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AN EXCITON IN THE SUPERSTRONG AND HYPERSTRONG MAGNETIC FIELDS

S.A. Vaganov, A.G. Panfilov, R.P. Seisyan

Ioffe Physical-Technical Institute of the Russian Academy of Sciences,

St. Petersburg, Russian Federation

The transformation of excitonic states in the raising magnetic field has been investigated beginning with the diamagnetic exciton (strong field) through excitonic magnetopolymer (superstrong field) to the QEL exciton (hyperstrong magnetic field). The effects induced by this transformation were considered. The capability of the exciton as a model object to study the hydrogen and positron atoms in superstrong and hyperstrong magnetic fields was also analysed.

Keywords: diamagnetic exciton, excitonic magnetopolymer, superstrong magnetic field, quantum electromagnetic limit, hyperstrong magnetic field

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Introduction

The Wannier–Mott exciton (large-radius exciton) is one of the most important objects of semiconductor physics. An immense number of studies and reviews are dedicated to this hydrogen-like formation consisting of an electron and a hole (see, for example, monographs [1, 2] and references therein).

The behavior of excitons in various semiconductor crystals, nano- and heterostructures has been described in detail for magnetic and/or electric fields, under intense laser excitation, etc. [1, 3].

The effect of a weak magnetic field on the Wannier–Mott exciton has been the subject of thorough theoretical and experimental studies. Substantial data have also been accumulated on the Zeeman effect (splitting of exciton energy levels, linear with the field) and the diamagnetic shift (states shifting to higher energies, proportionally to the squared field and radius of the corresponding exciton state).

The transition to strong field effects, such as Landau quantization and the production of diamagnetic excitons (DE) was considered in different semiconductor crystals and low-dimensional structures (see [2] for more details).

Thus, it can be argued that the behavior of the exciton in both weak and moderately strong magnetic fields is fairly well-understood. It would be therefore interesting to carry out experiments answering the following questions:

whether any new magnetic field effects are observed in exciton spectra in stronger fields;

which fields with the magnetic flux density $B > B_{ss}$ can be regarded as superstrong (B_{ss} here and below refers to the magnetic flux density of a superstrong magnetic field), radically changing the behavior of the exciton.

Since the Wannier–Mott excitons are atom-like, it would be natural to look for answers in atomic physics. However, direct comparisons cannot be drawn here, since the ‘hydrogen-like’ exciton is not completely similar to the hydrogen atom. The importance of this distinction is analyzed in in the first half of this study. The presence of a medium and a small reduced mass shift exciton effects by two to four orders of magnitude on the energy scale and by three to five orders of magnitude on the scale of magnetic fields relative to atomic ones. The role of the spin-orbit interaction turns out to be less significant, while the possibility of exciton production by light (or by electromagnetic field) and its short radiative lifetime in a crystal add important nuances to exciton behavior.

The second half of the study considers the hypothesized exciton effects in super- and hyperstrong magnetic fields. The present interest towards this problem is due to a number of circumstances.

On the one hand, considerable experimental progress has been achieved over the past 20–30 years: unique laboratory facilities have been created in Russia and in the world for obtaining super-strong magnetic fields [4]; at the same time, band and exciton parameters of many semiconductor materials (including new ones) were established or refined [5].

On the other hand, an obstacle faced by theoretical studies in magnetic-field atomic physics is that direct and even indirect experiments are impossible; for this reason, the fundamentals introduced half a century ago [6–9] saw no further development or conceptual modifications (see the review in [10] and references therein). Using increasingly powerful powerful mathematical methods (for example, second quantization) and exact analytical solutions practically does not alter the results obtained earlier.

Without attempting to consider theoretical models of atomic interactions in the magnetic field (since only experiments can ultimately confirm them), we believe that it is reasonable to apply these concepts to physics of atom-like quasiparticles in solids, i.e., excitons.

This paper is aimed at summarizing the data currently accumulated, making predictions for the new effects that can be expected to be observed in exciton optics and the conditions potentially favoring these effects. In particular, this concerns specific properties of semiconductor materials (existing, and most importantly prospective), as well as the magnitude of superstrong magnetic fields.

Effect of the medium. Scaling of magnetic-field effects

In contrast to an isolated hydrogen atom, the exciton exists in a medium with the dielectric constant $\varepsilon > 1$, reducing the Coulomb interaction by a factor of ε, and the exciton binding energy by a factor of ε^2 times. Screening of the Coulomb interaction also increases the exciton radius by a factor of ε, which determines many key properties of the exciton, for example, size quantization in nanostructures.

Furthermore, since the electron is not a particle with the mass m_e in a crystal, but rather a quasiparticle with an effective mass m^*_e, defined as

$$m^*_e = (2/\hbar^2)\, d^2E/dk^2$$

near the extremum on the dependence $E(k)$ for the corresponding conduction band, and the effective mass m^*_h of the hole, i.e., the 'nucleus' of a hydrogen-like exciton, is much smaller than the mass of a proton, the reduced exciton mass

$$\mu = m^*_e m^*_h/(m^*_e + m^*_h)$$

is much smaller than m_e, in contrast to the reduced mass of a hydrogen atom $\mu_H \cong m_e$.

Consequently, the Bohr radius of the exciton

$$a_{ex} = a_0\varepsilon/(\mu/m_e) \quad (1)$$

is 10–100 times larger than the Bohr radius of the hydrogen atom ($a_0 \cong 0.53$ Å), and the exciton binding energy

$$Ry_{ex} = Ry\,(\mu/m_e)/\varepsilon^2 \quad (2)$$

is 100–1000 times lower than the ionization energy of the hydrogen atom $Ry \cong 13.6$ eV.

Additionally, the mass of the nucleus for hydrogen and muonium atoms (proton p^+, mu meson μ^+) is much larger than the electron mass, and therefore the contribution of the magnetic moment of the nucleus to the energy of interaction of these atoms with the magnetic field can be neglected:

$$E_M(B) = e\hbar B/m_e + e\hbar B/m_{p,\mu+} \cong e\hbar B/m_e. \quad (3)$$

However, the contribution of the nucleus cannot be neglected for the exciton (as well as for the positronium atom), and the total energy of interaction of the exciton with the magnetic field becomes higher than that of the atom by a factor of m_e/μ:

$$E_M(B) = e\hbar B/m^*_e + e\hbar B/m^*_h = e\hbar B/\mu. \quad (4)$$

Summarizing all these characteristics, we find that the value of the magnetic flux density B_{Ryex} at which the interaction energy $E_M(B)$ between the magnetic moment of the exciton and the field becomes equal to the binding energy Ry_{ex} turns out to be lower by a factor of $\varepsilon^2(\mu/m_e)^{-2}$ for the exciton than for the hydrogen atom:

$$\begin{aligned} B_{Ryex} &= \mu Ry_{ex}/e\hbar = \\ &= m_e Ry\,(\mu/m_e)^2/\varepsilon^2 = B_{Ry}\cdot(\mu/m_e)^2/\varepsilon^2, \end{aligned} \quad (5)$$

with the difference reaching 3–5 orders of magnitude.

The values of the corresponding masses, binding energies and characteristic magnetic fields B_{Ry} for hydrogen-like atoms and fields B_{Ryex} for excitons in various semiconductor crystals are given in Tables 1 and 2. The values for excitons in Table 2 are approximate rather

Table 1

Values of basic physical quantities for hydrogen and hydrogen-like atoms

Atom	Nucleus	m_{nuc}/m_e	μ/m_e	Ry, eV	B_{Ry}, kT
Hydrogen H	Proton p^+	1836	1.000	13.6	117.5
Muonium Mu	μ meson μ^+	207	0.995	13.5	116.5
Positronium Ps	Positron e^+	1	0.500	6.8	29.0

Notations: m_{nuc}, m_e, μ are the masses of the nucleus, electron and muon, respectively; Ry is the binding energy, B_{Ry} is the corresponding characteristic magnetic field.

Table 2

Values of basic physical quantities for exciton in various crystals

Crystal	m^*_h/m_e	m^*_e/m_e	μ/m_e	a_{ex}, Å	Ry_{ex}, meV	B_{Ryex}, T
ZnS	1.76	0.340	0.285	16.5	49.00	120
ZnO	0.59	0.280	0.190	22.0	42.50	70
ZnSe	0.78	0.160	0.130	29.0	35.00	39.0
GaN	0.80	0.200	0.160	31.0	25.00	35.0
CdS	0.68	0.210	0.160	31.0	25.00	34.0
ZnTe	0.60	0.120	0.100	46.0	18.00	15.5
CdSe	0.45	0.110	0.088	61.0	11.50	8.70
CdTe	0.63	0.096	0.083	65.0	11.00	7.80
InP	0.60	0.079	0.070	95.0	6.00	3.60
GaAs	0.50	0.063	0.056	125	4.40	2.10
GaSb	0.28	0.041	0.036	231	2.00	0.62
InAs	0.41	0.024	0.023	350	1.35	0.27
HgTe	0.32	0.031	0.028	400	0.86	0.21
InSb	0.42	0.014	0.014	673	0.60	0.07

Notations: m^*_h, m^*_e are the effective masses of the hole and the electron, respectively; a_{ex} is the Bohr radius of the exciton; Ry_{ex} is the binding energy of the hole and the electron in the exciton, B_{Ryex} is the corresponding characteristic magnetic field.

than reference. The differences reach 10–15% in original studies; preference is given to self-consistent data: all values in Eq. (2) can be obtained experimentally, and they must be consistent, which is not observed in a number of publications.

The material medium in which the exciton exists ‘scales’ the magnetic-field effects: the action of the field B on the exciton is equivalent to the action of a much larger field,

$$B^* = \varepsilon^2/\mu^2 B,$$

on the hydrogen atom. As a result, the magnetic fields available in laboratories, equal to approximately 1 T, allow to simulate the behavior of hydrogen atoms in fields of 10^3–10^5 T in studies of excitons in semiconductor crystals.

There is another option for further reducing the “model” fields. Since optical methods are primarily employed to study diamagnetic excitons, the study of excited exciton states is the most informative. Applying mathematical processing to magnetoabsorption oscillations arising in the spectrum provides information about both the Landau levels and the parameters of the diamagnetic exciton in a specific material at a given field value [2]. The most pronounced oscillations correspond in this case to fieldless exciton states with the principal quantum number n = 3–5. At the same time, since there is vritually no difference in the symmetry and other properties of the states such as, for example, 1S, 2S, 3S, etc., another scaling is introduced: the effect of the magnetic field $B_{Ryex,n}$ on the exciton state nS is equivalent to the effect of a larger field

$$B_{Ryex,n=1} = n^2 B_{Ryex,n}$$

for state 1S.

Similar scaling takes place for states nP, nD, and so on.

Summarizing the above, we can assume that the behavior of the exciton state 4S, for example, in CdSe crystals in a magnetic field of 3–4 T (it is not at all a record) can be used to obtain information on the behavior of the ground state 1S of hydrogen atoms in megatesla-level magnetic fields that are superstrong for a hydrogen atom. Fig. 1 illustrates this point for a CdSe crystal (we previously published less detailed diagrams in [11]). It is evident that, for example, for exciton states n= 3, 4, the transition from the behavior characteristic for weak fields (Zeeman splitting, the energy shift of lines is quadratic (diamagnetic) with the field) to the behavior characteristic for strong fields (multiplet splitting, the shift is linear with the field (cyclotron)) occurs in cadmium selenide in fields noticeably smaller than B_{Ryex} = 9 T.

Thus, laboratory studies of the exciton using scaling of the magnetic field make it possible to simulate the behavior of the hydrogen atom in fields unattainable on Earth: fields of hundreds of thousands of Tesla can arise only in the vicinity of exotic objects of the Universe, such as neutron stars; constant magnetic fields up to 40 T (and pulsed fields up to 1500 T) can be generated in special laboratories.

In 1999, the State Prize of Russia in Science and Technology was awarded to physicists from the All-Russian Research Institute of Experimental Physics (Sarov, Russia) for their work on explosive generation of superstrong magnetic fields and solid state studies in magnetic fields of the 10-MG range [4]. A record field of 2800 T was reached, and the maximum field reached by the setup for optical experiments was 800 T.

There is another notable circumstance. The criterion for a strong field in atomic physics is the ratio between the interaction energy of the magnetic moment of an electron with an external magnetic field $E_M(B)$ and the energy of the spin-orbit interaction Δ_{SO}. The transition from a weak magnetic field to a strong one is accompanied by a transition from the Zeeman effect to the Paschen–Back effect in experiments. The criterion for a superstrong magnetic field B_{Ry} for a hydrogen atom is the ratio of the same interaction energy of the electron

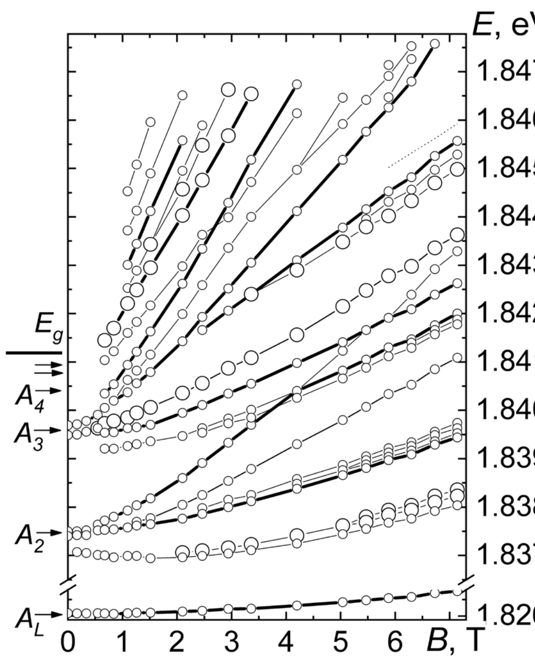


Fig. 1. Fan diagram of exciton states (series A) in a CdSe crystal in an external magnetic field $H \perp c$, $H \perp k$. Polarization $\boldsymbol{E} \parallel \boldsymbol{c}$.
The thicknesses of the lines and the radii of the circles reflect the intensities and widths of the absorption spectral lines, respectively

magnetic moment with an external field $E_M(B)$, and the binding energy of an electron with a proton in a hydrogen atom, Ry (the total potential energy of the hydrogen atom in the ground state, 1 Hartree = 2Ry, is often considered instead of Ry). Because the ionization energy of the hydrogen atom in the ground state is four orders of magnitude higher than the energy of the spin-orbit interaction, fields over 100,000 T are already superstrong for the hydrogen atom. Table 1 shows the values of B_{Ry} for hydrogen, for the case $E_M(B_{Ry}) = Ry$ and for hydrogen-like atoms with the corresponding values of the binding energy.

As a rule, $Ry_{ex} << \Delta_{SO}$ for an exciton, i.e., an exciton in a ‘superstrong magnetic field’ (from an ‘atomic’ standpoint) is essentially the same as in a ‘strong magnetic field’ (a diamagnetic exciton). The behavior of the DE is already sufficiently well understood, at least for magnetic fields that are transitional from weak to strong [2].

Since the most interesting features of the behavior of hydrogen-like atoms in superstrong magnetic fields (see below) are manifested in the range B = 10–1000B_{Ry}, in terms of our methodology, it is convenient to use the term 'superstrong field' for hydrogen-like muonium, positronium and exciton provided that $B > B_{ss} \approx 10\ B_{Ry}(B_{Ryex})$.

Hydrogen and hydrogen-like atoms in a superstrong field

Only theoretical representations exist for the behavior and transformations of hydrogen atoms in magnetic fields above 10^6 T ('super-strong field') [6–8]. A consistent point of different models is that there is a geometrical rearrangement of the atom: it is initially spherical, subsequently contracting along the axes perpendicular to the direction of the magnetic field and turning into an ellipsoid (spindle or cigar); see Fig. 4. The ionization potential of a hydrogen atom presumably increases to thousands of electron volts in gigatesla fields, i.e., grows by several orders of magnitude [6, 8].

It is worth repeating that the dependences obtained in [6] and [8] are almost identical, even though more than 20 years have passed between the publication of these studies.

Fig. 2 shows how the binding energy of excitons Ry_{ex} would change in magnetic fields lower than the atomic ones by many orders of magnitude. The given dependences are not the result of exact calculations: here we only assume that the dependence $Ry_{ex}(B)$ and the theoretically predicted dependence $Ry(B)$ of atomic physics considering magnetic fields [6] are similar (i.e., have the form ~$\ln^2(B/B_{Ry})$). For convenience,, only a few crystals are shown: from wide-gap semiconductors to narrow-gap ones.

It is even less important to refine the dependences $Ry_{ex}(B)$ than in atomic physics: although the changes shown in Fig. 2 are radical (the exciton binding energy should change by several times in experimentally achievable magnetic fields), it is still difficult to observe them experimentally. As as a matter of fact, these changes occur as the electronic spectrum is rearranged, with Landau levels forming for carriers of both types. In this case, the dependence of the energy of the Landau levels on the magnetic field is linear:

$$E_{L,n}(B) = E_M(B)\cdot(n_L + 1/2), \quad (6)$$

and it is much stronger than the logarithmic dependence of the binding energy of an exciton (in this case, already diamagnetic) on the field, so the dependence $Ry_{ex}(B)$ is actually imperceptible against $E_{L,n}(B)$.

Considering the behavior of several levels at once will hardly help explain the changes

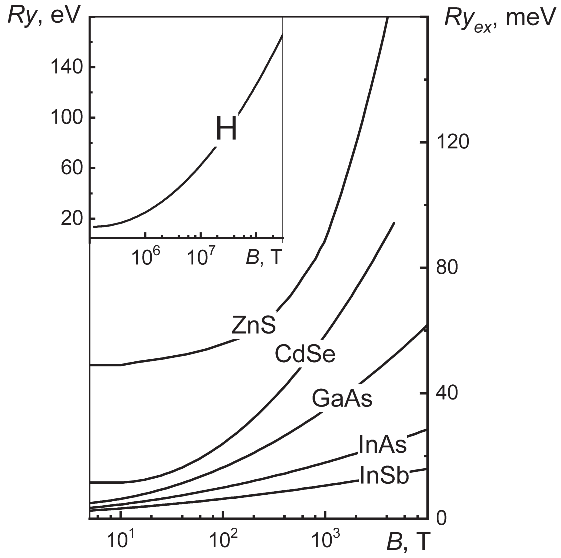


Fig. 2. Dependences of exciton binding energy on the magnetic field in various crystals (the inset shows a similar dependence for a hydrogen atom)

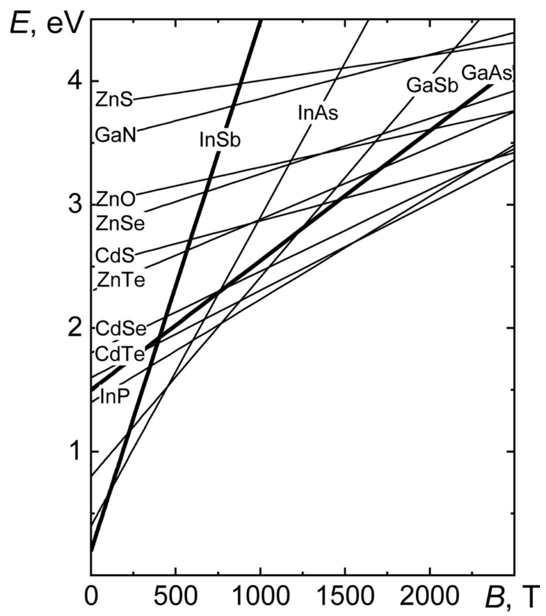


Fig. 3. Dependences of transition energy between the ground ('zero') Landau levels of an electron and a hole on the magnetic field (several dependences are highlighted for clarity)

occurring: the Landau levels are equidistant only in the first approximation. Furthermore, the binding energy of the exciton attached to each specific Landau level depends on the value of n_L [2]. Consider, for example, indium antimonide (InSb), one of the first materials studied (the effect of oscillations of magnetoabsorption in InSb was first analyzed in detail back in the 1950s–1960s): as the superstrong magnetic field B_{ss} amounts to several Tesla due to the extremely low binding energy of the exciton, the magnetic field dependences in InSb characterizing the energy of the diamagnetic exciton levels in fields up to 8 T (i.e., about 100 B_{Ryex}) were described without relying on the concept of increasing the exciton binding energy in such fields [2].

Interestingly, if the quantity E_M from Eq. (4) is used in Eq. (6), it is evident (Fig. 3) that for the value of the transition energy between the ground Landau levels of an electron and a hole ($n_L = 0$) for most semiconductors in an ultrastrong magnetic field is shifted to the visible part of the spectrum, convenient for experimental observation.

However, the most intriguing effect of a superstrong magnetic field is possibly the theoretically predicted modification of interatomic interaction which in the case of the exciton should manifest itself primarily in exciton luminescence rather than absorption spectra; the study of these spectra has received far less attention.

Interatomic (interexciton) interaction in a superstrong field

Neutral atoms strongly elongated along the direction of the field will have a large electric quadrupole moment, so they will begin to strongly attract each other along the magnetic field lines, weakly repelling in directions perpendicular to the field. The so-called magnetopolymers may form as a result, which are chains of spindle-like atoms modified by the field, arranged along the lines of the field (Fig. 4). Individual atoms disappear in such a polymer with a further increase in the field, while electrons are collectivized into a single elongated cloud, with protons that are atomic nuclei located along its axis [9].

This theoretical picture may have already been confirmed experimentally by the detected thermal emission spectra of neutron stars. A part of this radiation cannot penetrate the atmosphere of a neutron star, and absorption bands are observed in the X-ray region of the spectrum. The main band is typically located in the region $E_a \approx 200 - 700$ eV [12], and several less pronounced bands are sometimes observed at energies $2E_a$, $3E_a$ etc. Initial attempts to relate these bands to the cyclotron frequencies of electrons or protons, or to oxygen in the stellar atmosphere were unsuccessful.

However, this absorption can be explained

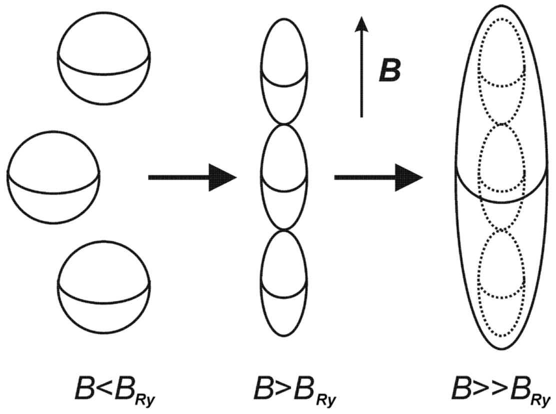


Fig. 4. Formation of atomic/excitonic magnetopolymers in superstrong magnetic fields

by ionization of hydrogen atoms in a superstrong magnetic field: an increase in the ionization potential to 200–700 eV (15 – 50*Ry*) is possible in fields of the order of $10^8 - 10^9$ T, i.e., specifically in the fields characteristic for old neutron stars (much smaller fields, namely, 10^6–10^7 T, are typical for young neutron stars (supernova remnants)).

An X-ray quantum can simultaneously ionize several atoms making up the magnetopolymer; moreover, collectivization of electrons occurs in such fields (as pointed out above). As a result, bands should be observed at energies E_a, $2E_a$, $3E_a$, etc., up to a relatively low binding energy of atoms into a polymer. This can explain the presence of additional absorption bands in the spectrum.

Interestingly, even though the surface temperature of neutron stars approaches a million degrees ($kT \approx 100$ eV), hydrogen atoms in their atmosphere should remain predominantly unionized, since $kT < E_a$. In other words, the atmosphere of an old neutron star hypothetically contains atoms (albeit unusual), molecules (as the atmosphere of the Sun), and magnetopolymers.

Going back from atomic physics to the physics of excitons, we should note that biexcitons have been studied only in several types of crystals and in fact in a relatively weak magnetic field, while triexcitons, etc., are merely a theoretical idea. It has already been discovered that, in contrast to an exciton, a biexciton does not transform into radiation in a one-photon process

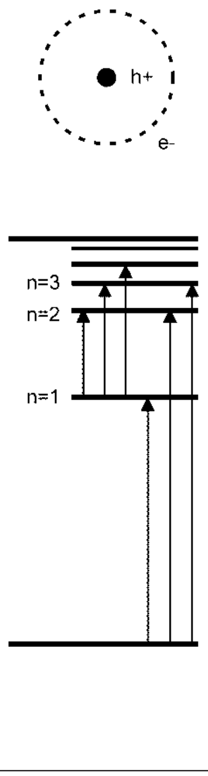
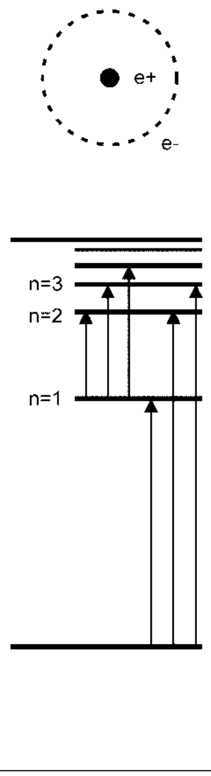
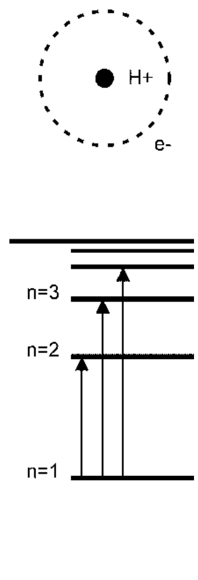
$$X_2 \rightarrow h\nu,$$

where X is an exciton, but rather does so in two stages [13]:

$$X_2 \rightarrow X_{n=2} + h\nu_1 \rightarrow h\nu_1 + h\nu_2.$$

Table 3

Comparison of exciton with hydrogen and positronium atoms

Characteristic	Exciton (X)	Positronium (Ps)	Hydrogen (H)
Structure	Hole + electron ($h^+ + e^-$)	Positron + electron ($e^+ + e^-$)	Proton + electron ($p^+ + e^-$)
Reduced mass	0.01 –0.30m_e	0.5m_e	$\approx m_e$
Size, Å	10–1000	≈ 2	≈ 1
Binding energy, MeV	1–100	≈ 6800	≈ 13600
Dimerization energy, eV	Biexciton X_2: 0.0001–0.0010	Dipositronium Ps_2: 0.4	H_2 molecule: 4.5
Annihilation (Energy)	1, 2,... photons; $h\nu \approx E_g \approx 0.1 - 10$ eV)	2, 3,... photons; $h\nu_\Sigma = 2m_ec^2 \approx 1$ MeV)	impossible; $H^+ + e^- = n^0 + \nu_e$ ($m_Hc^2 \approx 1$ GeV)
Lifetime, s	10^{-5}–10^{-7}	Parapositronium $1.25\cdot10^{-10}$ Orthopositronium $1.43\cdot10^{-7}$	Stable
Optical transitions			

Furthermore, annihilation of a positronium atom (and even more so a molecule) is also a multiphoton process (see Table 3).

Since the biexciton theory is poorly developed even for weak fields, not to mention strong and superstrong ones, studies of high-density excitons (which is a necessary condition for observing bi- and polyexcitons in weak magnetic fields) in superstrong magnetic fields can yield new, unexpected and interesting results.

Production of an exciton by light and its short lifetime

An extremely important difference between an exciton and a hydrogen atom is that the exciton can be produced by light.

In this aspect, exciton is much more similar not to the hydrogen atom but to the exotic positronium atom (Table 3). Table 3 also shows one more difference of an exciton (and a positronium atom) from a hydrogen atom, its (their) instability. Increasing the binding energy of the exciton, the magnetic field also increases the simultaneous lifetime of the electron and hole. However, it is unclear whether this trend will persist in extremely strong magnetic fields; if it does persist, it may contribute to producing a high-density exciton gas. In this case, the gas may possibly be obtained cold, i.e., the conditions for Bose–Einstein condensation of excitons can be generated [14].

A relative ‘overheating’ of the electronic subsystem usually takes place in the absence of a field or in magnetic fields that are not very strong. The reason for this is that the exciton gas excited by a short light pulse does not always have time to reach equilibrium, and the temperature of the exciton subsystem turns out to be uncoordinated with the temperature of the crystal lattice.

Aside from the exciton (electron-hole pair), many quasiparticle excitations can be generated in a crystal. They include, for example, phonons corresponding to the collective vibrations of the atoms in the crystal lattice. Since the energy of acoustic phonons has a lower limit of zero, these excitations can be produced in any magnetic field. Figuratively speaking, an electron and a hole moving along Larmor orbits emit acoustic phonons around them. The stronger the field, the more phonons are produced and the more the lattice overheats.

The larger the magnetic field that is to be generated, the less time there is for this task. The time for the field to reach its maximum value in the aforementioned record experiments was 5–20 μs; after that, a thermal explosion occurred, followed by destruction of both the magnet and the sample.

This means that the situation is different in hyperstrong magnetic fields. The exciton system may not have time to heat up before the sample is destroyed and will remain relatively cold. A high-density cold excitonic gas is an ideal situation for Bose-Einstein condensation [14].

Hyperstrong magnetic field

Theory [6–9] predicts for magnetic fields above $B_{hs} \approx 10^{10}$ T (B_{hs} here and below refers to the flux density of a hyperstrong magnetic field) that the hydrogen atom should be no longer spindle-shaped but rather needle-shaped, i.e., the ratio of the longitudinal to transverse linear dimensions of the ellipsoid should reach 200.

However, the proposed model is apparently no longer applicable in such a strong field. Evidently, the energy of interaction of the electronic magnetic moment of the hydrogen atom with the external magnetic field $E_M(B)$ exceeds the level of 10^6 eV in the given hyperstrong field, i.e., the energy becomes sufficient for the production of electron-positron pairs from vacuum, and the so-called quantum electrodynamic Schwinger limit (QEL, $B_{cr} = m_e^2c^2/e\hbar \cong 4.4\cdot10^9$ T) is reached, above which absolutely unpredictable effects can be observed (see below).

To produce an electron-positron pair due to the interaction of a magnetic field with a positronium atom rather than a hydrogen atom taking into account the Landau quantization (electrons and positrons are produced in a strong magnetic field only in states corresponding to the ground Landau level), the magnetic field B must become such that the energy of interaction with it is

$$2[E_{\mathrm{L},0e+}(B) + E_{\mathrm{L},0e-}(B)]$$

(compare Eqs. (3) and (6)) is higher than simply

$$m_ec^2 + m_{e+}c^2 = 2m_ec^2,$$

namely, it must follow the expression

$$E = 2m_ec^2 + E_{\mathrm{L},0e+}(B) + E_{\mathrm{L},0e-}(B) = 2m_ec^2 + E_M(B)/2. \quad (7)$$

In other words, the required interaction energy $E_M(B) = E$ must be higher than $4m_ec^2$, and the corresponding hyperstrong magnetic field must be

$$B_{hs} = B_{cr} \cong 4.4\cdot10^9 \text{ T}.$$

Since the production/annihilation of an electron-positron pair is a fundamentally multiphoton process (see Table 3), the required energy can either be ‘collected’ from two electrons of one atom, or obtained by exciting already existing bound electron-positron pairs (positronium atoms). In the latter case, cascade pair generation occurs. We should also note that a multiphoton process implies that 'subbarrier' production of an electron-positron pair is possible (similar to absorption of light below the absorption edge in semiconductors).

An energy that is a million times less than the energy for the production of an electron-positron pair in vacuum is required for the production of an electron-hole pair in a crystal. It is equal to the energy E_g that is the band gap, and varies from several electron volts for wide-gap semiconductors to tenths of electron volts for narrow-gap semiconductors. Similarly to Eq. (7), we have the following criterion for an exciton in the magnetic field:

$$E_M(B) > 2E_g,$$

$$B > B_{hs} = 2\mu E_g/e\hbar.$$

The corresponding dependence of the interaction energy on the magnetic field in the crystal, taking the form

$$E_M(B) = E_g + E_{\mathrm{L},0h}(B) + E_{\mathrm{L},0e}(B),$$

was already shown in Fig. 3 for different semiconductors.

The exciton in such a hyperstrong magnetic field should begin to emit not only phonons (see above) but also new electron-hole pairs.

However, before we consider this possibility, it should be borne in mind that there are also optical phonons, whose energy is comparable to the binding energy of an exciton (in contrast to acoustic phonons with their low energy). The interaction of excitons with optical phonons was analyzed in detail theoretically and experimentally as early as the 1950s–1970s (see, for example, [15]). The dependences of exciton-phonon interaction on the degree of ionicity of semiconductor crystals, their defect structure, temperature, etc. were described.

In fact, the actual motion of the Wannier–Mott exciton in the crystal was proved in a study of 'phonon repetitions', i.e., spectral lines caused by the emission of phonons by moving excitons (energy relaxation of excitons). A characteristic 'Maxwellian' shape of these lines corresponds to the energy

$$E_X - nE_{\mathrm{LO}},$$

where E_X is the energy of the ground exciton state ($E_X = E_g - Ry_{ex}$); E_{LO} is the optical phonon energy; $n = 1, 2$.

The given shape of the lines and their temperature dependence were described up to $n= 4$ [16].

However, the modification of exciton-phonon interaction in a magnetic field still remains virtually unexplored, especially for the case of a strong field, when the theoretical parameter $E_M(B)/Ry_{ex}$ grows large.

Thus, new exciton-phonon effects can be expected in superstrong and more so in hyperstrong magnetic fields. At the very least, numerous exciton lines of 'phonon repetitions' should appear in the luminescence spectra.

Going back to potential production of not only phonons but also new electron-hole pairs ('cloning' of excitons, so to speak) in a hyperstrong magnetic field, by analogy with optical phonons, '(e-h)- repetitions' of the ground exciton state bound to the zero Landau levels of the electron and hole can be expected to appear. However, the theory for this is completely untenable, leaving space for interpretation.

The energy $E_M = 2E_g$, for example, in an InSb crystal ($E_g \approx 0.2$ eV), corresponds to the magnetic field $B_{hs} \approx 50$ T, which is achievable in modern laboratories. Table 4 shows the values of B_{hs} for several other materials.

Even more surprising exciton effects are also possible. An attempt was made in [17] to hypothesize what happens to a substance at magnetic fields above the QEL. The vacuum supposedly becomes polarized, and the light entering the region with such a field changes its speed and wavelength. Consequently, birefringence is observed, similar to, for example, anisotropic crystals. Moreover, high-energy photons begin to spontaneously split into several quanta or merge into one.

Notably, such splitting/merging in a hyperstrong magnetic field can be quite simply described in terms of exciton physics as follows: the quanta $h\nu \approx 2E_g$ produce excitons ($e + h$ pairs), these pairs/excitons form a magnetic molecule (magnetopolymer, see Fig. 4) whose extended 'hypernucleus' is surrounded by collectivized electrons. This magnetopolymer of N excitons annihilates with a certain probability, i.e., a quantum of light an energy $Nh\nu$ is emitted. Or, vice versa, a quantum $Nh\nu$ generates a magnetopolymer of N excitons; each of these excitons annihilates, emitting a quantum of light with the energy $h\nu \approx 2E_g$.

Thus, the emission spectrum of semiconductors in a hyperstrong magnetic field excited by light with an energy of about N_1E_g should exhibit several bright emission lines separated by an energy of about N_2E_g, due to both the above-described 'splitting/merging' of light quanta, and by cascade generation of electron-hole pairs. The spectrum of the radiation appearing will be complex, with line shifts due to the binding energies of the biexciton, the magnetopolymer, reflecting transitions to the non-ground state of the exciton, etc.

It is evident that the QEL imposes a constraint on the increase in the binding energy of an electron in a hydrogen atom. A similar situation apparently happens in the case of an exciton. Unfortunately, the existing theories are not applicable near the QEL, and the quantitative results of the calculations are still unreliable.

Furthermore, it is notable that the ratio of superstrong and hyperstrong fields is hundreds of thousands for the positronium and hydrogen atoms, while for excitons it does not exceed 100 (see Table 4, where superstrong fields $B_{ss} > 10\ B_{Ry}$). Correspondingly, the radius of the hydrogen or positronium atom transverse to the field can decrease by two orders of magnitude upon reaching hyperstrong fields, and the exciton radius near $B \approx B_{hs}$ by only several times (by less than five times for narrow-gap materials and slightly more for wide-gap materials, according to the most optimistic estimates). However, we should note that the corresponding magnetic fields are far beyond the current experimental capabilities This somewhat weakens the research potential of the exciton as a model object for analyzing the behavior of hydrogen-like atoms in superstrong magnetic fields, although the effect of exciton magnetopolymerization should still be observed.

Energy density of a hyperstrong field

Another radical difference in the behavior of excitons in a hyperstrong magnetic field from the behavior of hydrogen and positronium atoms under the same conditions is in their characteristic energy densities of the given field.

The magnetic field is a special type of matter through which moving charged particles or bodies with a magnetic moment interact. Interacting with the magnetic moment of the electron, this field increases the electron's energy, generating by itself a certain energy density.

The record energy density currently achieved in a constant magnetic field is 6 kJ/cm^3, and 3 MJ/cm^3 in a pulsed one [4]. The energy density of an ultrastrong field with respect to the positronium atom is $W_{hs} \cong 8\cdot 10^{18}$ J/cm^3,

Table 4

Parameters of energy and magnetic field for some crystals and the positronium atom

Crystal	E_g, eV	B_{hs}, T	Ry_{ex}, meV	B_{ss}, T	B_{hs}/B_{ss}
Exciton					
InSb	0.2	47	0.60	0.7	67
InAs	0.4	160	1.35	2.7	59
GaSb	0.8	500	2.00	6.2	81
GaAs	1.5	1 400	4.40	21	67
InP	1.4	1 700	6.00	36	47
CdTe	1.6	2 300	11.00	78	30
CdSe	1.8	2 700	11.50	87	31
ZnTe	2.3	4 000	18.00	155	26
GaN	3.5	9 800	25.00	350	28
CdS	2.5	6 800	25.00	340	20
ZnSe	2.8	6 200	35.00	390	16
ZnO	3.0	9 900	42.50	700	14
ZnS	3.8	19 000	49.00	1200	16
Positronium Ps					
–	$\Sigma(h\nu)$, eV	B_{cr}, T	Ry_{Ps}, meV	B_{ss}, T	B_{cr}/B_{ss}
–	10^6	$9\cdot 10^6$	$7\cdot 10^3$	$3\cdot 10^5$	$3\cdot 10^4$

Notations: E_g, Ry_{ex} are the band gap and binding energy of the exciton; B_{hs}, B_{ss} are the corresponding characteristic magnetic fields; $\Sigma(h\nu)$, B_{cr}, Ry_{Ps}, B_{ss} are the respective quantities for the positronium atom.

or $1.3\cdot10^8$ MeV/Å^3 for $B_{hs} = B_{cr} \cong 4.4\cdot10^9$ T. At the same time, if we imagine a space that is densely (or completely) filled with positronium atoms in the absence of a field, the corresponding energy density will be $W_{ps} \approx 0.1$ MeV/Å^3 ($\Sigma(h\nu) \approx 1$ MeV per positronium volume $V_{ps} \approx 10$ Å^3), i.e., a billion times less. Taking into account the doubling of the positronium energy in a hyperstrong field and a simultaneous hundred-fold decrease in its transverse radius increases W_{ps} by four orders of magnitude, i.e., does not change the ratio $W_{hs} >> W_{ps}$. It is this circumstance that leads to the so-called boiling of vacuum, i.e., production of a large number of virtual particles in magnetic fields B_{Wps}, much weaker than the magnetic field B_{cr} that is a hyper-strong field of the quantum electrodynamic limit (subbarrier production, see previous section), materialization of these particles and their subsequent annihilation (de-excitation). Such light emission by production and subsequent annihilation of pairs can actually be interpreted as transformation of energy of a constant magnetic field into electromagnetic radiation. In any case, the main effect of a hyper-strong field that is the production of pairs should be already observed in characteristic fields an order of magnitude smaller than the hyper-strong ones.

The role of a vacuum in semiconductor crystals is played by a medium with a dielectric constant different from unity; instead of virtual electron-positron pairs, all kinds of virtual quasiparticles with different energies are produced, including virtual electron-hole pairs. Following the above 'atomic/positronium' concept, the characteristic values of B_W can be given depending on the band gap and the binding energy of the exciton (its radius) (Table 5). For simplicity, Table 5 shows the parameters of only three abstract semiconductors: wide-gap (CdS-ZnTe type), narrow-gap (InSb type) and 'intermediate' (GaSb type). Aside from the hyperstrong magnetic field density W_{hs}, the 'exciton energy' density W_1 is given for each material (the density is expressed as the ratio E_g values to the exciton volume V_{ex}, i.e., it is assumed that the given crystal is densely filled with excitons). The factor accounting for the change in the radius and energy of the electron-hole pair does not exceed 100 in a superstrong field. The energy W_2 was calculated assuming a superdense exciton gas with one exciton per crystal cell (or electron-hole liquid, see below). The magnetic fields whose energy density determines the values of W_1 and W_2 are also given.

We should note that a magnetic field equal to only 5 T corresponds to a field energy density of 10 J/cm^3, or 1.6 10^4eV/Å^3 in the case of a narrow-gap material, which corresponds to the exciton density $\rho \cong 8\cdot10^{20}$cm^{-3}, or one exciton per six cells. In this regard, it should be mentioned that the Wannier–Mott exciton (an exciton of large radius) is a quasiparticle, which

Table 5

Dependence of energy densities and corresponding characteristic magnetic fields on the parameters of semiconductor crystals

Quantity	Value for semiconductor		
	wide-gap	intermediate	narrow-gap
E_g, eV	2.4	0.8	0.2
V_{ex}, 10^6 Å^3	0.08	10	500
B_{hs}, T	5000	500	50
W_{hs}, J/cm^3	10^7	10^5	10^3
W_1, J/cm^3	2	$5\cdot10^{-3}$	$2\cdot10^{-5}$
B_1, T	2	0.1	0.007
W_2, J/cm^3	1000	250	60
B_2, T	50	25	12

Notations: E_g, V_{ex} are the band gap and the volume of the exciton; B_{hs} is its characteristic magnetic field; W_{hs} is the energy density corresponding to B_{hs}; W_1, W_2 are the energy densities of superstrong and hyperstrong fields; B_1, B_2 are the magnetic fields corresponding to W_1 and W_2.

is not bound by the condition for 'dense filling' of space; the wave function of such an exciton is distributed over the entire crystal (in contrast to the Frenkel exciton localized on the cell at each moment of time); therefore, the exciton density can hypothetically increase infinitely. However, as the density increases, on the one hand, the effects of mutual screening of Coulomb interaction by excitons begin within each of them, and as a result, the excitonic gas transforms not into an excitonic liquid but into an electron-hole one; on the other hand, an exciton is a particle described in the one-electron approximation, and as the exciton density increases, this description becomes less and less valid

It is clear that not all of the energy of the magnetic field is transferred to exciton excitations. Nevertheless, if electron-hole pairs are generated at a sufficiently high rate, then all the known effects of a high-density excitonic gas ($\rho > 10^{18}$ cm^{-3}) will become possible: the formation of biexcitons (triexcitons, etc., up to, again, magnetopolymers) or excitonic/electron liquid, Bose-Einstein condensation. In other words, even given weak laser excitation of the crystal (when the exciton density produced by the laser is extremely low), it is nevertheless possible to obtain a high exciton density in extreme magnetic fields.

Materials with potential applications in magneto-optical studies on superstrong and hyperstrong magnetic fields

It can be concluded that the lower the binding energy of an electron and a hole Ry_{ex} (the larger the radius of the Wannier–Mott exciton a_{ex}), the more pronounced the discussed effects are. Since Ry_{ex} is typically the lower, the smaller the semiconductor band gap E_g (see, for example, Table 4), we can assume that a narrow-gap material with the band gap E_g < eV has potential for applications.

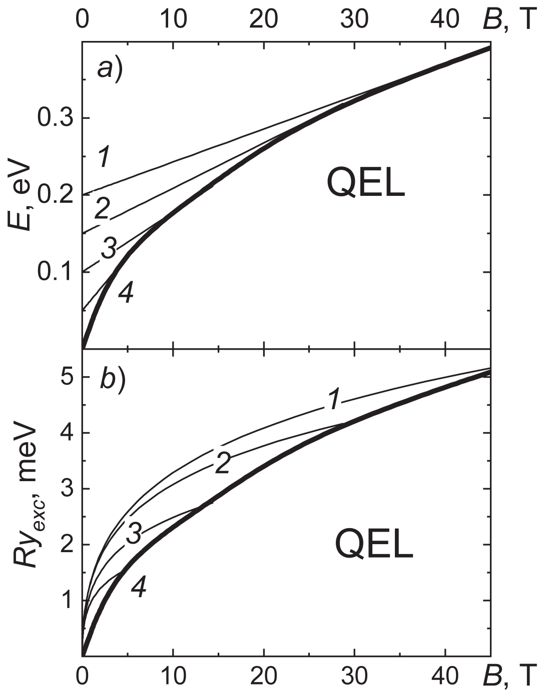


Fig. 5. Calculated dependences of ground Landau level energy (*a*) and binding energy of diamagnetic exciton (*b*) on the magnetic field in four semiconductors (numbers of the curves correspond to numbers in Table 6)

It seems reasonable to assess (if the current trends continue) the estimated parameters of such a hypothetical material (Table 6). Accordingly, it is possible to calculate the dependences of the energy of the ground ('zero') Landau level and the binding energy of a diamagnetic exciton on the magnetic field (compare Figs. 2 and 3). The calculated data obtained for three types of materials given in Table 6 (in addition to the well-known type *1*) are shown in Fig. 5.

Table 6

Parameters of extremely narrow-gap semiconductors

Number	E_g, meV	μ/m_e	ε	Ry_{ex}, meV	B_{ss}, T	B_{hs}, T
1	200	0.014	17.3	0.60	0.70	47.0
2	150	0.010	17.5	0.47	0.40	25.0
3	100	0.007	17.8	0.31	0.20	13.0
4	50	0.004	18.0	0.17	0.06	3.5

Notes. 1. The notations for the quantities correspond to those given in Tables 2 and 4, ε is the dielectric constant. 2. Variant *1* refers to, for example, InSb crystals.

Analyzing the data in Table and Fig. 5, we can see that even in the case of a material with E_g = 0.05 eV, in magnetic fields up to 10 T that are not at all record, the exciton line (or optical transition between the ground Landau levels of an electron and a hole) shifts from the infrared to the visible region of the spectrum, as in the vast majority of other semiconductors. Furthermore, increasing the exciton binding energy expands the temperature range where the exciton lifetime is sufficiently long: by 10 K (in a rough approximation), with Ry_{ex} increasing by 1 meV.

Unfortunately, the most narrow-gap compounds among homoatomic and binary semiconductors, for example, crystals such as InSb and mercury-based compounds with an inverted band structure have the values of $E_g \approx$ 0.2–0.3 eV: HgS, HgSe, HgTe.

Several extremely narrow-gap ternary (tertiary) compounds ($E_g \leq$ 0.1 eV) based on InBi are already known: $InAs_{1-x}Bi_x$, $InSb_{1-x}Bi_x$, as well as mercury chalgogenides but in the form of solid solutions, i.e., with a large number of structural defects that interfere with the existence of excitons (for example, the main problem in the case of the InBi compound is the formation of bismuth nanocrystals). We hope that further technological progress will nevertheless allow to obtain crystals of sufficient quality with the values of $E_g \leq$ 0.1 eV.

Conclusion

Let us now summarize the main effects expected for excitons in extremely strong magnetic fields.

The most interesting modification of the interexciton interaction in a superstrong field, with $B > B_{ss} \approx 10\ B_{Ryex}$, leads to greater stability of biexcitons and the appearance of exciton 'polymers'. In this case, the shift to the visible spectrum of the transition energy between the ground Landau levels of electron and hole in a superstrong field simplifies the experiments for most semiconductors. The rearrangement of the wave function of the exciton ground state from a spherical to a needle-like shape is not as substantial as that of a hydrogen atom in the corresponding superstrong atomic fields; therefore, the effect from increasing the binding energy is likely to be weak. At the same time, an increase in the exciton lifetime, which is still difficult to estimate, can result in generating a high-density cold exciton gas, i.e., produce the conditions for Bose-Einstein condensation of excitons.

In a hyperstrong magnetic field, given the magnetic flux density

$$B >; B_{hs} = 2\mu E_g/e\hbar$$

for an exciton in a solid or

$$B > B_{hs} = 4\mu m_e c^2/e\hbar$$

for hydrogen/positronium in a vacuum, the "cloning" of excitons (or, respectively, electron-positron pairs), the effect of magnetically induced birefringence and the effect of proportional multiplication/division of the frequency of the light incident on the crystal become possible.

Acknowledgments and closing remarks

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THE AUTHORS

VAGANOV Sergei A.
Ioffe Physical-Technical Institute of the Russian Academy of Sciences
26 Politekhnicheskaya, St. Petersburg, 194021, Russian Federation
SV.Exciton@mail.ioffe.ru

PANFILOV Andrei G.
Ioffe Physical-Technical Institute of the Russian Academy of Sciences
26 Politekhnicheskaya, St. Petersburg, 194021, Russian Federation
a.panfilov@mail.ioffe.ru

SEISYAN Ruben P.
Ioffe Physical-Technical Institute of the Russian Academy of Sciences
26 Politekhnicheskaya, St. Petersburg, 194021, Russian Federation
rseis@ffm.ioffe.ru