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THE FIBER-OPTIC INTERFEROMETRIC SCHEMES WITH MULTIPLEXED SENSITIVE ELEMENTS: AN ANALYSIS OF OUTPUT OPTICAL POWER LEVEL

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A concept for calculation of element parameters and analyzing the output power in the fiber-optic interferometric schemes with time-division multiplexing of the sensitive elements (TDM) has been put forward in the paper. The calculation procedure of element parameters allows ensuring the equality of the optical power from all multiplexed sensitive elements, as well as evaluating the effect of deviation of the optical scheme parameters from the calculated ones. Using two optical schemes as an example, the implementation of this calculation concept, the sequence of obtaining mathematical expressions, and examples of calculation results were presented. The proposed calculation method could be successfully applied in the design of interferometric meters with multiplexing of fiber-optic sensitive elements.

Keywords: fiber-optic sensor, fiber-optic splitter, optical power, optical loss, time-division multiplexing

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АНАЛИЗ ВЫХОДНОЙ МОЩНОСТИ ОПТОВОЛОКОННЫХ ИНТЕРФЕРОМЕТРИЧЕСКИХ СХЕМ С МУЛЬТИПЛЕКСИРОВАННЫМИ ЧУВСТВИТЕЛЬНЫМИ ЭЛЕМЕНТАМИ

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В статье предложена идеология расчета параметров элементов и анализа выходной мощности в волоконно-оптических интерферометрических схемах с мультиплексированием чувствительных элементов по времени (TDM). Метод расчета параметров элементов позволяет обеспечивать равенство оптической мощности от всех мультиплексированных чувствительных элементов, а также оценивать влияние отклонения параметров оптической схемы от расчетных. На примере двух оптических схем показана реализация такой идеологии расчета, последовательность получения математических выражений и примеры расчетных результатов. Описанный метод расчета предлагается применять при проектировании интерферометрических измерителей с мультиплексированием волоконно-оптических чувствительных элементов.

Ключевые слова: волоконно-оптический датчик, волоконно-оптический разветвитель, оптическая мощность, потери оптической мощности

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Introduction

Major advances are currently made in developing fiber-optic interferometric sensors and introducing them to measure different physical quantities [1]. Multiplexing a large number of fiber-optic sensitive elements (SE) in a single fiber-optic cable allows to create efficient quasi-distributed interferometric measuring systems, including long-distance ones. These technologies hold potential, for example, for constructing towed hydrophone arrays for seismic surveys of mineral resources in the shelf [2, 3], as well as many other similar systems.

There are several approaches to multiplexing in fiber-optic interferometric measuring devices separating signals from different SE: time-division multiplexing (TDM), frequency-division multiplexing (FDM), wavelength-division multiplexing (WDM), code-division multiplexing (CDM) or polarization-division multiplexing (PDM) [4]. The TDM technology, providing the maximum number of multiplexed elements using a single laser and photoreceiver, is the most widespread approach [5]. A TDM/WDM combination is often proposed, even though the TDM remains the primary technology in this case, while WDM technologies are used for secondary multiplexing of SE arrays separated in time, which allows reducing the number of fiber cables used [6].

An important issue related to installing fiber-optic systems with TDM is selecting reasonable parameters for the optical scheme elements providing multiplexing, estimating and optimizing the key parameters values of interference signals, such as relative level of interference signals and its difference for different SE, signal-to-noise ratio, contrast, etc.

However, studies considering fiber-optic multiplexing schemes for interferometric measuring devices practically never provide clear accounts of the procedures for calculating and estimating the parameters of such schemes in terms of methods for reasonably choosing optimal beam-splitting elements. While expressions for such calculations are occasionally presented [7, 8], they are usually obtained with many simplifications. It is often proposed to neglect the losses of optical power in the elements, or to approximate a large number of multiplexed SE [8], although systems with 4, 8 and 16 elements in one fiber-optic cable are often used in practice [9, 10].

Most studies give estimates for phase resolution depending on the number of sensitive elements N, i.e., the calculations rely on certain methods of auxiliary modulation and processing of interference signal [8, 9].

This paper presents a procedure for energy calculation of the parameters of the fiber-optic part, independent of the operating principles, considering two standard fiber-optic interferometric schemes with SE multiplexing. Formulated in this manner, the procedure can be applied for schemes with different types of auxiliary signal modulation and processing. The proposed approaches to calculations allow to take into account the influence of deviations from parameter values of passive fiber-optic elements on parameter values of interference signals formed in schemes from multiplexed SE.

Problem statement

Time-division multiplexing implies that short optical pulses with high duty cycle and optical power P_{in} are fed from a laser source to the input of an optical scheme with an array of N sensitive elements. The fiber optic scheme contains beam-splitting elements (splitters or semi-transparent reflectors) and should be constructed so that every input pulse passes through different paths and different combinations of SE, forming a sequence of $N+1$ output pulses with the power p_n (n is the number of an output pulse changing from 0 to N), delayed in time relative to each other. Most of the practical schemes (including those considered below) are organized so that every subsequent output pulse passes through one more SE than the previous one. The difference in the delay between the output pulses relative to each other is due to the difference in optical paths ΔL which the input pulse passes to form output pulses. These differences must be identical. The so-called compensated interferometer (CI) with the optical path difference also equal to ΔL is used to generate interferometric signal. When output pulses pass through the CI, they are split and combined in pairs with a shift by one pulse. As a result, a new sequence of N+2 pulses with the powers P_{rn} (it is convenient to number them from 0 to N+1), where each initial output impulse is combined with the previous one, is generated at the CI output and is subsequently transmitted to the photodetector. Each pulse P_{rn} is a result of interference of the pulses p_n and p_{n-1}. The only exceptions are the first and the last pulses, P_{r0} and $P_{r(N+1)}$, which are not combined with the previous and the subsequent pulse while passing through the CI due to lack thereof. Impacts on the n^{th} fiber SE change the phase delay $\Delta\varphi_n$ of light emission passing through this SE. For this reason, the interference of the pulses p_n and p_{n-1} is related to $\Delta\varphi_n$, as the pulse p_{n-1} passed through SE from the first to the $(n-1)^{\text{th}}$, and pulse p_n passed through the SE from the first to the n^{th}. Given that P_{rn} is defined by the interference of two output pulses, they have the form

$$P_{rn}(t) = C\{P_{0n} + P_{mn}\cdot\cos[\Delta\varphi_n(t)]\}, \quad (1)$$

where $P_{0n} = p_n + p_{n-1}$ is the constant component; $P_{mn} = 2(p_n p_{n-1})^{1/2}$ is the amplitude of the interference component.

The argument of the interference signal $\Delta\varphi_n$ contains target oscillations of the phase delay of the n^{th} SE, related to measured impacts, and can be determined during subsequent processing. The coefficient C is related to losses during passage through the CI, and ideally, $C = 1/2$. Notably, CI can be located at the input of a fiber optic scheme as well. In this case, the details differ for the pulses passing through the scheme, but interference signals taking the form (1) are also generated as a result.

Comprehensive analysis of fiber-optic interferometric schemes with SE multiplexing should consider different systems of relations including different types of parameters for the optic scheme elements, characteristics of other elements of the system and interrogation pulses. In terms of energy relations, one of the key problems is selecting the elements that provide optimal parameter values of interference signals P_{0n} and P_{mn}. The set of values of P_{0n} and P_{mn} plays an essential role in organizing correct signal recording, estimating the signal-to-noise ratio achieved, and, consequently, resolution of a system.

An important result of energy calculation from the standpoint of scheme design is finding the required ratios for light beam splitting in the splitting elements of a fiber-optic scheme. Depending on the elements used in the scheme, these parameters include the splitting ratios of fiber-optic splitters or the reflection coefficients of semi-transparent reflectors.

If identical splitting elements are used, inevitably, the values of p_n and P_{rn} greatly depend on n, and the problem of choosing optimal splitting ratios necessitates complex analysis of the criteria of optimality. A more attractive option in terms of the achieved effect and, at the same time, a simpler one in terms of the criteria of optimality entails choosing the splitting elements provided that all p_n are equal:

$$p_0 = p_1 = ... = p_n = \ldots = p_N = P_0. \quad (2)$$

In this case, $P_{0n} = P_{mn} = 2P_0$, the contrast of all interference signals equals unity (if polarization matching is ensured).

The scheme constructed in this study satisfies this condition specifically. At the same time, the normalized power level of pulses serves as an important indicator:

$$p_{norm} = P_0/P_{in}; \quad (3)$$

this indicator makes it convenient to compare the “energy efficiency” achieved in different schemes and at different values of N.

Generally, if condition (2) is fulfilled, it is evident that the higher the value of p_{norm}, the less influence different noises and fluctuations have on the output signals of a measuring device.

Clearly, a special set of splitting ratios for splitters or reflectors has to be used to satisfy condition (2), but since modern technologies allow to produce these elements with virtually arbitrary parameters, this approach to constructing optimal schemes based on criterion (2) can be put into practice. However, it is essential for designing such schemes that not only the optimal splitting ratios of the elements are found but various additional aspects can be analyzed in detail, including the influence of other parameters of splitting elements, their fluctuations and other factors on the parameters of interference signals, all of which must be taken into account at the stage of design.

General principles of the calculation procedure

Considering the calculation procedure, let us review different groups of parameters characterizing the scheme elements, which are used in analysis and calculations. First of all, these are transmittances (with respect to power) in fiber sections connecting the scheme elements including those in SE. These transmittances differ from unity due to optical power losses in the fiber-optic cable and additional conditions (fiber sections can be wound into a coil, contain connections, etc.). The transmittances defined initially are assumed to be given and are not supposed to be found in the calculations. The transmittances can actually be either identical or different for different SE but are regarded as known parameters.

Another type of parameters are the splitting ratios of optical power in splitters and semi-transparent reflectors (typically fiber Bragg gratings) if the schemes use the first or the second type of elements as optical power splitters. Optimal splitting ratios should be selected on the calculation procedure that satisfies condition (2).

We should note that splitters or fiber-optic Bragg gratings also incur internal losses which, strictly speaking, should be also taken into account in the calculations. In general, the losses of splitting elements may depend on splitting ratios and transmission coefficients. This can be included in the calculations if the dependence is known. To represent the specific results obtained in a clear and simple manner, the analysis below includes a case when these losses have fixed values, and are regarded as a known parameter.

Calculating optimal splitting ratios of splitters or reflectors, it is of course possible and feasible to take into account only regular components of transmittances in the fiber sections known in advance and losses in the splitting elements. However, the calculation procedure as a whole should provide a possibility to analyze the influence of potential deviations of the calculated and initially given parameters of the elements from the actual ones. These phenomena can be caused by both regular deviations from reference values and by fluctuations of parameters during operation. The changes may occur, for example, due to aging, unstable temperature and polarization of optical emission.

The procedure for energy calculation and analysis of fiber optic elements of the scheme involves obtaining and applying two systems of relations:

firstly, the equations of multiplicative structure for calculating the values of p_n taking into account all key parameters characterizing the elements of the fiber optical scheme;

secondly, recurrence relations connecting the selected parameters of splitting elements of adjacent links in the scheme and allowing to calculate the splitting ratios of all splitting elements taking into account certain conditions for boundary elements.

The first system of relations is formulated by considering a light pulse related to the n^{th} splitting element passing from the input to the output.

The second system of relations requires considering the condition for power balance $p_{n-1} = p_n = P_0$ and solving the balance equation with respect to the splitting element parameter.

Regarding the choice of schemes discuss further in this paper, we should note that different types of optical schemes with TDM can be divided into two types: reflective schemes or passage schemes.

In case of the reflective scheme, a scanning pulse passes through the scheme from the first to the n^{th} SE, then travels in the opposite direction and is fed as an n^{th} output pulse into the same part of the scheme (or directly to the same fiber-optic line) as the input pulse, but in the reverse direction. In this case, the scheme requires mirrors (the so-called Faraday mirrors are often used to suppress polarization fading).

In case of the passage scheme, an input pulse is fed from one end of the scheme and then, passing from the first to the n^{th} SE, forms an n^{th} output pulse at the opposite end of the scheme. It is typically assumed for the schemes considered below that the decrease of p_{norm} with the growth in N is described by ~ $1/N^2$ provided there are no losses [9].

There are different types of schemes where the decrease is described by $1/N$, but they may contain multiple passes through the SE and aliasing of different pulses, as well as crosstalk [8]. The schemes with crosstalk have their own peculiarities, but they are not considered in this paper.

Analysis of the power of output pulses in a reflective-type scheme

Let us consider a standard scheme of the reflective type (Fig. 1). The scheme includes N coils of sensitive elements (SE) numbered $n = 1,2, ..., N$, as well as (N+1) Ysplitters (Y) and mirrors (M) numbered $n = 0, 1, 2, ..., N$.

It seems helpful to introduce the direct (K_d) and cross (K_c) gains of the splitter, the gain K_{sf} of

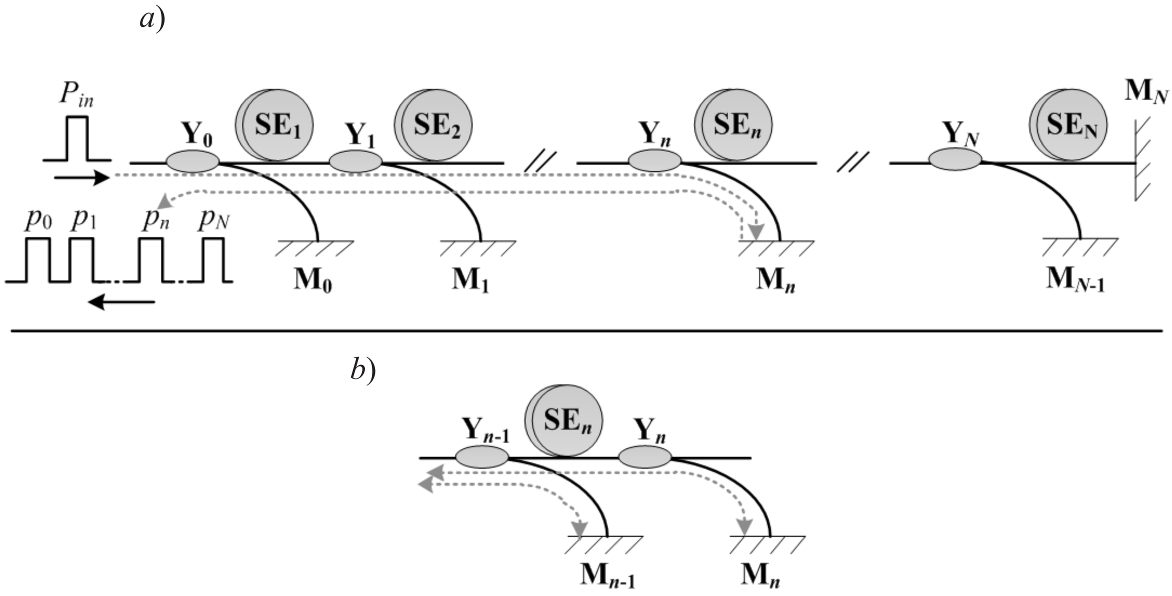

Fig. 1. Reflective scheme and generation of output pulses (*a*), n^{th} link of scheme (*b*): sensitive elements SE_i; mirrors M_i; splitters Y_i; input pulse P_{in}; output pulses p_i

a fiber section with the sensitive element, gain K_f of a service fiber section between the splitter and the mirror, the reflection coefficient R (ideally, $R = 1$, however, the actual reflection coefficient may be less than unity) as the key parameters of the scheme. If the scheme contains connections, the losses in them should be taken into account in the fiber section gains K_{sf} and K_f. The gains K_d and K_c are rigidly bound to the splitting ratio D and the parameter of internal losses of the splitter α, as described in Appendix 1.

Considering the path of an input pulse from the n^{th} mirror and back (see Fig. 1), it is easy to formulate multiplicative equations for p_n:

$$p_n = P_{in} K_{fn}^2 K_{cn}^2 R_n \cdot \prod_{q=1}^{n} (K_{d(q-1)}^2 K_{sfq}^2). \quad (4)$$

Eq. (4) implies that if the upper limit of the product is less than its lower limit, which occurs at $n = 0$, then the product equals unity. Furthermore, the case when $n = N$, related to the difference of the final link in the scheme from others, is different because there is no need to direct the optical power further for the last N^{th} SE, and it is not practical to use a splitter between the N^{th} SE and the N^{th} mirror. However, expression (4) is relevant for all n, if the presence of formal coefficients $K_{cN} = K_{fN} = 1$ is taken by definition. In practice, it can be often assumed that all SE are equivalent and K_{sf} does not depend on n. Then this parameter can be excluded from the product in Eq. (4) and the multiplier $(K_{sf})^{2n}$ can be used.

Analyzing one link of the scheme and comparing the difference in the paths of the $(n-1)^{th}$ and n^{th} pulse (see Fig. 1,*b*) we can obtain an equation corresponding to the balance $p_{n-1} = p_n$. For the given scheme (if the definition $K_{cN} = K_{fN} = 1$ is preserved), this equation has the form

$$K_{c(n-1)}^2 K_{f(n-1)}^2 R_{n-1} = \\ = K_{d(n-1)}^2 K_{cn}^2 K_{sfn}^2 K_{fn}^2 R_{n-1}\ . \quad (5)$$

The recurrence relation for the parameters of the splitters is obtained by taking into account the connection between K_d and K_c. In view of the explanations in Appendix 1, we can use a model of splitter parameters that has the form

$$K_d = (1-\alpha_{el}) \cdot D/(1 + D) \\ \text{and } K_c = (1 - \alpha_{el})/(1 + D), \quad (6)$$

then Eq. (6) is transformed directly transforms to the recurrence form:

$$D_{n-1} = A_n(1+ D_n), \quad (7)$$

based on the assumption that the parameter of excess splitter losses α_{el} does not depend on D and is the same for any n, introducing a constant

$$A_n = K_{f(n-1)} \sqrt{R_{n-1}} \times \\ \times [(1-a_{el}) K_{sfn} K_{fn} \sqrt{R_n}]^{-1}. \quad (8)$$

It is commonly acceptable for calculations of practical schemes to assume the gains K_{sf}, K_f and the coefficient R to be identical for all n. In this case, the calculation of optimal values of D_n does not depend on the values of K_f and R, while the constant A_n does not depend on n and is simplified:

$$A = 1/[(1-\alpha_{el})K_{sf}, \tag{9}$$

(here the calculations of the optimal values of D_n are affected by the excess losses of the splitters and the SE).

If it is acceptable to neglect the excess losses of the splitters and the SE, it can be assumed that $\alpha_{el} = 0$.

To use Eq. (7), we need to define the initial condition for recursive calculation of optimal values of D_n. For this scheme, this condition is a direct result of the absence of a splitter with the number N. A different connection of the last SE would definitely impair the obtained values of p_0 and P_{norm}. At the same time, considering the final link which contains the last SE provides a condition of power balance (5), if $K_{cN}=K_{sfN}=1$ is substituted in the right-hand side. Then, taking into account Eqs. (6) for the $(N-1)^{th}$ splitter leads to a simple relation:

$$D_{(N-1)} = \frac{K_{f(N-1)}}{K_{sfN}}\sqrt{\frac{R_{N-1}}{R_N}}, \tag{10}$$

which corresponds to the recurrent expression (7), excluding the parameters K_{cn}, K_{fn} and α_{el} from Eq. (8) for determining the constant A_n.

Evidently, expression (10) gives D_{N-1} . 1 for small losses in the elements, when the gains $K_{f(n-1)}$ and K_{sfN} and the coefficients R_{N-1} and R_N approach unity, which is a logical result for the balance of such a link with splitting close to a 50:50 splitter, regardless of the losses in the splitter.

Furthermore, recurrent expression (7) can be used to successively obtain values for the rest of the splitters numbered from $n = N - 2$ to $n = 0$, forming a set of values $\{D\}$, and then recalculating the values of $\{D\}$ into sets of values $\{K_d\}$ and $\{K_c\}$ for all splitters based on Eq. (6) and a given α_{el}.

Substituting sets of values $\{K_d\}$ and $\{K_c\}$ in expressions (4), we obtain the same value of p_0 because of the method by which these sets of values were obtained; moreover, this value of p_0 is the largest possible for any n with the given parameters used in the calculation.

However, an important result of the calculation is the actual value for the level of p_{norm}, as well a possibility to analyze its dependence on N and other parameters used in the calculation.

The calculated sets of values $\{D\}$, $\{K_d\}$ $\{K_c\}$ for the splitters are given in Appendix 2 for $N = 8$ and

$$\alpha_{el[\mathrm{dB}]} = \alpha_{sf[\mathrm{dB}]} = 0.10 \text{ dB},$$

$$\alpha_{f[\mathrm{dB}]} = 0.05 \text{ dB and } R = 0.99,$$

where it is taken that

$$\alpha_{sf[\mathrm{dB}]} = -10\lg(K_{sf}),$$

$$\alpha_{f[\mathrm{dB}]} = -10\lg(K_f),$$

$$\alpha_{el[\mathrm{dB}]} = -10\lg(1-\alpha_{el}).$$

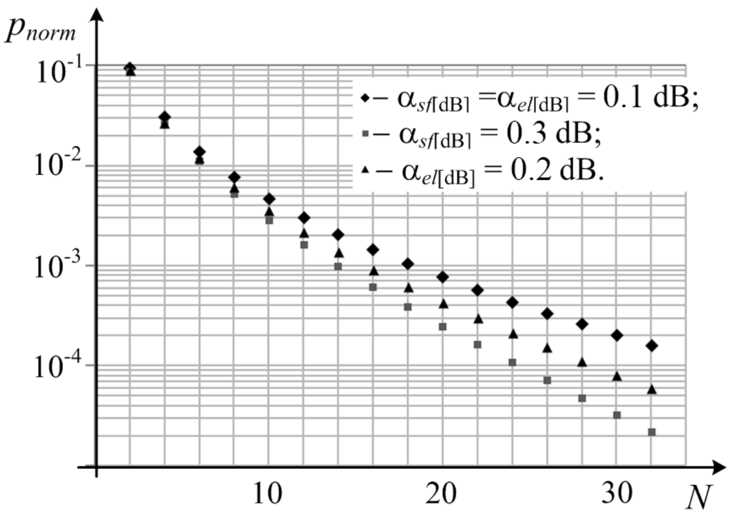

Fig. 2. Example of reflective-type scheme. Normalized power level of pulses depending on the number of sensitive elements (SE) for different values of losses α_{sf} and α_{el}

These values of the splitting ratios are important for practical implementation of this scheme, as they have to be known in order to install the appropriate splitters. However, the examples shown in Fig. 2 for p_{norm} depending on N for the same set of parameters, as well as for cases when either of the parameters α_{sf} or α_{el} have different values are more important for analysis of energy efficiency of the scheme.

An example of the dependences in Fig. 2 shows the achieved levels of relative power for the schemes with the given parameters and with optimal splitting ratios chosen for the splitters, the exponential form of the dependences $p_{norm}(N)$, indicating that it is possible to study the influence of other parameters of the scheme elements on the achieved level of p_{norm}.

Importantly, the given systems of expressions allow not only to analyze the influence of scheme element parameters on the achievable level of p_{norm}, but also take into account and study the influence of deviations of actual parameters from ideal ones for the values of p_{n}. If the ratios D are given as (1− d)/d with the accuracy in selecting d ranging, for example, to 1 or 2% in production of real splitters, the set of optimal values $\{D\}$ value set obtained by a recurrent procedure can then be rounded up. Next, we can substitute the rounded values in expressions (4) and, calculating p_{0n}, estimate their variance and deviation from the calculations without the round-off.

Similarly, we can take into account the influence of both fixed and random deviations of element parameters from the initially calculated ones.

The first case concerns precision measurements of the parameters of an actual set of splitters produced for the scheme, which are then used in calculations.

The second case implies that the element parameters may fluctuate during operation. Then, after the initial calculations of optimal sets $\{D\}$ for the splitters using regular parts of α_{el}, K_f, K_{sf} and R, the parameters containing, aside from the regular component, random additions are substituted in Eq. (4) at the second stage of the calculations. Then the calculations will give a set of values of p_n with random deviations with respect to the estimate obtained for p_0 in the calculations with regular parameters.

Those are important aspects of the proposed calculation procedure, although considering specific examples is outside the scope of this study.

Analysis of the power of output pulses in a passage-type scheme

Let us consider a standard scheme of the passage type (Fig. 3). The scheme includes pairs of Y-splitters in the “upper” and the “lower” lines. The sequence of output pulses is formed by means of the n^{th} pulse passing a part of the path through the “upper” line, splitting into the lower part through the n^{th} couple of splitters and then propagating towards the output through the “lower” line. We can confirm that both of the n^{th} splitters must have the same splitting ratios within the scope of the given problem. The situations differ for $n = 0$ and $n = N$, when a pulse passes to the “lower” line only through the zero or only through the N^{th} splitter which have no pair.

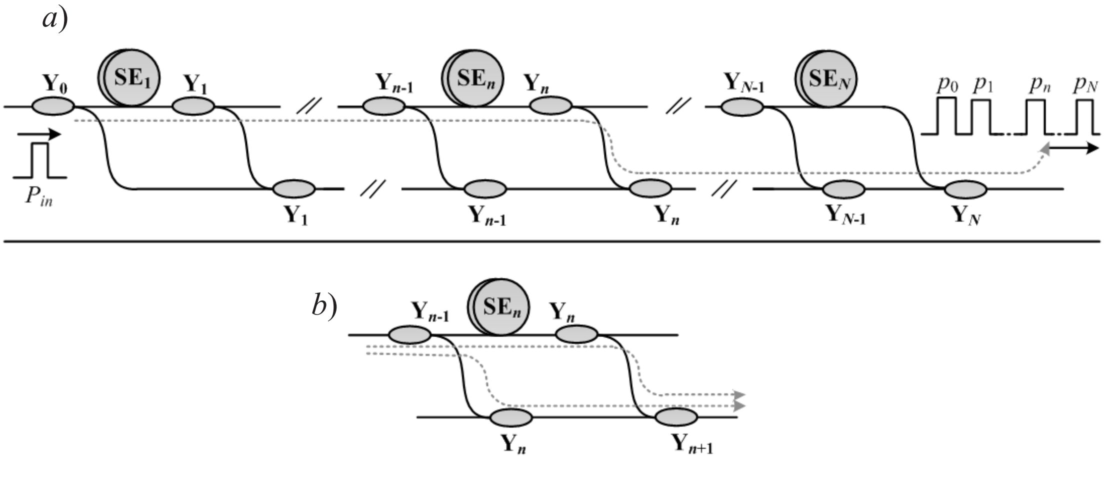

Fig. 3. Passage-type scheme and generation of output pulses (*a*), n^{th} link in given scheme (*b*); the notations are the same as in Fig. 1

We also need to take into account the transmittance of fiber sections: K_{sfn} for a fiber section with the n^{th} sensitive element; K_{fn} for service fiber sections connecting the $(n-1)^{th}$ and the n^{th} splitters in the "lower" line; K'_{fn} for "vertical" sections between a pair of the n^{th} splitters. The difference in optical paths DL is formed by the difference in the lengths of fiber sections between adjacent splitters in the "upper" and in the "lower" lines (the fiber in the SE is typically larger than the service section of the "lower" line). The scheme could be constructed symmetrically, with the SE located in the "lower" line but the principles of operation and calculation would remain the same.

Considering the path travelled in the scheme by the n^{th} pulse, it is easy to compose multiplicative equations to find p_n. As the structures of the first and the last links differ from the structure of the central links, the expressions are different for n=0 and n=N:

$$p_0 = P_{in} K_{f0} K_{c0} K_{d1} \cdot \prod_{q=1}^{N} K_{dq} K_{fq} \quad \text{for } n = 0;$$

$$p_n = P_{in} K'_{fq} K_{cn}^2 \cdot \prod_{q=1}^{n} K_{d(q-1)} K_{sfq} \cdot \prod_{q=n+1}^{N} K_{dq} K_{fq} \quad \text{for } n = 1, ...(N-1); \tag{11}$$

$$p_N = P_{in} K_{cN} \cdot \prod_{q=1}^{n} K_{d(q-1)} K_{sfq} \quad \text{for } n = N.$$

Equations for the power balance of adjacent pulses for the first and the last links also differ from the equation for the central links and, as follows from the difference in the paths of the $(n-1)^{th}$ and the n^{th} output pulses (Fig. 3,*b*), take the following forms for these three case

$$K_{c0} K_{d1} K_{f1} = K_{d0} K_{c1}^2 K_{sf1} K'_{f1} \quad \text{for } n = 1;$$

$$K_{c(n-1)}^2 K_{dn} K'_{f(n-1)} K_{fn} = K_{d(n-1)} K_{cn}^2 K_{sf1} K'_{fn} \quad \text{for } n = 2, ... (N-1); \tag{12}$$

$$K_{c(N-1)}^2 K_{dN} K'_{f(N-1)} K_{f(N-1)} = K_{cN} K_{d(N-1)} K_{sfN} \quad \text{for } n = N;$$

where n corresponds to the link covering the n^{th} SE.

Notably, Eqs. (11) imply equal ratios for the n^{th} splitter in the "upper" and the "lower" lines.

Based on Eq. (12), we can obtain recurrence relations connecting the splitter parameters. In view of model (6), we can obtain an equation for the first link (for n=0) from Eq. (12), which has the following form:

$$D_1^2 + D_1 - A_0 D_0 = 0, \tag{13}$$

where we introduce a constant

$$A_0 = (1 - \alpha_{el}) K_{sf1} K'_{f1} / K_{f0}. \tag{14}$$

The solution to this quadratic equation (only one of the two roots is positive and acceptable) has the form

$$D_1 = 0.5\ [(4D_0 A_0 + 1)^{1/2} - 1]. \tag{15}$$

Using model (6) and Eq. (12), we obtain a relation for the subsequent links (except the last one):

$$D_n^2 + D_n - A_n[(D_{n-1})^2 + D_{n-1}] = 0, \tag{16}$$

introducing a constant

$$A_n = K_{sf1} K'_{fn} / (K'_{f(n-1)} K_{fn}). \tag{17}$$

The solution to this equation provides a recurrence relation taking the form

$$D_n = 0.5\ \{[4((D_{n-1})^2 + D_{n-1}) A_n + 1]^{1/2} - 1\}. \tag{18}$$

And, finally, the equation for the last link ($n = N$) follows from model (6) and Eq. (12):

$$(D_{N-1})^2 + D_{N-1} = A_N D_N, \tag{19}$$

introducing a constant

$$A_N = (1 - \alpha_{el}) K_{f(N-1)} K'_{f(N-1)} / K_{sfN}. \tag{20}$$

In this case, we need to determine D_N, so the solution takes the form

$$D_N = [(D_{N-1})^2 + D_{(N-1)}] / A_N. \tag{21}$$

We should note that we formulated the expressions for the passage scheme assuming that $N > 2$. The case $N = 2$ has to be considered separately to obtain the corresponding expressions but because it has little practical value, it was not included in this study.

If we give a certain value for D_0, then, based on Eqs. (15), (18), (21), we can derive the

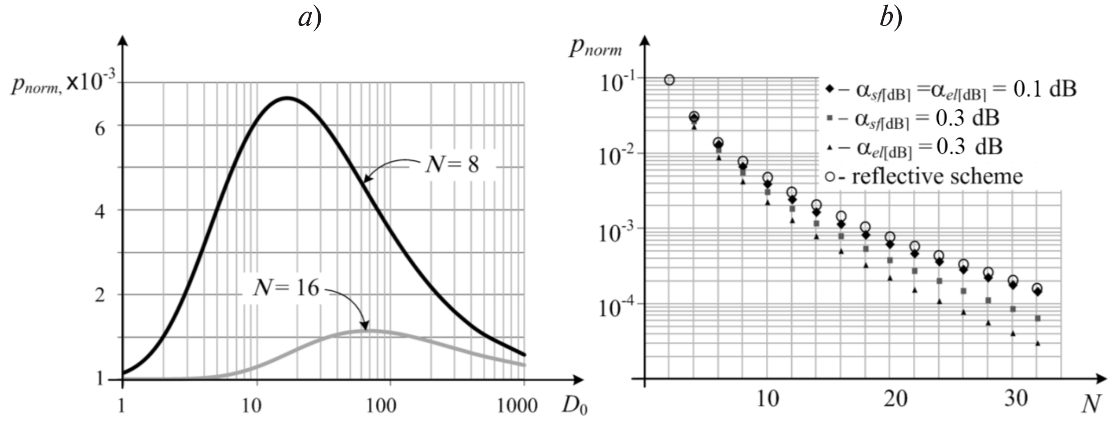

Fig. 4. Example of passage-type scheme. Calculated normalized power level of pulses p_{norm} depending on splitting ratio of first splitter for two values of N (a) and on number of sensitive elements N for two different values of losses in SE and in splitters.
Dependence of p_{norm} for the reflective scheme is shown for comparison (b)

values of D_n for all the remaining splitters, i.e., a complete set $\{D\}$. Clearly, if we recalculate the set $\{D\}$ into the sets $\{K_d\}$ and $\{K_c\}$, and then calculate p_n based on Eq. (10), condition (2) is satisfied, and we obtain a certain value of p_{norm}, which does not depend on n. However, this value does depend on the initial choice of D_0.

Thus, the condition for obtaining the maximum value of p_{norm} in this scheme is the choice of the optimal value of D_0. A direct and simple approach to solving this problem consists in enumerating the values of D_0 and choosing an optimal value D_{opt} of such a value of D_0 for which p_{norm} reaches the greatest level. Evidently, a specific value of D_{opt}, as well as maximum achieved by p_{norm}, obviously depends on N and on the values of other parameters used in the calculations.

Let us provide examples of the calculations, where, as before, we assume for simplicity that K_f,K'_f and K_{sf} do not depend on n. Fig. 4,a shows examples of dependences of p_{norm} on D_0 with $\alpha_{el[dB]} = \alpha_{sf[dB]} = 0.1$ dB and $\alpha_{f[dB]} = \alpha'_{f[dB]} = 0.05$ dB for the cases when $N = 8$ and $N = 16$. In the first case, it follows from the calculation that $D_{opt} = 16.86$, providing $p_{norm} = 6.63 \cdot 10^{-3}$. In the second case, $D_{opt} = 62.67$, providing $p_{norm} = 1.15 \cdot 10^{-3}$.

Calculated sets of value for $\{D\}$, $\{K_c\}$ and $\{K_d\}$ are also given in Appendix 2 for $N = 8$, which can be used for practical applications of the scheme with these initial data.

From the standpoint of energy analysis, this scheme provides a good illustration of the dependences of p_{norm} on N (shown in Fig 4,b), obtained selecting $D_0 = D_{opt}$ for each N. As before, aside from calculation of the dependences with the above parameters of losses, two additional curves are given for the cases when the parameter α_{fs} and α_{el} have different values. To compare this scheme with the reflective one, Fig. 4 shows a dependence obtained earlier for the main sets of parameters.

Comparing the calculation results in Figs. 2 and 4, we can see that the levels achieved by p_{norm} with the optimal choice of the splitting ratios of the splitters are almost the same for both schemes given equal losses in the SE, with a small advantage in case of the reflective scheme (increasing with greater values of N). This is an expected result because, despite the different configurations of the schemes, a pulse passes through an equal number of splitters forwards, the same number of branches and sections of SE in both of them.

Each link of the second scheme contains an additional connecting section but the first scheme includes losses due to reflection from the mirror (the second scheme can be improved slightly in terms of power by changing the ratio of losses in these elements).

Fig. 4,b also shows that the dependence on N has an exponential behavior of $p_{norm}(N) \sim N^{-q}$, and the value of q is close to 2 if the losses decrease but increases if the losses increase. For the given dependences, q has values in the range of 2.5–3.3. We should note here that approximation yields different results and better accuracy if limited ranges of N are analyzed. For example, the values of the parameter for the curves in Figs. 2 and 4 lie between 2.4 and 2.8 in the range $4 \le N \le 16$, and between 2.8 and 3.7 in the range $10 \le N \le 32$.

However, it was not our intention to carry out comprehensive studies of such patterns as we focused instead on the procedure for correct calculation of this kind of dependences and the expressions required for this purpose. At the same time, correct calculation implies that the optimal choice of parameters is provided for the splitters.

Similarly to the reflective scheme, the principle proposed for the calculations of the passage scheme allows (except for the choice of the optimal system of splitters and estimation of the p_{norm} value) to analyze the influence of various additional factors: limitations in the accuracy with which the splitters can be made, random fluctuations of the element parameters, etc. However, we should also consider the fact that multiplicative expressions (11) restrict such analysis, assuming equal parameters in the pairs of splitters in the "upper" and the "lower" lines. Eqs. (11) are simplified because of this, and, most importantly, we can obtain a simple recursive expression (18) that is easy to interpret. To analyze the influence of the rounded splitting ratios, regular or random deviations of these ratios and similar factors, we need to use multiplicative equations of the following form:

$$p_0 = P_{in}K_{f0}K_{c0}K_{d1}\cdot\prod_{q=1}^{N}k_{dq}K_{fq}$$

$$\text{for } n = 0;$$

$$p_n = P_{in}K'_{fq}K_{cn}k_{cn}\cdot\prod_{q=1}^{n}K_{d(q-1)}K_{sfq}\cdot\prod_{q=n+1}^{N}k_{dq}K_{fq}$$

$$\text{for } n = 1, \ldots(N-1); \tag{22}$$

$$p_N = P_{in}k_{cN}\cdot\prod_{q=1}^{n}K_{d(q-1)}K_{sfq}$$

$$\text{for } n = N,$$

where the gains K_{dn}, K_{cn} of the splitters on the "upper" line are separate from the gains k_{dn}, k_{cn} of the splitters on the "lower" line.

Conclusion

We have proposed a procedure for calculating the parameters of elements in optical schemes with multiplexed fiber optic sensors, allowing to optimize the scheme in terms of achieving the maximum level and contrast of the generated interference signals, taking into account the optical power losses in splitting elements as well as in fiber sections and mirrors included in the fiber optic scheme.

The procedure for obtaining expressions for calculating the element parameters is described for two optical schemes.

We have given examples of the calculations of the element parameters in the scheme under consideration for $N = 8$ sensitive elements and the dependences of the normalized power level of an optical pulse at the output of the schemes on the number N for certain sets of element parameters.

The principles proposed for organizing the calculations allow not only to calculate the optimal splitting ratios of the scheme splitters and the power achieved by the output pulses but also to analyze the influence from varying the parameters of individual elements of the optical scheme (including random ones) on the characteristics of the system as a whole.

The calculated expressions formulated for the given schemes illustrate how similar calculations can be organized for other configurations of similar schemes.

The methods and results presented can be applied in design of fiber-optic interferometers based on multiplexing of sensitive elements.

Appendix 1

Parameters of Y-splitter

A Y-splitter has three terminals and is formally described by nine power gains K_{ij}. In view of symmetry, which is easy to achieve in practice, K_{ij} . K_{ji}. Lets us choose numbering so that when light is submitted to the first terminal, it is then transmitted to the second and the third terminals. Then, due to directivity, K_{23} . 0, and the coefficients of reflection from the splitter, K_{ii} . 0, are small as well (in reality these coefficients correspond to attenuations by several tens of dB). Then two coefficients are significant: K_{12} and K_{13}. Assuming $K_{12} \geq K_{13}$ (the connection between terminals 1 and 2 is direct, and $K_{12} = K_d$, while terminals 1 and 3 are cross-connected, and $K_{13} = K_c$), the key parameter of the splitter, its splitting ratio, is given by the relation $D = K_{12}/K_{13}$ ($D > 1$). If K_{23}, $K_{ii} \ll K_{13}$, then based on the condition imposed for the power balance, $K_{12} + K_{13} = 1$. However, taking into account the internal (excess) losses of optical power, $K_{12} + K_{13} = 1 - \alpha_{el}$ for an actual splitter (where α_{el} is a small parameter characterizing the losses). The last equality from the definition of D gives

$$K_{12} = K_d = (1-\alpha_{el})D/(D+1);$$

$$K_{13} = K_c = (1-\alpha_{el})/(D+1),$$

introduced as expression (4).

Table

Calculated parameters {D}, {K_c} and {K_d} for reflective and passage schemes with optical power losses given in scheme elements

n	Reflective scheme			Passage scheme		
	D	K_d	K_c	D	K_d	K_c
0	9.848	0.887	0.09	16.857	0.923	0.055
1	8.405	0.873	0.104	3.543	0.762	0.215
2	7.026	0.855	0.122	3.520	0.761	0.216
3	5.71	0.832	0.146	3.498	0.76	0.217
4	4.453	0.798	0.179	3.475	0.759	0.218
5	3.253	0.747	0.23	3.453	0.758	0.219
6	2.106	0.663	0.315	3.43	0.757	0.221
7	1.012	0.491	0.486	3.408	0.756	0.222
8	–	–	–	15.374	0.918	0.06

Notations: n is the number of the Y-splitter.

Giving the parameters D and α_{el} rather than K_{12} and K_{13} is often clearer and more widespread for real splitters ($\alpha_{el[\mathrm{dB}]} = 10\cdot\lg(1-\alpha_{el})$ is usually taken).

Appendix 2

Examples of calculating the parameters {D}, {Kc} and {Kd}

The summary Table below presents the calculation results for the splitting ratios D and gains K_d, K_c for the schemes described in case $N = 8$, $\alpha_{el} = 0.977$, $K_{sf} = 0.977$, $K_f = 0.989$, and $R = 0.99$ for the reflective scheme and $K'_f = 0.989$ for the passage scheme (the given ratios and gains correspond to the levels $\alpha_{el[\mathrm{dB}]} = \alpha_{sf[\mathrm{dB}]} = 0.1$, $\alpha_{f[\mathrm{dB}]} = \alpha'_{f[\mathrm{dB}]} = 0.05$).

While the values listed in the Table are not particularly illustrative or interesting for the considered dependences of p_{norm} on N and other parameters of similar characteristics, splitters with the calculated set of parameters {K_c} and {K_d} should be chosen for practical implementations of the optimal scheme satisfying condition (2).

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