ge 0,\ A \ge 0, N \ge 0\}$$

is a part of the attraction domain of asymptotically stable stationary solution X_{st}.

P r o o f. Take an arbitrary trajectory $X(t, X_0)$ of system (2), (3) originating in the R_+^3 set. As we noted in the beginning of the section, in finite time, it either end up in R_+^N or in R_-^N. Let's employ further reasoning in three steps as well.

Step 1. Let us show that any solution

$$X(t) = (C(t), A(t), N(t))$$

out of the R_+^N set in finite time falls into the R_-^N set.

Consider surface (8) (Fig. 4), where $\dot{A} = 0$, and function $V_1(X) = \dot{A}$. In the Λ_2 parameter domain, keeping in mind Eq. (8), if condition (12) is satisfied, for a certain positive volume δ we have:

$$\dot{V}_1(X)\Big|_{X:V_1(X)=0} = \\ = \ddot{A} = \rho\dot{C} - \eta\gamma\dot{N}A = \\ = \left(\frac{\rho C - \lambda A}{\eta\gamma}\right)(\rho\alpha - \mu\eta\gamma - \beta\eta\gamma + \\ + \eta\gamma^2 A) + \rho\mu C_* - \lambda\mu A \ge \\ \ge \left(\frac{\rho C_* - \lambda\dfrac{\beta}{\gamma}}{\eta\gamma}\right)(\rho\alpha - \mu\eta\gamma - \\ - \beta\eta\gamma + \eta\gamma^2 A) + \rho\mu C_* - \lambda\mu\frac{\beta}{\gamma} = \\ = \left(\frac{\rho\gamma C_* - \lambda\beta}{\eta\gamma^2}\right)(\rho\alpha - \mu\eta\gamma -$$

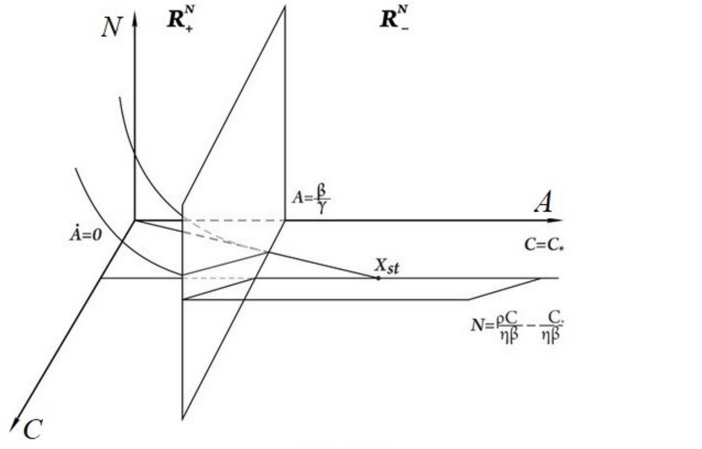

Fig. 4. Key surfaces (8) and (13) in sets R_+^N and R_-^N respectively

$$-\beta\eta\gamma+\eta\gamma^2 A\big)+\frac{\rho\mu\gamma C_*-\lambda\mu\beta}{\gamma}>\delta>0.$$

Thus, the trajectory $X(t)$ falls into the domain, where $\dot{A}>0$, from surface (8), and, respectively,

$$N<\frac{\rho C}{\eta\gamma A}-\frac{\lambda}{\eta\gamma}.$$

If $V_1(X) < 0$ in a certain part of the R_+^N space, then the search for derivative $V_1(X)$ sign, due to system (2), comes down to calculating the sign of a derivative on the surface $V_1(X) = 0$, since

$$\dot{V}_1(X)_{X:V_1(X)<0}=$$
$$=\ddot{A}=\rho\dot{C}-\lambda\dot{A}-\eta\gamma\dot{N}A-\eta\gamma\dot{A}N\geq$$
$$\geq\rho\dot{C}-\eta\gamma\dot{N}A=\dot{V}_1(X)\big|_{X:V_1(X)=0}>0.$$

This means that all trajectories out of the R_+^N set fall in the part, where $\dot{A}>0$.

Due to smoothness of function $V_1(X)$, we can find such a positive number ε, so that the component $N(t)$ from some finite point of time starts satisfying a relation

$$N<\frac{\rho C}{\eta\gamma A}-\frac{\lambda}{\eta\gamma}-\varepsilon,$$

only if the $X(t)$ trajectory is in the R_+^N set.

Next, for the component $A(t) > 0$, we have:

$$\dot{A}=\rho C-\eta\gamma AN-\lambda A>\rho C-$$
$$-\eta\gamma A\left(\frac{\rho C}{\eta\gamma A}-\frac{\lambda}{\eta\gamma}-\varepsilon\right)-\lambda A=$$
$$=\eta\gamma A\varepsilon>0.$$

The last inequality ensures that any $X(t)$ trajectory of system (2) after finite time falls into R_-^N, where $A > \beta/\gamma$ and, consequently, $\dot{N}<0$.

Step 2. Let us prove that falling from the R_+^N set into the R_-^N set, the $X(t)$ trajectory at $t \to +\infty$ approaches the stationary solution X_{st}.

Let us set a direction for a vector field at the surface

$$N=\frac{\rho}{\eta\beta}(C-C_*)+q,\ q-\text{const.} \tag{13}$$

Scalar product of vectors

$$\mathbf{n}=\left(\frac{\partial N}{\partial C};\frac{\partial N}{\partial A};-1\right)=$$
$$=\left(\frac{\rho}{\eta\beta};0;-1\right),$$

$$\frac{dX}{dt}=\left(\frac{dC}{dt};\frac{dA}{dt};\frac{dN}{dt}\right)$$

has the following form:

$$\frac{\rho}{\eta\beta}\left(C-C_*\right)\times$$
$$\times\left[\left(\alpha\frac{\rho}{\eta\beta}+\gamma\right)A-\left(\mu+\beta\right)\right].$$

This expression is positive when

$$A>\frac{\mu+\beta}{\alpha\frac{\rho}{\eta\beta}+\gamma}=\frac{\left(\mu+\beta\right)\eta\beta}{\alpha\rho+\gamma\eta\beta}=\bar{A}.$$

Because of the fact that in the Λ_2 parameter domain $\bar{A} < \beta/\gamma$, the $X(t)$ trajectory, falling from the R_+^N set into R_-^N (see Fig. 4), can no longer go back from R_-^N to R_+^N. And since in R_-^N $\dot{N} < 0$, after finite time, the $X(t)$ trajectory of system (2) arrives in the arbitrarily small neighborhood of plane $N = 0$. For system (2), this plane is invariant, and according to system of equations (5), all trajectories on it approach the stationary X_{st}. Therefore, based on the theorem of continuous dependence of system (2), (3) solutions on the initial data [8], the $X(t)$ trajectory of this system from the small neighborhood of plane $N = 0$ also approaches the stationary X_{st}, when $t \to +\infty$.

Step 3. Take the $X(t, X_0)$ trajectory of system (2), (3), originating in the R_-^N set. At any t moment of time, a point of this trajectory is located on surface (13) for some $q = q_0$. But in R_-^N, as shown in the second step of the proof, with the growth of time t, $X(t, X_0)$ "descends" onto a surface, where $q < q_0$. Therefore, the $X(t, X_0)$ solution originating in the R_-^N set arrives either in the R_+^N set via the plane $A = \beta/\gamma$ or on surface (13), where $q \leq 0$. In both cases, this guarantees, in line with the reasoning of the previous steps of the proof, that $X(t, X_0) \to X_{st}$, when $t \to +\infty$.

Thus, Theorem 3 is proved.

The phase pattern of system (2), (3) is shown in Fig. 5.

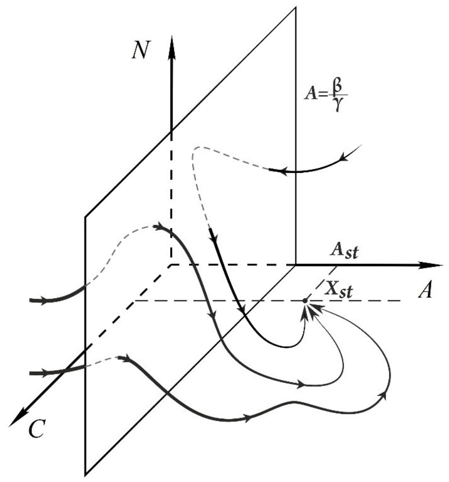

Fig. 5. Phase pattern of system (2), (3) in the parameter domain Λ_2

The interpretation of the obtained results is of interest. They show that with the indicated relations of system (2) parameters the society (or its segment) is completely dominated by a certain concept (for example, ideological or technological). The cause of this might be full acceptance of the processes occurring in the society or a failure to change the established views. This could also happen due to high efficiency of the censor bodies which do not allow new information to fully fill the information space.

Conclusion

The study performed in this paper significantly expands the domain of investigated system characteristics which allows predicting the behavior of solutions depending on the initial data.

1. In the parameter space, we identified two important domains Λ_1 and Λ_2, in which we mathematically justified certain global properties of the phase pattern of the studied dynamic system.

2. For each case, we provide an interpretation of the obtained results. In one instance, it is preparedness of the society to replace the dominating concept completely (for example, ideological or technological). In another case, on the contrary, it is a failure to accept new provisions due to various reasons.

The authors consider the results obtained in this article to be a continuation of the system research presented in papers [1, 9–11]. This project is aimed at studying mass media as a dynamic system with a high velocity of changes. The use of the methods of non-linear dynamics allows most complete and thorough investigation of the structure and properties of the processes unfolding in such a system.

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