DOI: 10.18721/JPM.13406

INFLUENCE OF THE SLOPE ANGLE OF A LOCALLY HEATED SURFACE ON THE HEAT TRANSFER DURING BIFURCATION OF A PURE THERMAL PLUME

E.F. Khrapunov, Yu.S. Chumakov

Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russian Federation

The results of numerical simulation of the puffing regime of a pure thermal plume, which forms above a slightly inclined (with respect to the horizon) surface have been presented. The flow structure in the near-wall region was investigated and the fields of the velocity components were analyzed. Particular attention was paid to the analysis of the distribution of instantaneous, time- and surface-averaged values of the heat transfer coefficient α. Based on the presented results, it was concluded that there was an optimal slope angle of the surface maintaining the most efficient heat transfer with a developed periodic flow regime.

Keywords: natural convection, numerical simulation, bifurcation, heat transfer, puffing

Citation: Khrapunov E.F., Chumakov Yu.S., Influence of the slope angle of a locally heated surface on the heat transfer during bifurcation of a pure thermal plume, St. Petersburg Polytechnical State University Journal. Physics and Mathematics. 13 (4) (2020) 59–67. DOI: 10.18721/JPM.13406

Introduction

Analyzing the studies available to us on free-convection flows, we can see that significant progress has been made, first of all, regarding flows formed along vertical heated surfaces, when the vector of the free-fall acceleration **g** is parallel to the heated surface [1, 2]. This is largely due to the possibility to describe such flows within the boundary layer approximation that has appeared in the second half of the 20th century. Tools for numerically solving equations were constructed, allowing for detailed analysis of the characteristics of both laminar and fully developed turbulent boundary layer.

In turn, studies of flows formed over horizontal locally heated surfaces perpendicular to vector **g**, the so-called free-convection plumes, have long presented considerable difficulties [3, 4]. In this case, upward flow is observed over a small part of the surface (near its center), and a wall layer forms over the rest of the surface under the action of a favorable pressure gradient produced by the upward jet. Obviously, the presence of upward flow, as well as the section with a smooth transition from the wall layer to the upward jet does not allow the boundary layer approximation to be used to describe the plume as a whole. Such an approximation proves to be valid over a small section of the surface away both from its edge and from its center, as well as at a considerable distance from the disk surface in the region of fully developed upward flow. The section of the plume between these two regions (the transition region) cannot be fundamentally described within the above approximation. For a long time, the only way to numerically determine the flow characteristics in this region was to ‘merge’ the solutions obtained in the regions where the boundary layer equations were applicable.

At present, the significantly improved range of experimental and numerical methods allows to describe the flow as a whole, without dividing it into fragments, in particular focusing on analysis of changes in not only the averaged but also the instantaneous characteristics of the plume. For instance, a bifurcation of the solution was discovered in [6–9], as a result of which a stable steady regime is replaced by a stable self-oscillatory one, or puffing. The essence of puffing, which is a particular case of Hopf bifurcation, is periodic destruction of the near-wall layer by the evolving toroidal vortex structures [10]. While some studies [7, 9] considered the changes in some characteristics of the plume during bifurcation, the influence of parameters of the heated surface (for example, the incline relative to the horizontal position) on the flow as a whole is still poorly understood. The question about the mechanism of bifurcation remains unclear. The inverse effect of fluctuations in the plume on the characteristics of heat transfer with the heated surface is practically unstudied. At the same time, the structure of the flow near an inclined heated surface in the absence a puffing flow regime is described in sufficient detail [11–15].

The first study considering the influence of the slope angle of the heated surface on heat transfer was apparently [11], discussing the flow evolving near a heated rectangular plate tilted relative to the vertical position. It was found that tilting the surface to angles up to 40° had practically no effect on the characteristics of the formed free-convection boundary layer. As a result, it was concluded that the characteristics of heat transfer for an incline can be determined by simply replacing the modulus of vector **g** by the modulus of its component parallel to the surface in the well-known dependences obtained for vertical plates.

It was found in subsequent works that the flow characteristics vary only slightly at large slope angles as well [12]. However, there is a limiting angle, at which a cardinal change in the flow structure is observed: the flow is detached from the surface at a certain distance from the lower edge, and a pure thermal plume is formed. In particular, this change in flow structure does not allow using the dependences obtained earlier to describe the heat transfer at large surface slope angles relative to the vertical position.

One of the first works studying the structure of the flow over a heated surface slightly inclined with respect to the horizon is [14]. The surface temperature varied over a wide range, but since the experimental study was conducted in water, the maximum temperature did not exceed 368 K. The streamlines were visualized using aluminum powder illuminated by a special source. It was found that the transition point of the near-wall layer into an ascending jet remains on the surface at slope angles less than 10° relative to the horizontal position. At larger angles, the transition point coincides with the plate's boundary point. The most recent results (of those known to us) were presented in [15], also indicating that the surface inclination relative to the horizon has the greatest influence on the flow structure at angles up to 10° in the case of a steady pure thermal plume.

In our opinion, the key reason why it is small slope angles that have a major influence on characteristics of the plume is the weakening of upward flow during formation of the flow in the near-wall region. As the slope angle increases, the transition region from the near-wall to the upward flow shifts towards the edge of the surface elevated above the horizon, and relatively stable near-wall flow is formed over most of the surface at an angle of about 10°. The slope angle gradually ceases to affect its characteristics. A further increase in this angle in this case ultimately leads to the near-wall flow becoming predominant.

The goal of this study was to obtain and analyze separate instantaneous pictures of the dynamic and thermal fields of the puffing flow regime of a pure thermal plume formed over a slightly inclined (in the slope range from 0° to 5° with respect to the horizon) surface.

In view of this goal, we consider in detail the fragments of flow in the near-wall region and analyze the fields of velocity component distributions within one puffing period. A particular focus was on analysis of the distribution patterns of instantaneous (within a single oscillation period) and time-averaged values of the heat transfer coefficient α.

Problem statement

We simulated free-convection flow over a heated disk of radius R = 95 mm in an unbounded space. The angle γ, which makes the free-fall acceleration vector **g** with the disk surface, varied from 90° (corresponding to the case of a horizontal surface) to 85°. A schematic of the computational domain and the reference system of angle γ are shown in Fig. 1,*a*. The ANSYS Fluent software allowing to solve the Navier–Stokes equations in the Boussinesq approximation for a compressible medium was used for numerical simulation. The height of the computational domain was H = 1.5 m, the diameter of the domain was D = 1 m.

Based on the results of preliminary experimental studies, partially given in [10], it was concluded that the contribution of turbulent mixing to the processes occurring near the disk surface at moderate heating temperatures was insignificant. For this reason, laminar 3D flow was simulated in the presence of conjugate heat transfer between the heated disk and the ambient. The thermal properties of air in the computational domain remained constant (their values were taken at room temperature T_a), since their variation seems insignificant in the given temperature range.

A quasi-structured mesh including $1.1 \cdot 10^6$ elements was used for the main computations, refined towards the assumed boundaries of the plume and to the lower surface of the computational domain. A fragment of the computational grid is shown in Fig. 1,*b*.

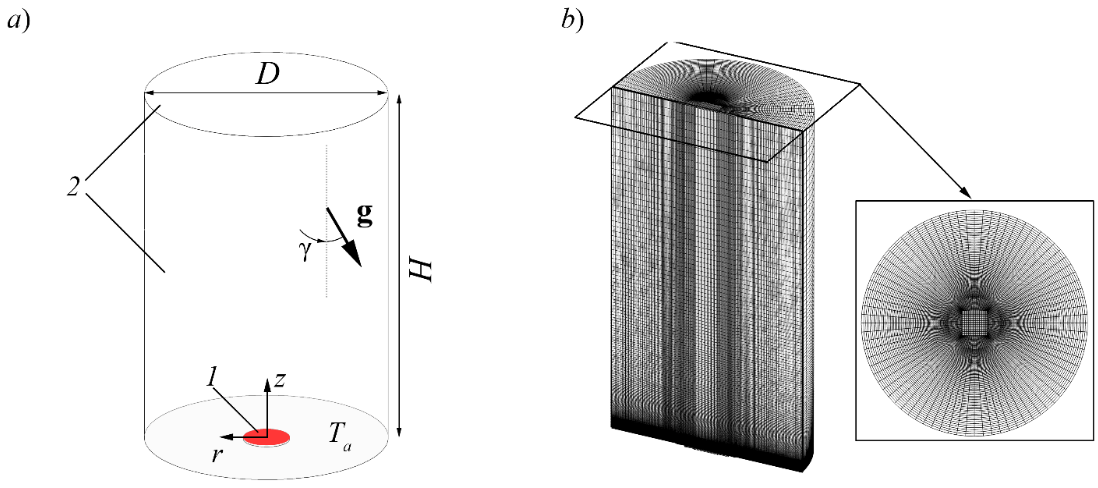

Fig. 1. Schematic of computational domain (*a*) and fragment of computational mesh (*b*): heated disk *1* (conjugate heat transfer conditions); free boundaries *2*

The boundary conditions were as follows (see Fig. 1, *a)*. The temperature of the disk's bottom surface was set equal to T_w = 383 K and remained constant during the simulation, the temperature of the horizontal surface of the computational domain also remained constant, equal to the ambient air temperature (T_a = 293 K). The temperature of the disk's upper surface was found from the condition of conjugate heat transfer with the air adjacent to the surface. Zero overpressure conditions were set at all other boundaries (side and top). Restrictions were imposed on the air flowing into the computational domain through these boundaries: the air temperature remained constant, equal to 293 K, and the velocity vector was always normal to the corresponding flow boundary.

In this study we analyzed a periodic flow regime stable over time. As a criterion that the regime was stabilized, we used data on the time variation of the temperature at the control points of the near-wall region, as well as the time variation of the heat transfer coefficient averaged over the disk surface (as shown, for example, in [10]).

The coordinate system used for representing the results is given in Fig. 1,*a*. The dimensionless components v_z* and v_r* are used to describe the velocity fields. The ratio of the kinematic viscosity coefficient ν to the disk radius *R* is used as the velocity scale.

Traditionally, the Grashof number Gr, determined by the disk radius and the characteristic temperature difference between the surface and the ambient air, is used as the main parameter to describe pure thermal plumes. Since the temperature of the disk's upper surface is determined by conjugate heat transfer conditions and is not known in advance, the temperature of the disk's lower surface T_w, set as boundary conditions within the framework of numerical simulation, is used to construct the Grashof number. The Grashof number is constructed accounting for the slope of the surface with respect to the free-fall acceleration vector **g**:

$$Gr_\gamma = \frac{g \cdot \beta \cdot (T_w - T_a) \cdot R^3}{\nu^2} \cdot cos(\gamma), \qquad (1)$$

where β is the coefficient of thermal expansion of air, taken as $3.6 \cdot 10^{-3}$ K^{-1}.

The given results correspond to the Grashof number Gr = $8.65 \cdot 10^6$, determined from dependence (1) in the case of a zero slope angle of the surface.

Analysis of results

Fairly detailed analysis of the characteristics of steady free-convection flows near inclined surfaces, as well as the fundamentals on the bifurcation of the solution, given in [10] for the case γ = 0°, allows to formulate the proposed scheme for the flow under consideration. The presence of a slope should lead to an acceleration of air in the near-wall region [16] and a shift of the transition region from near-wall to ascending flow toward the corresponding surface boundary. In this case, it is unclear whether the solution bifurcation will persist. At the same time, ascending flow shifted relative to the center of the heated surface may prove to be a noticeable obstacle to accelerated near-wall flow.

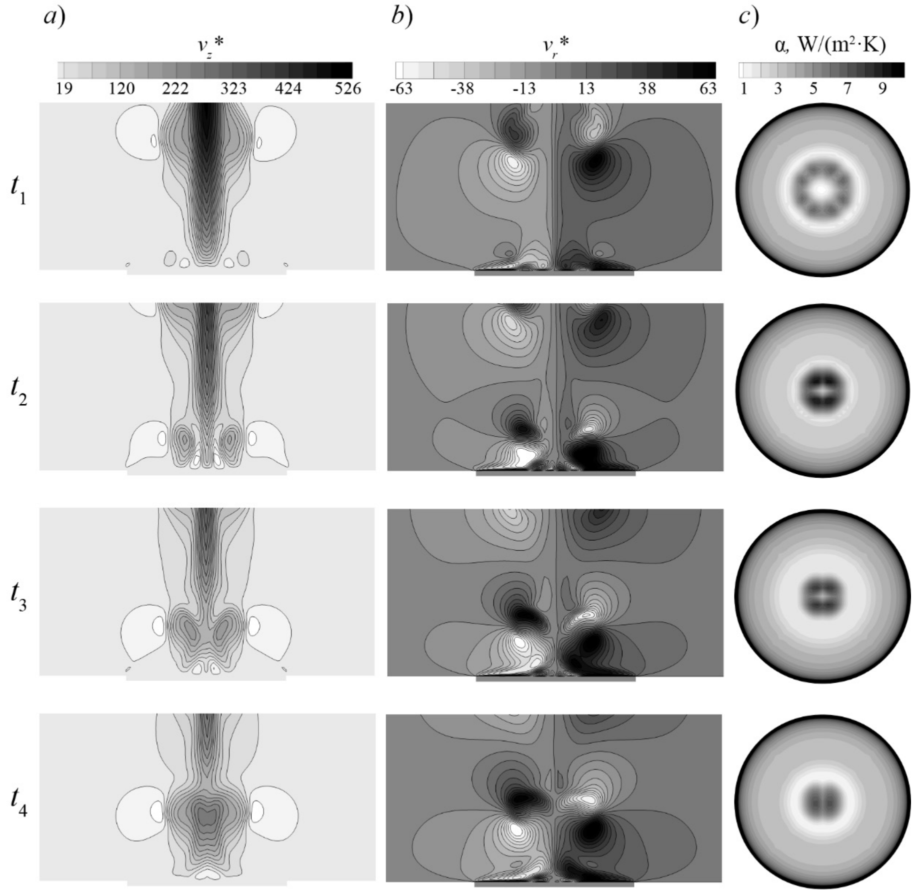

Fig. 2. Changes in axial (*a*) and radial (*b*) velocity components; changes in heat transfer coefficient (*c*) within one oscillation period when $\gamma = 0^o$

To analyze the effect of surface inclination on bifurcation, let us briefly consider the features of the flow for the case where the angle $\gamma = 0^o$ (see Fig. 1).

Fig. 2 shows the change in the flow characteristics in the *Orz* plane for $\gamma = 0^o$ (the case of a horizontal surface) within one plume oscillation period, as well as the distribution of instantaneous values of the heat transfer coefficient over the disk surface. It can be clearly seen that symmetric fields of the distribution of the axial and radial velocity components were obtained near the disk surface. It is also shown in [9, 10] that the flow in the near-surface region is axisymmetric. Axial symmetry is violated downstream (upstream of the disk surface) as a result of the development of instability during the rise of heated vortex structures in the surrounding cold ambient.

The formation of symmetric vortex structures leads to the emergence of regions where the heat transfer coefficient is increased over the disk surface, which can be clearly seen in Fig. 2.*c*. A region of increased heat transfer coefficient emerges because cold (temperature T_a) air is 'trapped' near the heated surface where the toroidal vortex separates. A similar conclusion is confirmed by the results of studies in [6, 8].

Notice that at each given moment of time, it is possible to detect a region on the surface of the disk where the heat transfer coefficient increases up to the values α^t_{max}, the maximum value of the coefficient on the disk surface at a given moment of time. In this case, the maximum value of the coefficient, α^t_{max}, for the full period of oscillations is over twice the α^t_{max} values of the coefficient in the case of a stable plume [17]. Within a single oscillation period, the region of increasing heat transfer coefficient is transformed: the radius of this region decreases, the region shrinks toward the center of the disk.

It should also be noted that an unusual structure that does not possess axial symmetry (see Fig. 2,*c*) appears on the disk surface (near the center) in visualizations of the heat transfer coefficient. This structure appears near the central prismatic block of the mesh (see Fig. 1,*b*), but this may be due to the general topology of

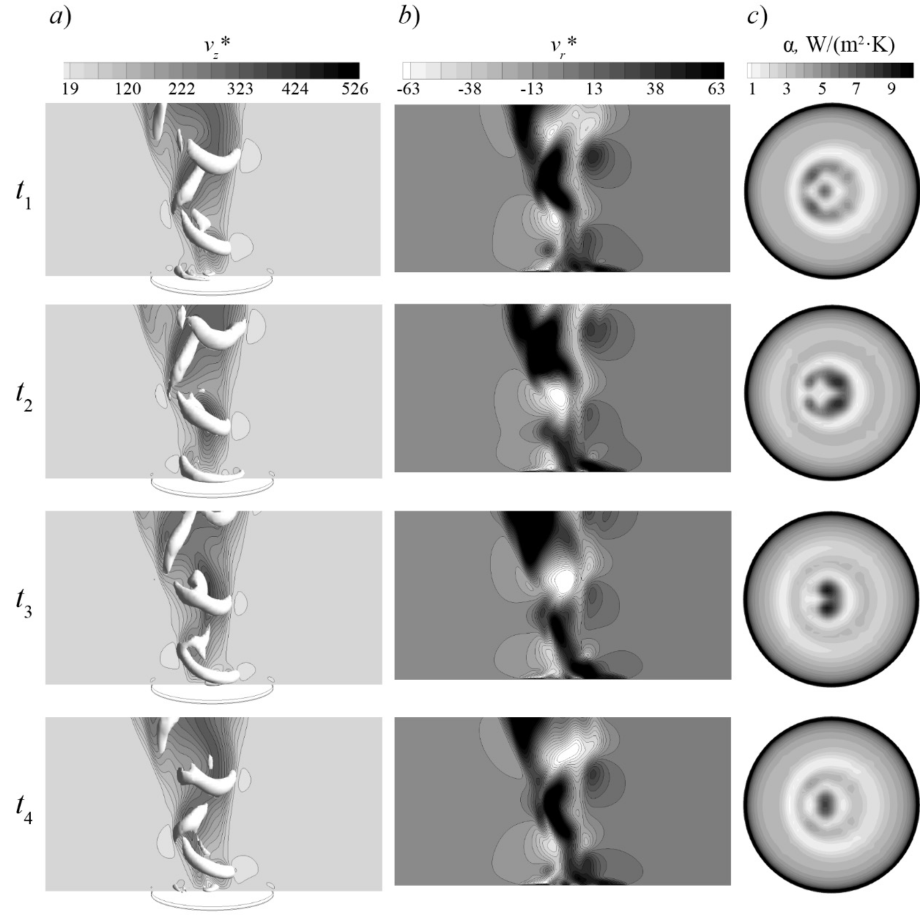

Fig. 3. Variations of axial (*a*) and radial (*b*) velocity components, as well as the heat transfer coefficient (*c*) at $\gamma = 1^o$ within one oscillation period

the mesh. We did not estimate the influence of mesh topology on the resulting solution as it is beyond the scope of our study.

Fig. 3 shows the change in the flow characteristics in the *Orz* plane for $\gamma = 1^o$ within one oscillation period. It can be seen that even a small surface slope angle leads to a noticeable change in the flow structure. For visual representation of the vortex structures, *Q*-criterion isosurfaces are shown on the fields of the v_z* component. Evolution of these isosurfaces for the case $\gamma = 0^o$ is described in [10]. We should emphasize that isosurfaces do not visualize the exact shapes and sizes of the vortex, only the part of the flow that has the highest vortex intensity. If we consider regions with different levels of vortex intensity, it is possible to carry out a general analysis of the changes in the characteristics of the pure thermal plume for an inclined surface.

The flow patterns shown in Fig. 3 allow us to conclude that when the horizontal surface is inclined, instead of an axisymmetric vortex, two areas with increased vortex intensity form, separated by the ascending flow. In this case, a vortex structure is formed at the initial moment of time, located ‘under’ the ascending flow. The air trapped in this area is heated stronger than the air in the near-wall layer forming on the other side of the ascending plume. As a result, there is a local increase in the Archimedean force, and, as a consequence, an increase in the axial velocity v_z*.

As vortices develop further, a ‘vortex track’ is formed in the space above the heated surface, where the vortex structures with different slopes alternate. Notably, at all moments of time within one period, the presence of multidirectional velocities v_r* is observed in the plume formation region. In this case, the surface inclination indeed, leads to flow acceleration and to the general dominance of positive radial velocities (see the coordinate and angle reference systems in Fig. 1) in the near-wall region.

Let us turn to the analysis of the peculiarities of the distribution of the heat transfer coefficient over the disk surface inclined by 1°. The size and shape of the region where the coefficient α increases significantly change as a result

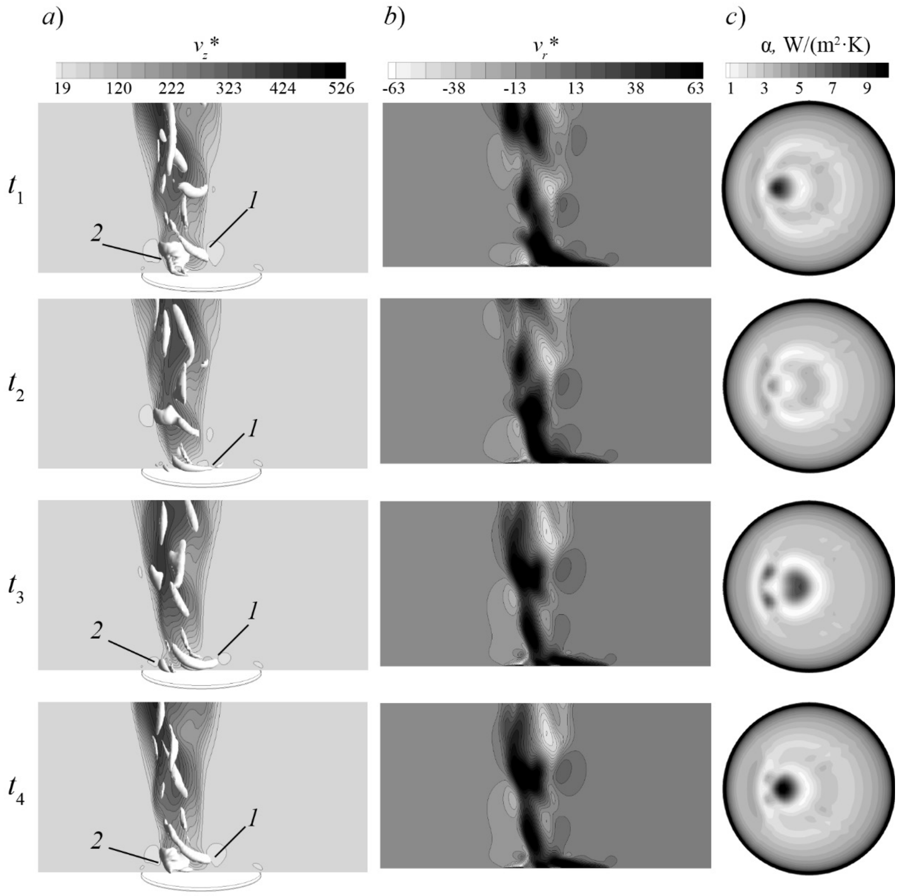

Fig. 4. Changes in the axial (*a*) and radial (*b*) velocity components, as well as the heat transfer coefficient (*c*) within one oscillation period at $\gamma = 3^o$. The vortex structures bending around the plume (*1*) and localized (*2*) are shown

of violating the symmetry of the flow in the near-wall region. Thus, in Fig. 3,*c*, the region with the maximum value of the heat transfer coefficient is observed first on the left and then on the right from the center of the disk.

Fig. 4 shows the change in the flow characteristics in the *Orz* plane for the value of angle $\gamma = 3^o$ within one oscillation period. Compared to the previous case ($\gamma = 1^o$), the delay in the formation of two vortex structures, separated by the ascending flow, becomes more noticeable. An elongated vortex structure can be distinguished in the plume, which bends around the ascending flow and a localized vortex, which appears to be ‘trapped’ between the inclined plume and the disk surface. The corresponding vortex structures are marked by numbers *1* and *2* in Fig. 4. Note that the transition point of near-wall into ascending flow at $\gamma = 3^o$ is visibly shifted from the center of the disk.

Analysis of the distribution fields of the heat transfer coefficient suggests that an increase in the surface inclination leads to a decrease in the size of regions with elevated values of the coefficient α. Within a single puffing period, it is possible to distinguish moments of time when there is no region of a pronounced increase in the coefficient α on the disk surface, which is not characteristic of smaller surface slope angles. Note also that the region of significant growth of α in this case is relatively small.

The value of the angle $\gamma = 5^o$ between the free-fall acceleration vector **g** and the disk surface (Fig. 5) is interesting because it seems to be a limiting case of a stable periodic flow regime. The transition point from the near-wall to ascending flow shifts practically to the disk boundary. A similar conclusion is drawn in [14]. Complex vortex motion is formed in the near-wall region, which, using the *Q*-criterion, can be represented as separate regions of increasing vortex intensity. As a result, there is reason to assume that a large number of small disconnected vortex structures are formed above the disk, which weakly interact with each other as they ascend in the surrounding air.

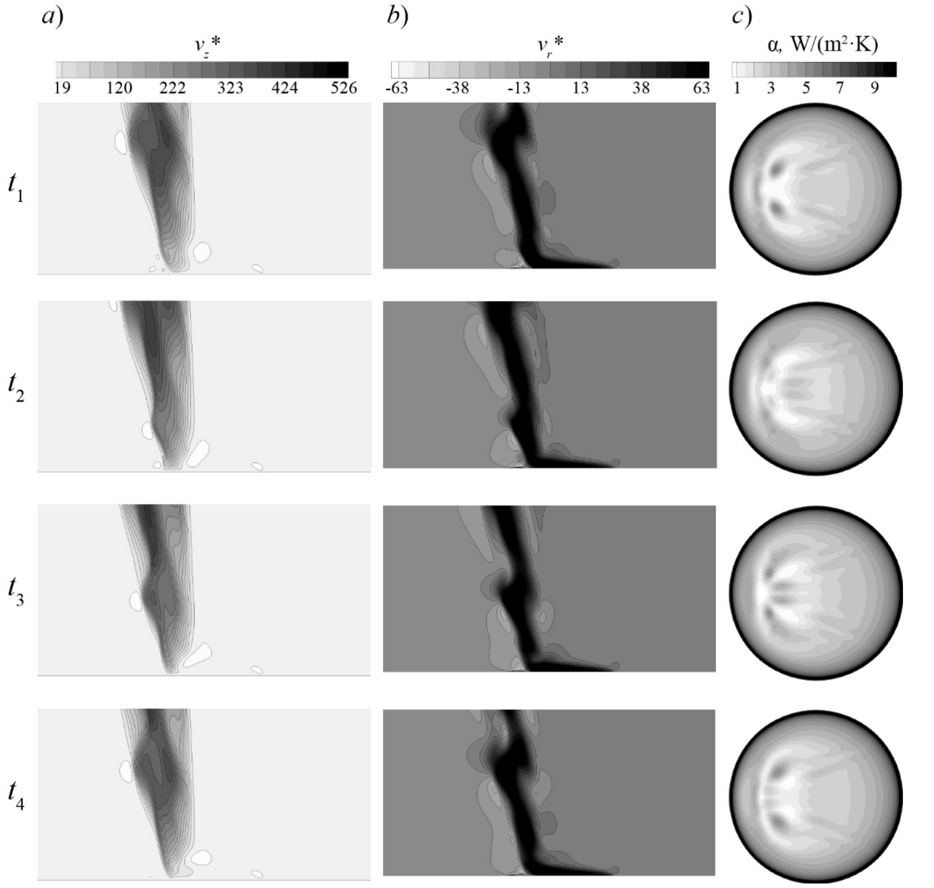

Fig. 5. Variations of axial (*a*) and radial (*b*) velocity components, as well as the heat transfer coefficient (*c*) at $\gamma = 5^o$ within one oscillation period

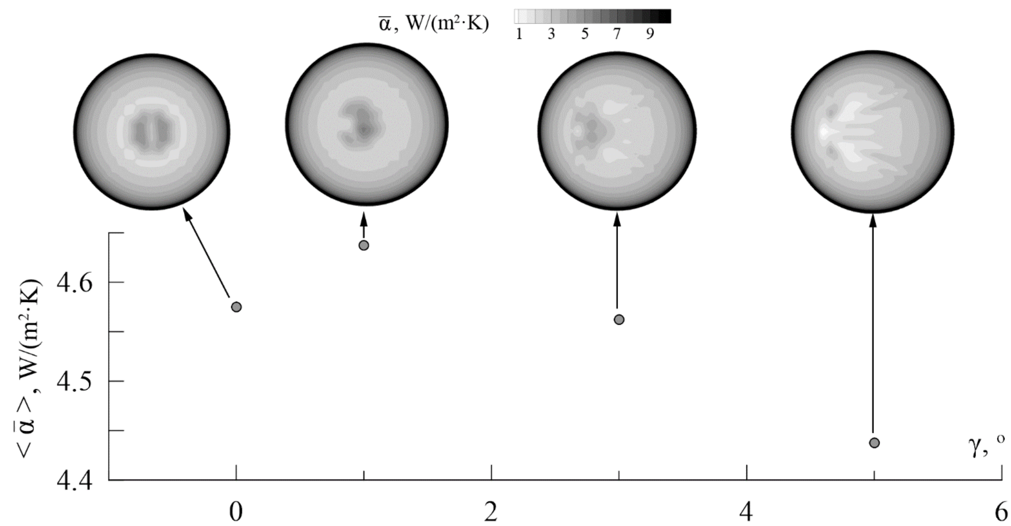

Fig. 6. Effect of surface slope angle γ on the values of the averaged heat transfer coefficients $\langle\bar{\alpha}\rangle$;
the distribution of $\underline{\alpha}$ over the disk surface is shown for each value (symbols $\langle\bar{\alpha}\rangle$)

Violation of the original (observed at $\gamma = 0^o$) toroidal vortex structure, and, consequently, the patterns by which cold air is forced towards the disk surface leads to the fact that it is impossible to identify a localized region on the disk surface where there is an increase in the coefficient α. Indeed, the regions with increasing values of the heat transfer coefficient turn out to be distributed over the surface (see Fig. 5,*c*). At the same time, there is no sharp increase in the values of α, which is characteristic of smaller angles.

Typically, it is not so much the instantaneous distributions of the local heat transfer coefficient but rather the distribution of the time-averaged coefficient $\underline{\alpha}$ over the disk surface, as well as the value of the time-averaged coefficient $\langle\bar{\alpha}\rangle$ integral over the disk surface that are of interest for practical applications. In this case, brackets denote averaging over the disk surface. The effect of the slope angle of the horizontal surface on the value $\langle\bar{\alpha}\rangle$ is shown in Fig. 6. In addition, the distribution of $\bar{\alpha}$ over the disk surface is shown for each value of $\langle\bar{\alpha}\rangle$.

It can be noted that the dependence of the coefficient $\langle\bar{\alpha}\rangle$ on the slope angle γ is not monotonic, there is a pronounced maximum in the vicinity of the slope angle $\gamma \approx 1^o$, i.e., an increase in the intensity of heat transfer is observed at this value.

Conclusion

In this paper, the influence of the slope angle on the characteristics of puffing in a pure thermal plume was investigated for the first time. It is established how exactly the change in the properties of the formed vortex structures affects the characteristics of heat transfer.

In our opinion, analysis of unsteady regime of a pure thermal plume over an inclined surface is of interest because such studies provide additional data on the flow model and establish the degree of influence of the plate orientation relative to the free-flow acceleration vector **g** on the characteristics of the unsteady process, whose main peculiarity is puffing.

The study revealed that a small (up to five degrees) inclination of the heated surface has a noticeable effect on the puffing process, which, ultimately, affects the heat transfer between air and the heated surface. There is reason to assume that further study of the influence of the orientation of the heated surface relative to the vector **g** on the given process will allow to develop effective ways to ‘control’ puffing, influencing the values of the local heat transfer coefficient over the required region of the heated surface.

REFERENCES

1. **Cheesewright R.,** Turbulent natural convection from a vertical plane surface, J. Heat Transfer. 90 (1) (1968) 1–6.

2. **Tsuji T., Nagano Y.,** Characteristics of a turbulent natural convection boundary layer along a vertical flat plate, Int. J. Heat and Mass Transfer. 31 (8) (1988) 1723–1734.

3. **Batchelor G.K.,** Heat convection and buoyancy effects in fluids, Quarterly Journal of the Royal Meteorological Society. 80 (345) (1954) 339–358.

4. **Lewandowski W.M., Kubsky P., Khubeiz J.M.,** Natural convection heat transfer from round horizontal plate, Heat and Mass Transfer (Warme- und Stoffubertragung). 27 (5) (1992) 281–287.

5. **Morton B.R., Taylor G., Turner J.S.,** Turbulent gravitational convection from maintained and instantaneous sources, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences (London). 234 (1196) (1956) 1–23.

6. **Jaluria Y.**, Natural convection: Heat and mass transfer, Pergamon Press, Oxford, 1980.

7. **Plourde F., Pham M.V., Doan K.S., Balachandar S.,** Direct numerical simulations of a rapidly expanding thermal plume: structure and entrainment interaction, J. of Fluid Mechanics. 604 (10 June) (2008) 99–123.

8. **Lopez J. M., Marques F.,** Instability of plumes driven by localized heating, J. of Fluid Mech. 736 (10 December) (2013) 616–640.

9. **Hattori T., Bartos N., Norris S.E., et al.,** Experimental and numerical investigation of unsteady behavior in the near-field of pure thermal planar plumes, Exp. Thermal Fluid Science. 46 (April) (2013) 139–150.

10. **Khrapunov E., Chumakov Y.,** Structure of the natural convective flow above to the horizontal surface with localized heating, Int. J. Heat and Mass Transfer. 152 (May) (2020) 119492 (12 p.)

11. **Rich B.R.,** An investigation of heat transfer from an inclined flat plate in free convection // Transactions – ASME: Journal of Heat Transfer. 75 (May) (1953) 489–499.

12. **Kierkus W.T.,** An analysis of laminar free convection flow and heat transfer about an inclined isothermal plate, Int. J. Heat Mass

Transfer. 11 (2) (1968) 241–253.
13. **Vliet G.C.,** Natural convection local heat transfer on constant-heat-flux inclined surfaces, J. Heat Transfer. 91 (4) (1969) 511–516.
14. **Fujii T., Imura H**., Natural-convection heat transfer from a plate with arbitrary inclination, Int. J. Heat Mass Transfer. 15 (4) (1972) 755–767.
15. **Kimura F.,** Fluid flow and heat transfer of natural convection at a slightly inclined, upward-facing, heated plate, Heat Transfer – Asian Research. 31 (5) (2002) 362–375.
16. **Shaukatullah H., Gebhart B.,** An experimental investigation of natural convection flow on an inclined surface, Int. J. Heat Mass Transfer. 21 (12) (1978) 1481–1490.
17. **Khrapunov E.F., Potechin I.V., Chunakov Y.S.,** Structure of a free convective flow over a horizontal heated surface under conditions of conjugate heat transfer, J. Physics: Conf. Series. 891 (2017) 012081.

Received 08.08.2020, accepted 05.10.2020.

THE AUTHORS

KHRAPUNOV Evgenii F.
Peter the Great St. Petersburg Polytechnic University
29 Politechnicheskaya St., St. Petersburg, 195251, Russian Federation
hrapunov.evgenii@yandex.ru

CHUMAKOV Yuriy S.
Peter the Great St. Petersburg Polytechnic University
29 Politechnicheskaya St., St. Petersburg, 195251, Russian Federation
chymakov@yahoo.com