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## CURRENT CONFIGURATIONS IN THE LONG JOSEPHSON CONTACT IN AN EXTERNAL MAGNETIC FIELD

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Current configurations in a periodically modulated long Josephson contact located in an external magnetic field are considered for values of the pinning parameter  $I$  greater than and less than the critical one ( $I_c$ ). It is shown that, if  $I > I_c$ , the maximum value of the non-quenching current is determined by the contact length and does not depend on the value of the external magnetic field. In the case  $I < I_c$ , the critical current is determined by the value of the magnetic field at which the vortices begin to fill the entire length of the contact, and does not depend on the length of the contact. At the same time, with the growth of the external magnetic field, the critical value of the current decreases.

**Keywords:** long Josephson contact, magnetic field, undimmed current, vortices

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## ТОКОВЫЕ КОНФИГУРАЦИИ В ДЛИННОМ ДЖОЗЕФСОНОВСКОМ КОНТАКТЕ ВО ВНЕШНЕМ МАГНИТНОМ ПОЛЕ

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Рассмотрены токовые конфигурации в периодически модулированном длинном джозефсоновском контакте, находящемся во внешнем магнитном поле, для значений параметра пиннинга  $I$  больше и меньше критического ( $I_c$ ). Показано, что при  $I > I_c$  максимальное значение незатухающего тока определяется длиной контакта и не зависит от величины внешнего магнитного поля. В случае  $I < I_c$  критический ток определяется значением магнитного поля  $H_{\max}$ , при котором вихри начинают заполнять всю длину контакта, и не зависит от длины контакта. При этом с ростом внешнего магнитного поля критическое значение тока снижается.

**Ключевые слова:** длинный джозефсоновский контакт, магнитное поле, незатухающий ток, вихри

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## Introduction

The studies of the recent years brought us much closer to room-temperature superconductivity [1, 2]. Therefore, the problem of critical fields and currents, essential for practical application of superconductors, is now of utmost importance. For the classic superconductors, this problem is solved on the basis of Ginzburg – Landau equations. High-temperature superconductors (HTSC) are for the most part represented by granulated ceramics. They consist of adjoining granules separated by a dielectric. At the points where the granules touch each other, Josephson contacts are formed. All these Josephson contacts are nonlinear elements, which highly complicates the analysis of such media.

Moreover, the macrostructure of the HTSC is a cellular medium which leads to vortex pinning. Such complexities exclude the possibility of using Ginzburg – Landau equations to calculate current states in the HTSC. Other approaches have to be found to analyze currents in such media.

Recently, long Josephson contacts are in the focus of scientists' attention. On the one hand, this is connected with the possibility of developing artificial structures of such a type [3 – 5] that could allow testing theoretical predictions. At the present time, such structures in which the dielectric in the layer between the superconductors is replaced by a ferromagnetic are under study [6]. This keen interest to the indicated structures is also caused by the physical phenomena observed in them attributable to three-dimensional superconductors: the Meissner effect, emergence and interaction of vortices, vortex lattice formation, etc. Periodic modulating of a long Josephson contact allows us to study both the problems of vortex pinning, as well as the profile of the magnetic field penetrating the contact in a form of vortices. In this case, the mathematical problem is much simpler, than that for the three-dimensional superconductor, and can have an exact solution. Therefore, many works are devoted to the research of the long Josephson contacts

[7 – 12]. For example, papers [10, 11] considered a periodically modulated long contact placed into a constant external magnetic field parallel to the contact plane in case when the total current through the contact equals zero. Next, paper [12] presents a calculation of the current distribution in a contact with a given total current in a zero external magnetic field. However, in many cases the external field is not equal to zero, thus we should evaluate its influence on the superconductor current, in particular, on the critical current value.

The purpose of this study is to analyze a general case of a long Josephson contact with a non-dissipative supercurrent placed into an external magnetic field.

## Problem setting

An artificial periodically modulated long Josephson contact (PMLJC) (Fig. 1, *a*) is a thin dielectric layer (plane  $xz$ ) between two superconductors crossed by dielectric strips  $2l$  thick along the axis  $y$  and  $d$  wide along the axis  $x$ ; the strips are parallel to each other, infinite along the axis  $z$  and are periodically located along the axis  $x$ , at a distance of  $L$  from each other. The external magnetic field, as well the axes of the vortices are directed along the axis  $z$ . Fig. 1, *b* depicts the structure of the artificially created PMLJC [3]. In the regions between the strips, the value of the phase jump  $\varphi$  between the sides of the contact is changing slowly in the coordinates, while it changes dramatically in the strip.

In Fig. 1, *a*, the value of the phase jump mean with respect to the  $k$ th region between the strips is denoted as  $\varphi_k$ . Assume the phase jump in the region closest to the contact boundary equals  $\varphi_1$ , and as it advances deeper into the long contact the phase jumps is denoted as  $\varphi_2$ ,  $\varphi_3$ , etc. The distribution of the  $\varphi_k$  values describes the steady-state current.

We should consider the penetration of the magnetic field in the presented model of the contact at zero and nonzero currents, as well as the processes in the absence of any external magnetic field.

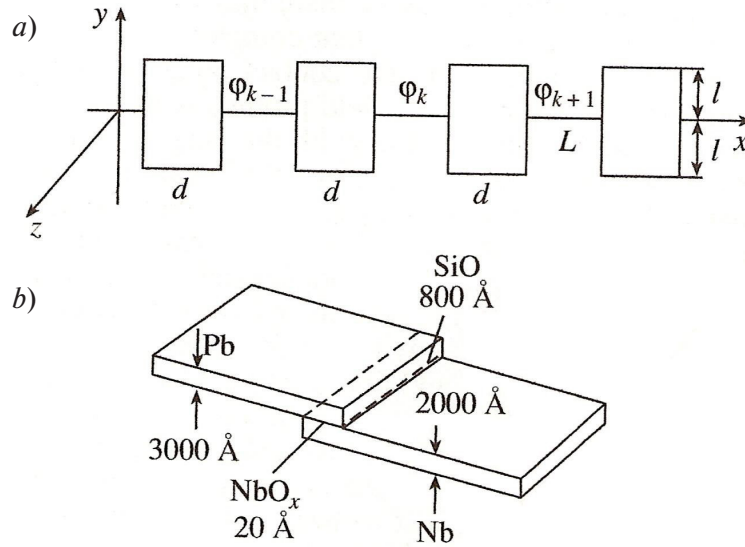


Fig. 1. Model of a periodically modulated long Josephson contact (a) and an example of the structure of such a contact created (b);  $\varphi_k$  — is the value of the phase jump mean with respect to the  $k^{\text{th}}$  region between the strips

### Penetration of magnetic field in a PMLJC at zero total current

First, let us consider a case when the total current through the PMLJC equals zero. We should account for the fact that in the presence of pinning, the distribution of the phase jumps and currents throughout the contact is ambiguous. The cause of this ambiguity lies in the obvious “hysteresis” of the situation: the form of the established distribution configuration depends on the previous history of the PMLJC operation, i.e. on how the contact achieved this state. For example, if it was placed in any arbitrarily small field before the cool down and the transition into the superconducting state, then the magnetic fluxes will penetrate the inner cells of the contact as well. If it was placed in a magnetic field in the superconducting state, then at small fields, it will have the Meissner configuration, i.e. the field penetrates only a narrow boundary layer. There is an enormous range of variations.

Let us solve the problem for the case of adiabatic switching of the field. The contact is already in the superconducting state, while the external field  $H_e$  is slowly growing from the zero value.

Since the current is equal to zero and due

to symmetry, we can conclude that near both boundaries of the contact, the currents are distributed similarly, but have opposite directions.

At small values of  $H_e$ , there is the Meissner configuration near the contact boundary, when the  $\varphi_k$  values decrease as the number goes up and equal zero in the depth of the contact. At the same time, the magnetic field created by the boundary currents completely compensates the external one in the depth of the contact. Such a situation can take place until the external field reaches a certain maximum possible value  $H_s$ , moreover, up until the moment the Meissner state is stable [13]. In the Type-I superconductors, the limit of the indicated state is defined by the equality of the energies of the normal and the superconducting states taking into account the energy of the shielding currents. If the external field exceeds  $H_s$ , then the sample goes into a normal state. In the case of the Josephson contact under consideration, this reasoning is inapplicable.

It is interesting to learn how the contact behaves, when the external field exceeds the  $H_s$  value and the Meissner state is impossible. As it is known, when the pinning is absent, there would be a periodic sequence of vortices in the contact.

In this case, we need to account for the occurrence of the pinning. The authors of papers [10 – 12] demonstrated (including the authors of this article) that the character of the vortex picture depends on the value of the so-called pinning parameter [12]:

$$I = 4\pi\mu_0 j_c Lld / \Phi_0,$$

where  $j_c$ , A/m<sup>2</sup>, is the critical current density of each point Josephson junction;  $\Phi_0$ , Wb, is the magnetic flux quantum; the sense of the geometric variables  $L, l, d$  is clear from Fig. 1.

At small values of the pinning parameter  $I$ , the situation is the same as with zero pinning, i.e. when the external field exceeds a certain value  $H_{\max} > H_s$ , vortices fill the entire contact from its boundary and to infinity. This is similar to the situation of the Type-II superconductor. At great values of the parameter  $I$ , with the growth of the field vortices advance from the boundary inside the contact, while the magnetic field in the depth of the contact remains equal to zero, i.e. the situation is similar to the Type-III superconductor behavior. On the basis of an approach developed in nonlinear dynamics [13, 14], paper [12] shows that there is a critical value of the pinning parameter  $I_c = 0.9716$  which divides these two cases.

In the frame of works [10, 11], we calculated the magnetic field profile inside the contact based on the analysis of continuous configuration transformation which takes place in the direction of its energy decreasing (to be precise, Gibbs energy decreasing). The process of reconstructing the configuration is regarded as a continuous transformation of the currents and phase jumps distribution. As the external magnetic field increases gradually starting from zero, there is a continuous transformation of the transient current distribution. In addition, in some configuration regions the currents decrease, while in the other they rise, i.e. the vortices are not like hard particles “pushed” inside by the field, it is as if they are “flowing” inside the contact.

An algorithm proposed in papers [10, 11] allowed finding such a configuration, into which the Meissner state transits when the external field exceeds the  $H_s$  value to a small degree, and ob-

serving its development during any further growth of the field. This method answers the question of the state stability as well.

The calculations showed that there is such a critical value of the pinning parameter  $I_c$  in the range of 0.95–1.00 which divides two possible modes of the magnetic field penetrating the contact. This result correlates with the critical value of the pinning parameter  $I_c = 0.9716$  found in papers [12 – 14]. In case of  $I > I_c$ , at any value of the external field there is a boundary current structure of finite length which completely compensates the external field in the depth of the contact. In our article [10], we present a detailed research of this case. In the depth of the contact, the magnetic field equals zero, at the border it decreases with the depth almost linearly, with some more or less considerable oscillations. Values of the slope coefficient are rational fractions that remain constant in the finite intervals of the parameter  $I$ . If a value of  $I$  goes beyond the upper limit of such an interval, the slope coefficient spikes and assumes the value of another rational fraction

As it was mentioned before, in the considered case of zero total current through the contact, the currents are distributed identically near both of its ends, but go in opposite directions. Moreover, the magnetic fields generated in the points symmetrical with respect to the middle of the contact are identical in value and direction.

Let us introduce normalized strength of the external field  $h = H_e / H_0$ , where  $H_0 = \Phi_0 / \mu_0 S$  is a value of the external field at which one quantum of the magnetic flux  $\Phi_0$  passes through each cell with the area  $S$ . Then the magnetic field inside the  $m$ th cell can be calculated using the formula [10]:

$$h_m = (\varphi_{m+1} - \varphi_m) / 2\pi.$$

Fig. 2 presents magnetic field reliefs inside the contact for different values of the  $I$  parameter at certain values of the external normalized magnetic field  $h$ . The calculation was based on the assumption that right and left structures do not intersect, i.e. the contact length was considered infinite. If the length is finite, then the right and the left diagrams intersect, a recalculation should

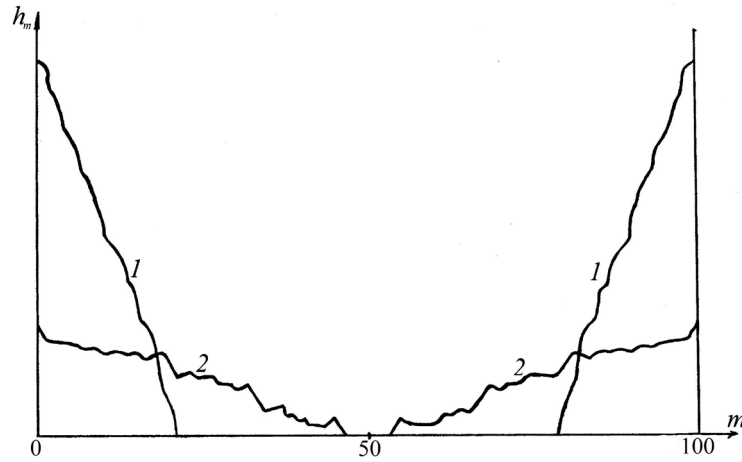


Fig. 2. Profiles of the magnetic field inside a long ( $m = 100$ ) contact for two values of the pinning parameter: 5 and 2 (curves 1 and 2 respectively) at certain values of the external magnetic field ( $m$  is the cell number)

be made to account for that. However, in any case the picture is symmetrical, and the vortex system is at a standstill.

If  $I < I_c$ , then the boundary structure can only exist till the value of the external field  $H_{\max}(I)$ . At  $H_e > H_{\max}$ , the length of the boundary configuration, calculated by the method used in paper [10] (i.e. assuming infinite contact length), grows continuously in the process of the calculation. This means that the calculation process can have infinite duration, while the field penetrates the contact to infinite depth. For a detailed analysis of the  $I < I_c$  case, we used the indicated method in paper [11] for the option of the limited contact length, as two symmetrical sequences of vortices coming from different ends of the contact must stop after they meet at its center.

Just as for  $I > I_c$ , in any case the picture is symmetrical, and the vortex system is at a standstill.

#### Nonzero current in the absence of the external magnetic field

At the zero external magnetic field, due to symmetry, one can argue that the currents are distributed in a similar manner near both ends and flow in the same direction. It results from the Ampère's circuital law that the field outside is expressed as

$$H = J/2b,$$

where  $J$ ,  $A$ , is the total current through the contact;  $b$ ,  $m$ , is the length of the contact along the axis  $z$ .

As in the “Penetration of magnetic field in a PMLJC at zero total current” section, we should find the configuration of the phase jump distribution in the boundary region minimizing Gibbs energy at the given fields on both sides of the contact (see this calculation in the aforementioned section). The critical value of the pinning parameter dividing the modes is the same, i.e.  $I_c = 0.9716$ . The difference in this case consists in the fact that the total current serves as the given parameter instead of the external magnetic field. If the value of the pinning parameter is less than critical ( $I < I_c$ ), then the boundary structure can exist only till the external field value  $H_{\max}(I)$ , i.e. till the value of the total current

$$J_{\max}(I) = 2b H_{\max}(I).$$

However, the main difference of the option under study from the one in the aforementioned section lies in the fact that vortices at different ends of the contact have opposite orientations. Until the contact length is great enough for these vortex picture not to overlap (either at  $I > I_c$  or at  $I < I_c$  and  $J < J_{\max}$ ), the whole configuration is static, the vortices are still. But, if the contact length is such that the pictures overlap, then in this region of overlap, the vortices of the oppo-



site orientations are attracted to one another and “annihilate”, i.e. are mutually eliminated. At the same time, the vortices that were contributing to the force holding the vortex configuration due to pinning disappear, and the picture stops being static. New vortices enter the overlap region, and the same happens to them as well. As the vortices move, the energy transfers into heat, and the currents stop being nondissipative.

The same phenomenon occurs, if  $I < I_c$ , but  $J > J_{\max}$ , when the vortex sequences from both sides tend to fill the entire length of the contact. These sequences have opposite orientations, and so they are mutually eliminated. They are replaced by new vortices, which motion leads to a transfer of the energy into heat.

### Contact in external magnetic field at nonzero current

Let us consider a general case. A contact is in an external magnetic field which was switched on adiabatically, i.e. the magnetic field strength was growing slowly and monotonically from zero to  $H_e$ . After that, an external current is passing through the contact which is growing slowly and monotonically from zero to  $J$ . The resultant field outside the contact on one side (to be specific, let us say, the right one) equals  $J + J/2b$ , and on the other side is equal to  $H_e - J/2b$ .

Let us denote the vortex orientation in the right structure as positive, while the reverse one is negative. Again, we face the problem of calculating the boundary region configurations.

**Case of  $I > I_c$ .** Introduce dimensionless parameters

$$j = \frac{J}{bH_0}, l_{\text{cont}} = \frac{L_{\text{cont}}}{L+d},$$

where  $l_{\text{cont}}$  is the contact length expressed as the number of cells.

Fig. 3 shows the magnetic field profiles inside the contact for the case when it is long enough, so prior to switching the  $j$  current the boundary structures (in dashed lines) do not overlap. With the growth of the  $j$  current the situation inside the contact near its opposite ends starts to differ dramatically. On its right end, in the neighborhood

of which the field equals  $H_e + J/2b$ , the configuration corresponds the minimum of Gibbs energy at the adiabatic increase of the magnetic field from  $H_e$  to  $H_e + J/2b$ , i.e. the field linearly decreases with the depth into the contact starting from  $H_e + J/2b$ . The length of this boundary structure equals  $(h + j/2)/k$  cells.

On the other end, the field was decreasing from  $H_e$  to  $H_e - J/2b$ , i.e. there was a counter motion of the external magnetic field. Therefore, due to hysteresis, the relief is no longer linear.

The following cases are possible.

1. If  $j > 4h$ , then  $h - j/2 < -h$ . In addition, the field subdues the pinning of the vortices already existing on the left end, as a result of which there is a vortex structure consisting of oppositely oriented vortices establishing on the left end (Fig. 3,a). The field profile on the left end of the contact is also linear, its length is equal to  $(j/2 - h)/k$ . If the contact length  $l_{\text{cont}}$  is less than the sum of the boundary structure lengths, in particular

$$(h + j/2)/k + (j/2 - h)/k = j/k,$$

then in their overlap region the oppositely oriented vortices are annihilated and replaced by new ones which cause the energy transfer into heat, and the currents stop being nondissipative.

2. If  $2h < j < 4h$ , then the vortices closest to the left end of the contact are negatively oriented, while the farthest are positively oriented (Fig. 3,b). In addition, the length of the negative part of the structure equals  $(j/2 - h)/k$  cells. With the current growth, the right structure expands, but the vortices are not annihilated on the boundary with the left one, since they have the same orientation. The left configuration is shrinking under the pressure of the right one, and its vortices of the positive orientation are moving left and cancel out the negative ones. If the contact length is less than the sum of the lengths, i.e.

$$(h + j/2)/k + (j/2 - h)/k = j/k,$$

then all positive vortices of the left structure are eliminated, and the negative vortices of the left structure are cancelling out the positive vortices

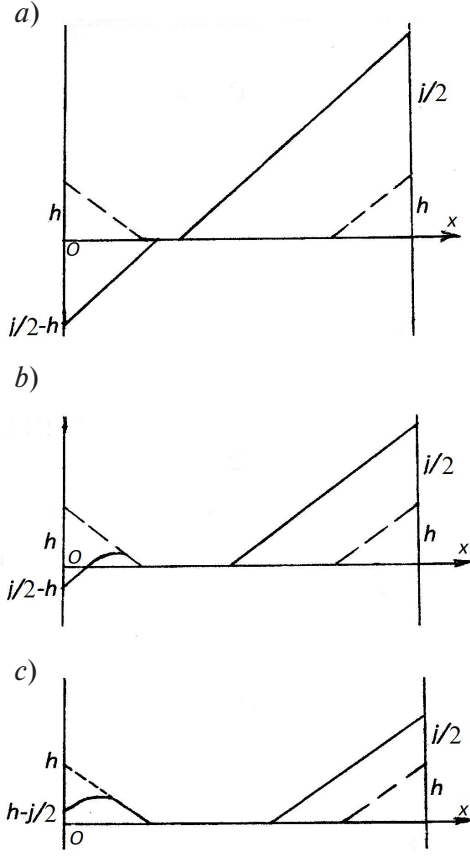


Fig. 3. Profiles of the magnetic field inside the long contact at the given external field  $h$  and different values of the current  $j$ :  $j > 4h$  (a),  $4h > j > 2h$  (b),  $j < 2h$  (c). Dashed lines show the profiles prior to the current switching ( $j = 0$ )

of the right one. Again, we find ourselves in the situation considered in clause 1.

Generalizing cases 1 and 2, we draw a conclusion that the continuous vortices motion with the energy-heat transfer occurs if the following conditions are met:  $j > kl_{cont}$ ,  $j > 2h$ , which can be written in a form

$$j > \max\{kl_{cont}, 2h\}. \quad (1)$$

This result includes the condition of the vortex picture motion  $j > kl_{cont}$ , obtained in paper [12], when  $h = 0$ .

3. If  $h > j/2$  (Fig. 3,c), then both vortex structures are oriented identically, therefore there is no mutual annihilation in case of them overlapping. But there is a question whether the picture

is stationary or not. If the boundary structures intersect at nonzero current, then the forces acting on a point of their intersection from both sides are equal. With the current increasing, the right end force grows, while the left end one drops, as the left structure was obtained due to a decrease in the magnetic field strength. Therefore, the right structure is moving the left one to the left. At the same time, the left configuration shrinks to such a state at which the force of its counteraction is maximal. This state corresponds to a linear dependence of the magnetic field on the depth with the same coefficient  $k$ .

As a result, boundary profiles of the magnetic field of finite length establish at both ends. The sum of the boundary structure lengths equals

$$\begin{aligned} & (h + j/2)/k + \\ & + |h - j/2|/k = 2h/k \text{ cells.} \end{aligned}$$

Fig. 4 demonstrates various possible cases. In case shown in Fig. 4,a, the structures don't intersect, the situation is stationary. In case presented in Fig. 4,b, the structures  $AFG$  and  $CMN$  would be at a standstill. We need to find whether the structures  $DBFG$  and  $BDMN$  would be at a standstill, and if the force exerted by  $BDMN$  on  $DBFG$  is sufficient to hold it at rest. It follows from the equilibrium of  $CMN$  that the force exerted by  $BDMN$  is equal to the force of  $CBD$ , which in turn equals the  $ABD$  force. The latter compensates the  $BDFG$  force, i.e. the  $BDFG$  structure would be at rest. The same goes for the  $BDMN$  structure. Similar consideration shows that in the case displayed in Fig. 4,c the structures are at a standstill as well.

However, in the case shown in Fig. 4,d (see its description below), there is no equilibrium, thus the vortices move right to left, and their energies transfer to heat.

Let us find such a ratio of parameters at which the situation  $d$ ) occurs:

$$\begin{aligned} FG &= h + j/2, \quad MN = h - j/2, \\ FP &= FG - MN = j. \end{aligned}$$

It follows from  $FP > RP$  that  $j > kl_{cont}$ . Taking account, that  $h > j/2$ , we come to a relation

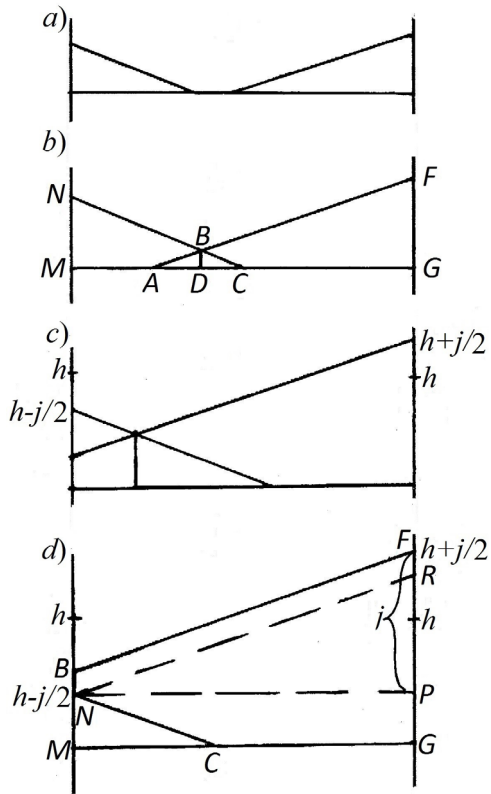


Fig. 4. Possible profiles of the normalized magnetic field strength in the contact. In the cases *a*, *b* and *c*, the structures are stationary; in the case *d* there is a right-to-left motion (letters denoting the points are introduced for convenience of reference in the text)

$$kl_{cont} < j < 2h. \quad (2)$$

By combining conditions (1) and (2), we obtain the condition for the nonstationary state at  $I > I_c$ :

$$j > kl_{cont}$$

(at any  $h$ ).

**Case of  $I < I_c$ .** In case of the indicated condition, various situations may occur.

If  $h + j/2 < h_s$ , then there are Meissner configurations lining up at both ends. If the contact is not too short, they do not overlap and the picture is stationary.

However, if  $h + j/2 < h_{max}$ , then at the right end of the contact, a sequence of vortices forms that tends to seize the entire length of the contact. The left structure is either a Meissner configura-

tion, or a finite boundary structure, or a chain of vortices, which also tends to seize the whole contact. But, since the field strength at the right end is greater than that at the left one, then the left structure cannot counteract the push from the right, the vortices move right to left with their energy transferring into heat.

In another case, when the conditions

$$h_s < h + j/2 < h_{max},$$

$$h_s < h - j/2 < h_{max},$$

are simultaneously satisfied, there are structures of finite length at both ends.

Nevertheless, their lengths are not proportional to the field values in the neighborhood of the boundary, as it was in the case of  $I > I_c$ . One can analyze the behavior of the boundary structures depending on the ratio of the structure and contact lengths. However, since the range of the magnetic fields from  $h_s$  to  $h_{max}$  is rather narrow, then we can confidently neglect this situation and assert that the picture stops being stationary as soon as the greater field  $H_e + J/2b$  exceeds the  $H_{max}$  value, i.e.  $h + j/2 > h_{max}$ .

In paper [12], at  $h = 0$  the condition of the vortices motion has the form  $j > 2h_{max}$ , which is a subresult of the obtained formula.

We can consider the case of 3D Josephson medium in a similar manner with some suppositions [15].

### Conclusion

Current configurations in a periodically modulated long Josephson contact located in an external magnetic field are considered for values of the pinning parameter  $I$  greater than and less than the critical one ( $I_c$ ). The study is based on the results of an analysis of continuous configuration transformation which takes place in the direction of its energy decreasing (to be precise, Gibbs energy decreasing). The process of reconstructing the configuration is regarded as a continuous transformation of the currents and phase jumps distribution.

It is shown that, if  $I > I_c$ , the maximum value of the nondissipative current is determined by the contact length and does not depend on





the value of the external magnetic field. In case  $I < I_c$ , the critical current is determined by the value of the magnetic field  $H_{\max}$  at which the vortices begin to fill the entire length of the contact,

and does not depend on the length of the contact. At the same time, with the growth of the external magnetic field, the critical value of the current decreases.

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