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## A MODEL OF NEW INFORMATION DISSEMINATION IN THE SOCIETY

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In the article, a basic mathematical model of new information dissemination in the society is constructed and studied. The suggested model has been described using the system of four ordinary differential equations with square nonlinearity in the right parts. Two stationary solutions furnishing quite logical interpretation for this system were found. Two areas with various properties of stationary solutions were separated in the parameters' space of the system. The global properties of a phase pattern of the constructed dynamic system were investigated by qualitative methods of the differential equations theory. The obtained results allowed finding several possible scenarios of new information dissemination in the society.

**Keywords:** dissemination of new information, stationary solution of system, invariant set, asymptotic stability

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## МОДЕЛЬ РАСПРОСТРАНЕНИЯ НОВОЙ ИНФОРМАЦИИ В ОБЩЕСТВЕ

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В статье строится и исследуется базовая математическая модель распространения в обществе новой информации. Предлагаемая модель представлена системой четырех обыкновенных дифференциальных уравнений с квадратичной нелинейностью в правых частях. Для данной системы найдены два стационарных решения, допускающие вполне логичную интерпретацию. В пространстве параметров системы выделены две области, в которых стационарные решения обладают разными свойствами. С помощью качественных методов теории дифференциальных уравнений изучены глобальные свойства фазового портрета построенной динамической системы. Это позволило выделить несколько возможных сценариев распространения новой информации в обществе.

**Ключевые слова:** распространение новой информации, стационарное решение системы, инвариантное множество, асимптотическая устойчивость

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## Introduction

Mass media have a great influence on all spheres of society, playing a major role in shaping the public opinion. Every day people receive huge amounts of new information affecting their choices and preferences, regardless of whether the media are perceived as a source of news, educational information, entertainment, or simply as a means to keep in touch with the external world.

There is clear movement towards the so-called information society. For example, this phenomenon has already been given a definition within Russian legislation. According to Article 3 of the Strategy of the Information Society Development in the Russian Federation for 2017–2030, the information society is a society where information, its applications and access to it fundamentally affect the economic and socio-cultural aspects of life<sup>1</sup>. Thus, information and knowledge are the main resources in this type of society [1]. The mass media play an increasingly pivotal role in shaping public opinion and consciousness. They are the original source of news, providing the latest information from anywhere in the world, sometimes in real time. People are well within their right to trust or mistrust the information disseminated by journalists and their assessments of what is happening. In the age of advanced information technologies, any news can be distributed in the society or its segment. Modern technologies for influencing the public consciousness through mass media can be used with equal success for unifying and stabilizing the society or for alienating and destabilizing it. The decisive factors are the goals of those initiating the information impact and the potential of the objects of this impact, either willing to accept these goals or to protect themselves from external pressure [2]. Success in introducing a new concept in society largely depends on the positions of influential mass media capable of shaping the public opinion (on the one hand), and the subjects of society, such as expert communities or executive bodies, capable of using mass media to cover alternative viewpoints and promote their own concepts in society (on the other hand) [3]. This informational confrontation is characterized by common factors; therefore, formalizing and studying the patterns governing this process is a challenging issue.

<sup>1</sup> On the Strategy of the Information Society Development in the Russian Federation for 2017–2030. Decree of the President of the Russian Federation of May 9, 2017 No. 203 // Collected legislation of the Russian Federation. 2017. No. 20, Article 2901.

## Construction of the model

We have constructed and analyzed the basic mathematical model for dissemination of new information in society. We should note that the model proposed is fairly generalized and should be further refined. However, even in this form it allows to combine the factors required for promoting news information into a system and may be useful for studying the overall picture.

We assume that the main factors for dissemination of new information are the following quantities, depending on the time  $t$ :  $N(t)$ ,  $C(t)$ ,  $A(t)$  and  $i(t)$ . They express the following concepts:

$N(t)$  (News) is the amount of news information (different kinds of messages) contributing to dissemination of a new concept in society (or a segment of society);

$C(t)$  (Censorship) is the number of entities with their own information resources in the structure of society (or a segment of society) interested in preserving previously adopted concepts;

$A(t)$  (Alternative view) is the amount of information (different kinds of messages) hindering dissemination (including on behalf of agencies of censorship) of a new concept in society (or a segment of society);

$i(t)$  (index) is a relative characteristic for acceptance of a new concept at time  $t$ ,

$$i = 1 - \frac{I^*}{I},$$

where  $I$ , % is the characteristic of society fully accepting a certain idea, which is replaced by a new concept;  $I^*$ , % is the corresponding characteristic for acceptance of this idea in dissemination of a new concept.

Evidently,  $i = 0$  before dissemination starts, and  $i = 1$  as the new concept is fully accepted.

Let us construct the corresponding relations for the model. The first equation describes the dynamics of the number of messages  $N(t)$  in mass media:

$$dN = \beta N dt - \gamma A N dt.$$

The expression  $dN$  on the left-hand side corresponds to the numerical change in news information promoting dissemination of a new concept in society over the time interval  $dt$ . Non-negative parameters  $\beta$ ,  $\gamma$  characterize the intensity of information dissemination through mass media and the likelihood that the impact of the message is neutralized by an alternative point of view presented, respectively. Dividing the ratio by  $dt$ , we ultimately obtain the equation



$$\frac{dN}{dt} = \beta N - \gamma AN. \quad (1)$$

The next equation describes the response of different censorship agencies to new information emerging, related to dissemination of new ideas in society. It is assumed that the administrative resource in the amount of  $C_*$  is always used to support concepts in the social environment. Therefore, as new information is disseminated, the activity of information protection agencies  $C$  and, accordingly, the numerical value of the resource may change compared with  $C_*$ :

$$d(C - C_*) = \alpha AN dt - \mu(C - C_*) dt.$$

The non-negative coefficient  $\alpha$  characterizes the reaction to the intensity of the confrontation of alternative viewpoints; the positive parameter  $\mu$  is the coefficient equal to the reciprocal of the time that the additionally established agencies operate.

Given that  $d(C - C_*) = dC$ , we obtain:

$$\frac{dC}{dt} = \alpha AN - \mu(C - C_*). \quad (2)$$

The third equation is used to calculate the balance of the number of alternative news as an opportunity for society to influence the dissemination of a new, unfamiliar concept through mass media. It is proposed to rely on the following ratio:

$$dA = \rho C dt - \eta \gamma AN dt - \lambda A dt, \quad (3)$$

where  $dA$  is the number of current news appearing in the information environment as an alternative to news  $N$  in the time interval  $dt$ ; the first term  $\rho C dt$  on the right-hand side describes the news produced in  $dt$ , while  $\rho \geq 0$  is the average rate of news from one information agency  $C$ ; the second term  $\eta \gamma AN dt$  describes the decrease in the number of current news due to targeted impact on news  $N$  for  $dt$ ,  $\eta \geq 0$  is the average amount of news information  $A$  for neutralizing the effect of message  $N$ ; the third term  $\lambda A dt$  describes how the news information is forgotten in  $dt$ ,  $\lambda > 0$  is the coefficient inversely proportional to the time in which information is forgotten.

Dividing the ratio (3) by  $dt$ , we obtain the equation

$$\frac{dA}{dt} = \rho C - \eta \gamma AN - \lambda A.$$

To characterize the acceptance of a new concept, let us consider the following equation:

$$\frac{di}{dt} = \sigma N - \omega i. \quad (4)$$

Eq. (4) indicates that the rate by which the acceptance of a new idea changes is proportional to the amount of new information  $N$  with a proportionality coefficient  $\sigma > 0$  given the inertia and suspicion towards new information with the corresponding coefficient describing how the acceptance of the old concept is restored,  $\omega \geq 0$ .

As a result, we obtain the following system of nonlinear ordinary differential equations:

$$\begin{aligned} \frac{dN}{dt} &= \beta N - \gamma AN, \\ \frac{dC}{dt} &= \alpha AN - \mu(C - C_*), \\ \frac{dA}{dt} &= \rho C - \eta \gamma AN - \lambda A, \\ \frac{di}{dt} &= \sigma N - \omega i. \end{aligned} \quad (5)$$

From now on, we shall write system (5) in a form that is more convenient for analysis:

$$\begin{aligned} \frac{dC}{dt} &= \alpha AN - \mu(C - C_*), \\ \frac{dA}{dt} &= \rho C - (\lambda + \eta \gamma N) A, \\ \frac{dN}{dt} &= (\beta - \gamma A) N, \\ \frac{di}{dt} &= \sigma N - \omega i. \end{aligned} \quad (6)$$

Let us combine this system of equations (6) with the initial data at  $t = t_0$ :

$$\begin{aligned} C(t_0) &= C_0 \geq 0, A(t_0) = A_0 \geq 0, \\ N(t_0) &= N_0 \geq 0, i(t_0) = i_0 \geq 0, \end{aligned} \quad (7)$$

We shall define the system of equations (6) with initial conditions (7) as the basic mathematical model of propagation of new information in society.

Since system (6) is autonomous, we take  $t_0 = 0$ ; the functions  $C(t)$ ,  $A(t)$ ,  $N(t)$ ,  $i(t)$  are assumed to be continuous in their domain.

### Analysis of the model

**Statement 1.** *If for all  $t \geq 0$  there exists a solution to system (6) with initial conditions (7), then the set*

$$R_+^4 = \{(C, A, N, i) \in \mathbb{R}^4 : C \geq 0, A \geq 0, N \geq 0, i \geq 0\}$$

is invariant for this system.

**Proof.** Indeed, it follows from the third equation of system (6) that the following condition holds true for  $t \geq 0$

$$N(t) = N_0 \exp\left[\int_0^t (\beta - \gamma A) dt\right] \geq 0.$$

This condition preserves the function  $i(t)$  non-negative with  $t \geq 0$ . In fact, if  $N(t) = 0$ , then

$$i(t) = i_0 \exp[-\omega t] \geq 0.$$

If  $N(t) \geq 0$ , then  $i(t) > 0$  near the point  $t = 0$ . Indeed, if  $i_0 = 0$ ,

$$\left.\frac{di}{dt}\right|_{t=0} = \sigma N > 0,$$

$i(t)$  increases in the vicinity of  $t = 0$ .

Then, due to continuity,  $i(t)$  becomes negative if there exists a point  $t = t_1 > 0$ , where

$$i(t_1) = 0, \quad \left.\frac{di}{dt}\right|_{t=t_1} < 0.$$

But this is impossible, since

$$\left.\frac{di}{dt}\right|_{t=t_1} = \sigma N(t_1) - \omega i(t_1) = \sigma N(t_1) > 0.$$

Similarly, it is easy to prove that functions  $C(t)$  and  $A(t)$  are non-negative with initial conditions (7).

Statement 1 is proved.

**Corollary 1.** *If  $C_0 \geq C_*$  for the conditions of Statement 1, then the inequality  $C(t) \geq C_*$  is satisfied for all  $t \geq 0$ .*

The solution of system (6) is non-negative, which corresponds to the meaning of the described process, since the variables of the model are interpreted as quantities whose values cannot be negative.

Similarly, it is easy to prove [4–7] that system (6) has unique, infinitely extendable solutions, continuously depending on parameters.

System (6) admits two stationary solutions:

$$X_{1st} = (C_{1st}, A_{1st}, N_{1st}, i_{1st}) = (C_*, \frac{\rho C_*}{\lambda}, 0, 0);$$

$$X_{2st} = (C_{2st}, A_{2st}, N_{2st}, i_{2st}),$$

where

$$C_{2st} = \frac{\alpha\lambda\beta - \eta\mu\gamma^2 C_*}{\gamma(\alpha\rho - \mu\eta\gamma)}, \quad A_{2st} = \frac{\beta}{\gamma},$$

$$N_{2st} = \frac{\mu(\lambda\beta - \gamma\rho C_*)}{\beta(\alpha\rho - \mu\eta\gamma)}, \quad i_{2st} = \frac{\sigma\mu(\lambda\beta - \gamma\rho C_*)}{\omega\beta(\alpha\rho - \mu\eta\gamma)}.$$

Let us select two regions in the system parameter space, where  $X_{ist} \in R_+^4$ ,  $i = 1, 2$ :

$$\Omega_1 : \begin{cases} \gamma\rho C_* > \lambda\beta, \\ \mu\eta\gamma > \alpha\rho, \end{cases} \quad \Omega_2 : \begin{cases} \gamma\rho C_* < \lambda\beta, \\ \mu\eta\gamma < \alpha\rho. \end{cases}$$

**Interpretation:** Here  $X_{1st}$  can be defined as the state of society in which a certain concept dominates. The administrative resource  $C_*$  with the necessary amount of information  $\rho C_*/\lambda$  in mass media is used to support the concept in society.  $X_{2st}$  is characterized as a state of society where the familiar old concept and the new concept (represented by their shares) coexist, and the relative characteristic of acceptance of new ideas  $i_{2st}$  has a positive value.

To study the stability of stationary solutions of system (6), we linearize it in the vicinity of stationary points  $X_{ist}$ ,  $i = 1, 2$ , and analyze the characteristic equation of the system of its first approximation:

$$W(k) = \begin{vmatrix} a_{11} - k & \alpha N_{ist} & \alpha A_{ist} & 0 \\ \rho & a_{22} - k & -\eta\gamma A_{ist} & 0 \\ 0 & -\gamma N_{ist} & a_{33} - k & 0 \\ 0 & 0 & \sigma & a_{44} - k \end{vmatrix} = 0,$$

where

$$a_{11} = -\mu, \quad a_{22} = -\eta\gamma N_{ist} - \lambda,$$

$$a_{33} = \beta - \gamma A_{ist}, \quad a_{44} = -\omega.$$

For  $X_{1st} = (C_{1st}, A_{1st}, N_{1st}, i_{1st})$ , we have:

$$W_1(k) = \left(\beta - \frac{\rho C_*}{\lambda} - k\right)(\lambda + k)(\mu + k)(\omega + k) = 0.$$

For  $X_{2st} = (C_{2st}, A_{2st}, N_{2st}, i_{2st})$ , respectively,

$$W_2(k) = (\omega + k)(k^3 + ak^2 + bk + c) = 0,$$

where



$$\begin{aligned} a &= \mu + \eta\gamma N_{2st} + \lambda, \\ b &= \mu(\eta\gamma N_{2st} + \lambda) - (\alpha\rho + \eta\gamma\beta)N_{2st}, \\ c &= \beta N_{2st}(\alpha\rho - \mu\eta\gamma). \end{aligned} \quad (8)$$

**Statement 2.** *The stationary solution  $X_{1st}$  of system (6) is asymptotically stable in the parameter domain  $\Omega_1$ , and the solution  $X_{2st}$  is unstable.*

**Proof.** The roots of the characteristic equation for the stationary solution  $X_{1st}$  have the following form:

$$\begin{aligned} k_1 &= -\omega < 0, k_2 = -\mu < 0, \\ k_3 &= -\lambda < 0, k_4 = \beta - \frac{\gamma\rho C_*}{\lambda}. \end{aligned}$$

However,  $k_4 < 0$  in the parameter domain  $\Omega_1$ . Therefore, the roots of  $W_1(k)$  are negative, which actually means that the solution  $X_{1st}$  is asymptotically stable in the linearized system and, therefore, in system (6).

For  $W_2(k)$  in the parameter domain  $\Omega_1$ , the free term

$$c = \beta N_{2st}(\alpha\rho - \mu\eta\gamma) < 0,$$

which means there is a positive root for the corresponding characteristic equation. Consequently, the stationary solution  $X_{2st}$  of system (6) is unstable.

Statement 2 is proved.

**Statement 3.** *The stationary solution  $X_{1st}$  of system (6) is unstable in the parameter domain  $\Omega_2$ , and the solution  $X_{2st}$  is asymptotically stable under the additional condition  $ba - c > 0$ .*

**Proof.** The roots of the characteristic equation for the stationary solution  $X_{1st}$  have the following form:

$$\begin{aligned} k_1 &= -\omega < 0, k_2 = -\mu < 0, \\ k_3 &= -\lambda < 0, k_4 = \beta - \frac{\gamma\rho C_*}{\lambda}. \end{aligned}$$

However,  $k_4 > 0$  in the parameter domain  $\Omega_2$ . Thus, there is a positive root for  $W_1(k)$ , and, therefore, the stationary solution  $X_{1st}$  of system (6) is unstable.

We have  $k_1 = -\omega < 0$  for  $W_2(k)$ . To study the remaining roots of the characteristic equation, let us consider the polynomial

$$P(k) = k^3 + ak^2 + bk + c$$

with coefficients from expressions (8).

Since

$$\begin{aligned} a &= \mu + \eta\gamma N_{2st} + \lambda > 0, \\ c &= \beta N_{2st}(\alpha\rho - \mu\eta\gamma) > 0 \end{aligned}$$

in the parameter domain  $\Omega_2$ , then the condition

$$ba - c > 0 \quad (9)$$

of Statement 3 implies that  $b > 0$ . Along with condition (9) itself satisfied, the Hurwitz criterion [8] can be used to conclude that all real roots and real parts of complex roots of the polynomial  $P(k)$ , and, therefore, the characteristic equation  $W_2(k) = 0$ , are negative. Thus, the stationary solution  $X_{2st}$  is asymptotically stable in the linearized system and, therefore, in system (6).

Statement 3 is proved.

**Remark 1.** Notably, the variable  $i(t)$  appears only in the last equation of system (6), therefore, it makes sense to carry out analysis only for the system

$$\begin{aligned} \frac{dC}{dt} &= \alpha AN - \mu(C - C_*), \\ \frac{dA}{dt} &= \rho C - (\lambda + \eta\gamma N)A, \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{dN}{dt} &= (\beta - \gamma A)N, \\ C(t_0) &= C_0 \geq 0, A(t_0) = A_0 \geq 0, \\ N(t_0) &= N_0 \geq 0, \end{aligned} \quad (11)$$

extending the conclusions and results to the variable  $i(t)$ .

Stationary solutions of system (10), (11) have the form:

$$\begin{aligned} X_{1st} &= (C_{1st}, A_{1st}, N_{1st}) = (C_*, \frac{\rho C_*}{\lambda}, 0), \\ X_{2st} &= (C_{2st}, A_{2st}, N_{2st}) = \\ &= \left( \frac{\alpha\lambda\beta - \eta\mu\gamma^2 C_*}{\gamma(\alpha\rho - \mu\eta\gamma)}, \frac{\beta}{\gamma}, \frac{\mu(\lambda\beta - \gamma\rho C_*)}{\beta(\alpha\rho - \mu\eta\gamma)} \right). \end{aligned}$$

#### Analysis of model (10), (11) in the parameter domain $\Omega_1$ .

The properties of the auxiliary two-dimensional system of differential equations are largely used for analysis of three-dimensional system (10), (11)

$$\begin{aligned} \frac{dA}{dt} &= \rho C - (\lambda + \eta\gamma N)A, \\ \frac{dN}{dt} &= (\beta - \gamma A)N, \end{aligned} \quad (12)$$

obtained from subsystem (10) at  $\alpha = 0$  and  $C(t) = C_*$  at  $t \geq 0$ .



*Interpretation.* This system (12) simulates a situation when the information protection agency do not respond to new information introduced, assuming the previously required amount of administrative resource to be sufficient for supporting familiar concepts and neutralizing the reaction to new information appearing in mass media.

Evidently, system (12) has unique, infinitely extendable solutions continuously dependent on parameters, and the set

$$R_+^2 = \{(A, N) \in R^2 : A \geq 0, N \geq 0\}$$

is invariant for system (12).

System (12) has the following stationary solutions in the parameter domain  $\Omega_1$ :

$$X_{1st} = (A_{1st}, N_{1st}) = \left( \frac{\rho C_*}{\lambda}, 0 \right),$$

$$X_{2st} = (A_{2st}, N_{2st}) = \left( \frac{\beta}{\gamma}, \frac{\gamma \rho C_* - \lambda \beta}{\beta \eta \gamma} \right),$$

lying in  $R_+^2$ , where  $X_{1st}$  is a stable node, and  $X_{2st}$  is a saddle.

Using well-known techniques for qualitative analysis of two-dimensional systems of differential equations [9] and the result of Theorem 4.1 given in [10], we constructed and studied the phase portrait for the trajectories of system (12) (Fig. 1).

Based on construction and the studied properties of the trajectories, we can identify for this phase portrait the following areas of the subspace  $R^2$ :

$$Q_1 = \left\{ (A, N) \in R_+^2 : \frac{\beta}{\gamma} \leq A < \infty, 0 \leq N \leq N_{2st} \right\},$$

$$Q_2 = \left\{ (A, N) \in R_+^2 : 0 \leq A < \frac{\beta}{\gamma}, 0 \leq N \leq N_{2st} \right\},$$

$$Q_3 = \left\{ (A, N) \in R_+^2 : 0 \leq A < \frac{\beta}{\gamma}, N_{2st} \leq N < \infty \right\},$$

$$Q_4 = \left\{ (A, N) \in R_+^2 : \frac{\beta}{\gamma} \leq A < \infty, N_{2st} \leq N < \infty \right\}.$$

Four separatrices adjoin the saddle point  $X_{2st}$  of system (12) in the parameter domain  $\Omega_1$ : stable  $p(t)$ ,  $q(t)$  and unstable  $r(t)$ ,  $s(t)$ ; in this case,  $p \in (t) \in Q_2$ ,  $q(t) \in Q_4$  with  $t \geq 0$ , and  $p(t)$ ,  $q(t) \rightarrow X_{2st}$  with  $t \rightarrow +\infty$ ,  $r(t) \in Q_3$ ,  $s(t) \in Q_1$  and  $r(t)$ ,  $s(t) \rightarrow X_{2st}$  with  $t \rightarrow -\infty$ .  $Q_1$ ,  $Q_3$  are invariant sets with respect to system (12). The curve composed of stable separatrices  $p$ ,  $q$  of the saddle  $X_{2st}$  is the boundary of the domain of attraction of the stable node  $X_{1st}$ .

Since analytical description of the curves representing the separatrices  $p(t)$  and  $q(t)$  is difficult, the proposed statement gives the following estimate of the domain of attraction  $X_{1st}$ , whose equivalent is given below for system (10).

**Statement 4.** Let us give the sets

$$Q_1^* = Q_1 \setminus \{X_{2st}\},$$

$$Q_2^* = \left\{ (A, N) \in Q_2 : 0 \leq A < \frac{\beta}{\gamma}, \right.$$

$$\left. 0 \leq N \leq N_{2st} \exp\left(\frac{-\beta^2}{\rho C_* \gamma}\right) \exp\left(\frac{\beta A}{\rho C_*}\right) \right\}.$$

The set  $Q = Q_1^* \cup Q_2^*$  in the parameter domain  $\Omega_1$  is the estimate of the domain of attraction of the asymptotically stable stationary solution  $X_{1st}$  of system (12).

*Proof.* Since  $Q_1^*$  is an invariant set of system (12) lying in the domain of attraction of the stable node  $X_{1st}$ , it follows from the fact that  $X_0 = (A_0, N_0) \in Q_1^*$  that  $X(t, X_0) \in Q_1^*$  for all  $t \geq 0$ , and  $X(t, X_0) \rightarrow X_{1st}$  at  $t \rightarrow +\infty$ .

Let us prove that if

$$X_0 = (A_0, N_0) \in Q_2^*,$$

then there is a point in time  $t$  when

$$X(t_*, X_0) \in Q_1^*.$$

Let  $N = N(A)$  be the integral curve of the differential equation obtained from system (12):

$$\frac{dN}{dA} = \frac{(\beta - \gamma A)N}{\rho C_* - (\lambda + \eta \gamma N)A} = f(A, N), \quad (13)$$

and  $G = G(A)$  the solution of the equation

$$\frac{dG}{dA} = \frac{(\beta - \gamma A)G}{\rho C_* - (\lambda + \eta \gamma N_{2st})A} \equiv \frac{\beta G}{\rho C_*}. \quad (14)$$

Obviously, for any point  $(A, N) \in Q_2$ ,

$$f(A, N) \leq \frac{\beta N}{\rho C_*},$$

therefore, in accordance with the Chaplygin theorem on differential inequalities [11], if

$$N(A_0) \leq G(A_0),$$

$$(A_0, N(A_0)) \in Q_2,$$

$$(A_0, G(A_0)) \in Q_2,$$

then,

$$N(A) \leq G(A)$$

for those  $A > A_0$ , for which  $(A, N, (A)) \in Q$ .

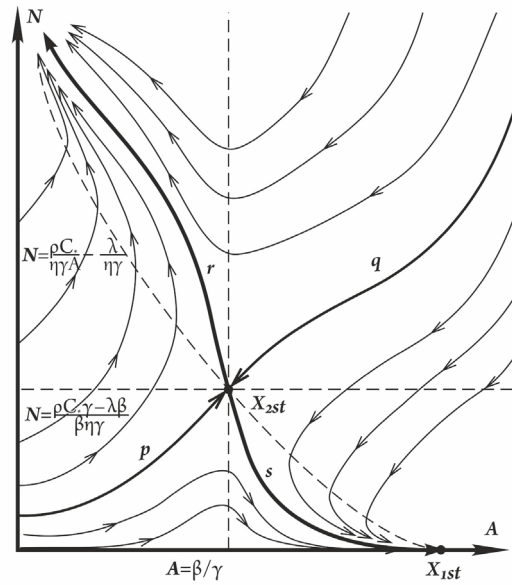


Fig. 1. Phase portrait of system (12)

Let  $G(0)$  be a point on the axis  $ON$ , starting from which the curve  $G(A)$  passes through the point  $(A_{2st}, N_{2st})$ . It follows from Eq. (14) that

$$G(A) = G(0) \exp\left(\frac{\beta A}{\rho C_*}\right),$$

therefore, if  $G(A_{2st}) = N_{2st}$ , then

$$G(0) = N_{2st} \exp\left(\frac{-\beta^2}{\rho C_* \gamma}\right).$$

Based on the Chaplygin theorem, any integral curve of equation (13) falls into the set  $Q_1^*$  with increasing  $A$  if  $N(0) \leq G(0)$ , and, therefore, tends to the stationary solution  $X_{1st}$ . Thus, each point of the set  $Q_2^*$  belongs to the domain of attraction  $X_{1st}$ , and the set  $Q = Q_1^* \cup Q_2^*$  is its estimate.

Statement 4 is proved.

*Interpretation.* Fig. 1 is a clear illustration of the situation evolving in the absence of proper attention to information leaks. If there are few messages about the new concept in mass media or the reaction of society to them is weak, the initial administrative resource may be sufficient to prevent new ideas from entering the public consciousness. However, if there is a substantial amount of new information, the traditional activity of information protection agencies may not be insufficient for neutralizing the public reaction. Without coordinated response from mass media, the new concept becomes dominant in society, because, as evident from

Fig. 1, not every reaction to suppression of a new idea leads to success.

Turning now to system (10), (11), we obtain a similar result. Let us prove the theorem.

**Theorem 1.** *The set  $D = D_1 \cup D_2$  of the phase space  $\{C, A, N\}$  of system (10) in the parameter domain  $\Omega_1$ , where*

$$D_1 = \{(C, A, N) : C_* \leq C < \infty,$$

$$\frac{\beta}{\gamma} \leq A < \infty, 0 \leq N \leq N_*\},$$

$$D_2 = \{(C, A, N) : C_* \leq C < \infty, 0 \leq A < \frac{\beta}{\gamma},$$

$$0 \leq N \leq N_* \exp\left(\frac{-\beta^2}{\rho C_* \gamma}\right) \exp\left(\frac{\beta A}{\rho C_*}\right)\},$$

$$N_* = \frac{\gamma \rho C_* - \lambda \beta}{\beta \eta \gamma},$$

*is the estimate of the domain of attraction of an asymptotically stable stationary solution  $X_{1st}$  of system (10), (11).*

*Proof.* It follows from Statement 4, the first and third equations of system (10), (11) that the set  $D_1$  is invariant.

$$\overline{D_1} = \{(C, A, N) \in \partial D_1 : N = 0\},$$

where  $\partial D_1$  is the boundary of the set  $D_1$ , is also obviously invariant for the solution of system (10), (11), and if

$$X_0 = (C_0, A_0, N_0) \in \overline{D_1},$$

then  $X(t, X_0) \rightarrow X_{1st}$  with  $t \rightarrow +\infty$ , since system (10) is given by linear equations on the set  $\overline{D_1}$

$$\begin{aligned} \frac{dC}{dt} &= -\mu(C - C_*), \frac{dA}{dt} = \rho C - \lambda A, \\ N(t, X_0) &\equiv 0, \end{aligned} \quad (15)$$

for which the singular point  $X_{1st}$  is globally uniformly asymptotically stable.

Let  $X_0 \in D_1 \setminus \overline{D_1}$ ,  $aX(t, X_0)$  be the solution to system (10), (11) starting at  $X_0$ . According to the third equation, the inequality with  $t \geq 0$  for the component  $N(t, X_0)$  of the vector  $X(t, X_0)$ , and only at an isolated point on the time semi-axis  $[0, \infty)$ , where

$$C(t, X_0) = C_*, A(t, X_0) = \frac{\beta}{\gamma},$$

$$N(t, X_0) \equiv N_*.$$

$$\text{Since the point } \left( C_*, \frac{\beta}{\gamma}, N_* \right) \in D_1$$

is not singular for system (10), (11), there exists such a point in time  $t_1 \geq 0$ , when we have  $N(t, X_0) \leq 0$  for all  $t \geq t_1 \geq 0$  for the non-negative function  $N(t, X_0)$ . But then

$$\lim_{t \rightarrow +\infty} N(t, X_0) = 0,$$

and, therefore,

$$X(t, X_0) \rightarrow \overline{D_1} \text{ at } t \rightarrow +\infty.$$

It follows from the theorem on continuous dependence of the solutions of system (10), (11) on the initial data (see [12]) that

$$X(t, X_0) \rightarrow X_{1st} \text{ with } t \rightarrow +\infty,$$

since every solution of system (15) has this property.

Let us consider the behavior of the trajectory of system (10), (11) on the set  $D_2$ . According to the above, when

$$X_0 \in D_2: C_0 \geq C_*$$

we have  $C(t) \geq C_*$  for all  $t \geq 0$ .

Let us consider a system of two equations:

$$\frac{dN_1}{dt} = (\beta - \gamma A_1)N_1, \quad (16)$$

$$\frac{dA_1}{dt} = \rho C(t) - (\lambda + \eta \gamma N_1)A_1,$$

which is equivalent to the equation

$$\frac{dN_1}{dA_1} = \frac{(\beta - \gamma A_1)N_1}{\rho C(t) - (\lambda + \eta \gamma N_1)A_1}. \quad (17)$$

It follows from Eqs. (17) and (13) in the region  $D_2$  that

$$\begin{aligned} \frac{(\beta - \gamma A)N}{\rho C(t) - (\lambda + \eta \gamma N)A} &\leq \\ &\leq \frac{(\beta - \gamma A)N}{\rho C_* - (\lambda + \eta \gamma N)A}. \end{aligned}$$

It follows from the Chaplygin theorem on differential inequalities [11] that if only  $N_1(0) \leq N(0)$ , then the inequality  $N_1(t) \leq N(t)$  holds true for all  $t \geq 0$  in the set  $D_2$  for systems (16) and (12). Therefore, if  $X_0 \in D_2$  for system (10), (11), then the solution  $X(t, X_0) \in D_2$  for all

$$A < \frac{\beta}{\gamma}. \text{ However, since the derivative } A(t) > 0$$

in the set  $D_2$ , and the variable  $C(t)$  is bounded, then  $X(t, X_0)$  falls into the region  $D_1$  in a finite period of time. Therefore, if  $X_0 \in D$ , then any solution

$$X(t, X_0) \rightarrow X_{1st} \text{ with } t \rightarrow +\infty,$$

if only  $X_0 \in D$ .

Theorem 1 is completely proved.

The following stronger theorem holds true.

**Theorem 2.** Let system (10) be given, whose parameters belong to the domain  $\Omega_1$ . Then for the unstable stationary solution  $X_{2st}$  there is a separatrix surface  $W^s$ , which is the exact boundary of the domain of attraction of the asymptotically stable stationary solution  $X_{1st}$ .

**Proof.** Indeed, since the parameters of system (10) belong to the region  $\Omega_1$ , a stable separatrix surface  $W^s(X_{2st})$  passes through the unstable stationary solution  $X_{2st}$ . Then, confirming that conditions A1–A3 of Theorem 4.1 given in [10] hold true, we obtain the result formulated in the theorem.

Theorem 2 is proved.

**Interpretation.** The results obtained indicate that with the relations given for the system parameters in the space  $\{C, A, N\}$ , there is a region from which the system tends to the steady state  $X_{1st}$ . In this state, the society (or its segment) is completely dominated by the familiar old concept. Therefore, a trajectory falling into the described domain can be achieved, theoretically, at any moment in time, by carefully controlling the system parameters.



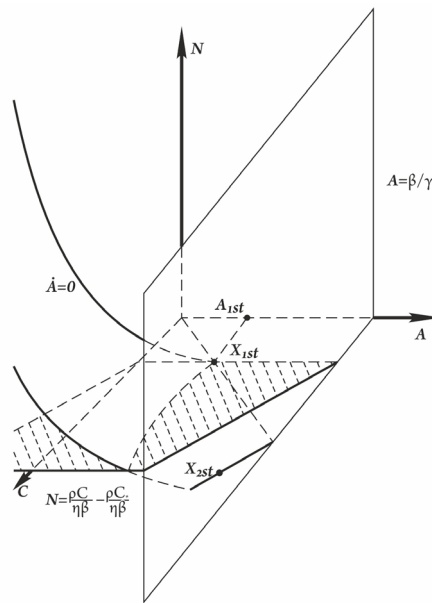


Fig. 2. Set  $G$  (see Eq. (20)) in phase space of system (10), (11)

### Analysis of system (10), (11) in the parameter domain $\Omega_2$

According to Remark 1, system (10), (11) is a reduction of system (5), (6). Therefore, by virtue of expression (3), the stationary solution  $X_{1st}$  of system (10) is unstable in the parameter domain  $\Omega_2$ , and the solution  $X_{2st}$  is asymptotically stable provided that additional condition (9) holds true. It is of interest to study the domain of attraction for the stationary solution  $X_{2st}$  of system (10), (11).

Let us consider a surface in the space  $\{C, A, N\}$ , where  $\dot{A}(t)$  is equal to zero:

$$N = \frac{\rho C}{\eta \gamma A} - \frac{\lambda}{\eta \gamma}. \quad (18)$$

We introduce an additional relation:

$$\rho C_*(\rho \alpha + \eta \beta \gamma) - \lambda \eta \beta (\mu + \beta) \geq 0 \quad (19)$$

and consider the following set (shown in Fig. 2):

$$G = \{(C, A, N) : 0 < N < \mathbf{p}(C - C_*), A_{1st} \leq A < \infty\}, \quad (20)$$

where

$$\mathbf{p} - \text{const.} : \frac{\mu \lambda + (\lambda \beta - \rho C_* \beta \gamma)}{\rho C_* \alpha} < \mathbf{p} < \frac{\rho}{\eta \beta}.$$

Fig. 2. Set  $G$  (see Eq. (20)) in phase space of system (10), (11)

**Statement 5.** Let condition (19) be satisfied for system (10), (11) in the parameter domain  $\Omega_2$ . Then the set  $G$  is invariant for this system.

**Proof.** Let us find the direction of the vector field on the surface  $N = \mathbf{p}(C - C_*)$ , defined in relation (19).

Scalar product of vectors

$$\mathbf{n} = \left( \frac{\partial N}{\partial C}, \frac{\partial N}{\partial A}, -1 \right) = (\mathbf{p}; 0; -1)$$

and

$$\frac{dX}{dt} = \left( \frac{dC}{dt}, \frac{dA}{dt}, \frac{dN}{dt} \right)$$

has the form

$$\mathbf{p}(C - C_*) [(\alpha p + \gamma)A - (\mu + \beta)].$$

This expression is greater than zero for

$$A > \frac{\mu + \beta}{\alpha p + \gamma} = \bar{A},$$

which means that the trajectories of the system, given such  $A$  from the plane

$$N = \mathbf{p}(C - C_*),$$

fall in the set  $G$ . However,  $A_{1st} \leq A$  in this set.

In other words, condition  $\bar{A} \leq A_{1st}$  should be satisfied for the set  $G$  to be invariant. But given  $\mathbf{p}$  from expression (20), the condition holds true only if relation (19) is satisfied (see Remark 2 below). The vector field on some part of the plane  $A = A_{1st}$  belonging to the set  $G$  is directed inside this set, since, given that  $N > 0$ , the inequalities hold true

$$\left. \frac{dA}{dt} \right|_{A=A_{1st}} > \rho(C - C_*) \frac{\lambda\beta - \rho C_* \gamma}{\lambda\beta} > 0.$$

Therefore, all trajectories of this system from the boundary  $A = A_{1st}$  fall in the set  $G$ . Since the plane  $N = 0$  is invariant, the trajectories cannot fall to this plane from the set  $G$  (the theorem on uniqueness is violated otherwise).

Statement 5 is proved.

**Remark 2.** The restriction that the parameter  $\mathbf{p}$  be bounded from below follows from the fact that the inequality  $\bar{A} \leq A_{1st}$  holds true only for

$$\frac{\mu\lambda + (\lambda\beta - \rho C_* \beta \gamma)}{\rho C_* \alpha} < \mathbf{p}.$$

The restriction that the parameter  $\mathbf{p}$  be bounded from above is required later in Statement 6.

If  $\mathbf{p}$ , condition  $\bar{A} \leq A_{1st}$  from expression (20) is indeed satisfied only with relation (19). The expression

$$q \frac{\rho}{\eta\beta} + (1-q) \frac{\mu\lambda + (\lambda\beta - \rho C_* \beta \gamma)}{\rho C_* \alpha},$$

$$q \in (0; 1)$$

describes the interval

$$\left( \frac{\mu\lambda + (\lambda\beta - \rho C_* \beta \gamma)}{\rho C_* \alpha}; \frac{\rho}{\eta\beta} \right).$$

Then, provided that condition  $\bar{A} \leq A_{1st}$

$$\mathbf{p} \in \left( \frac{\mu\lambda + (\lambda\beta - \rho C_* \beta \gamma)}{\rho C_* \alpha}; \frac{\rho}{\eta\beta} \right)$$

takes the form

$$q\rho^2 C_* \alpha + (1-q)\eta\beta[\mu\lambda + (\lambda\beta - \rho C_* \gamma)] + \rho C_* \eta\beta\gamma > \lambda\eta\beta(\mu + \beta).$$

This implies the inequality

$$(q-1)[\rho^2 C_* \alpha - \eta\beta(\mu\lambda + \lambda\beta - \rho C_* \gamma)] + \rho C_* \eta\beta\gamma + \rho^2 C_* \alpha - \lambda\eta\beta(\mu + \beta) > 0,$$

which is equivalent to the inequality satisfied

$$q[\rho C_* (\rho\alpha + \eta\beta\gamma) - \lambda\eta\beta(\mu + \beta)] > 0.$$

**Statement 6.** Let condition (19) be satisfied for system (10), (11) in the parameter domain  $\Omega_2$ . Then the trajectories of this system are bounded on the set  $G$ .

**Proof.** Let us prove the statement in two stages.

*Stage 1.* Let  $G_1$  be a subset of  $G$ , where

$$A_{1st} \leq A \leq \frac{\beta}{\gamma}.$$

Let us confirm that the trajectories of the systems are limited in the subset  $G_1$ .

For this purpose, let us consider the plane

$$\mathbf{a}A - C + \mathbf{b} = 0 \quad (21)$$

with the normal  $\mathbf{n}_1 = (-1; \mathbf{a}; 0)$ . Let us select the coefficients  $\mathbf{a}, \mathbf{b} > 0$  so that plane (21) intersects the subset  $G_1$ . Let us consider the vector field on this plane in  $G_1$

$$\frac{dX}{dt} = \left( \frac{dC}{dt}; \frac{dA}{dt}; \frac{dN}{dt} \right)$$

of system of equations (10). Namely, let us find the part of plane (21), where the scalar product of the vectors  $\mathbf{n}_1$  and  $d\mathbf{X}/dt$  is greater than zero. This product has the form:

$$\begin{aligned} & \mu(C - C_*) - \alpha AN + \alpha\rho C - \\ & - \mathbf{a}\lambda A - \mathbf{a}\eta\gamma AN > \\ & > \mu(C - C_*) - \alpha \frac{\beta}{\gamma} N + \\ & + \alpha\rho C - \mathbf{a}\lambda A - \mathbf{a}\eta\gamma \frac{\beta}{\gamma} N. \end{aligned}$$

Given this estimate and equation of plane (21), the scalar product is guaranteed to be positive with

$$N < \frac{\mathbf{a}\gamma(\mathbf{a}\rho + \mu - \lambda)}{\beta(\mathbf{a}\eta\gamma + \alpha)} A - \frac{\gamma(\mu C_* - \mathbf{a}\beta\rho)}{\beta(\mathbf{a}\eta\gamma + \alpha)}.$$

Let us find the ratio for which this product is greater than zero at any point on plane (21) from the subset of the  $G_1$ . For this purpose, let us find the intersection of the plane

$$N = \mathbf{p}(C - C_*)$$

with plane (21). The equation of this straight line has the form:

$$N = \mathbf{p}\mathbf{a}A + \mathbf{p}\mathbf{b} - \mathbf{p}C_*. \quad (22)$$

If the coefficient at  $A$  in the equation for line (22) is less than the corresponding coefficient of the line

$$N = \frac{\mathbf{a}\gamma(\mathbf{a}\rho + \mu - \lambda)}{\beta(\mathbf{a}\eta\gamma + \alpha)} A - \frac{\gamma(\mu C_* - \mathbf{a}\beta\rho)}{\beta(\mathbf{a}\eta\gamma + \alpha)}, \quad (23)$$

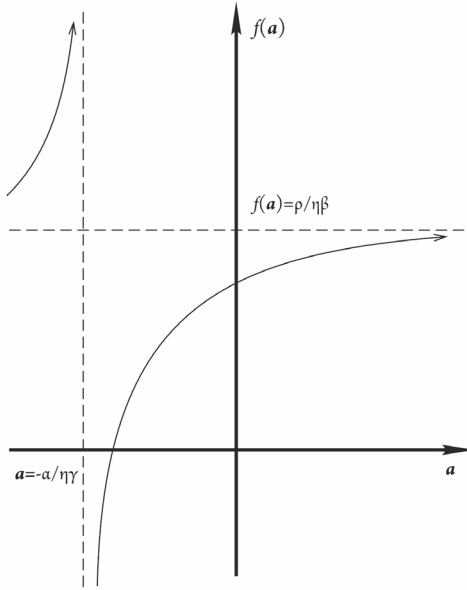


Fig. 3. Plotted function  $f(a)$

then it is possible to find such a constant  $\mathbf{b}$  in Eq. (21) that straight lines (22) (23) will intersect at point  $A_{1st}$ , and with the condition

$$A_{1st} \leq A \leq \frac{\beta}{\gamma} \quad (24)$$

the vector of system on the plane (21) in the subset  $G_1$  will be co-directed with the vector  $\mathbf{n}_1$  from the plane (21). This will mean that for any trajectory from the set  $G$  with the condition (24) there is a “partition”, which does not make possible for it to extend to infinity on the subset  $G_1$ .

It remains to prove that it is always possible to select  $\mathbf{a}$  such that for the the relationship

$$\mathbf{p} < \frac{\gamma(\mathbf{a}\rho + \mu - \lambda)}{\beta(\mathbf{a}\eta\gamma + \alpha)}.$$

will hold true for the coefficients with  $A$  in equations (22), (23).

Qualitative behavior of the function

$$f(a) = \frac{\gamma(\mathbf{a}\rho + \mu - \lambda)}{\beta(\mathbf{a}\eta\gamma + \alpha)}$$

is shown schematically in Fig. 3. It is evident from its analysis that there exists any

$$0 < \mathbf{a} < \infty : f(a) > \mathbf{p}$$

for  $\mathbf{p} < \rho/\eta\beta$ .

Thus, after falling into the set  $G$  with the condition (24), the trajectory of system (10), can extend to infinity, if only it crosses the plane  $A = \beta/\gamma$ .

*Stage 2.* Beginning in the subset of set  $G$ , where  $A > \beta/\gamma$ , trajectory also cannot extend to infinity. Actually, with  $A > \beta/\gamma$  it follows from the third equation of system (10) that  $\dot{N}(t) < 0$ . Therefore, if we rely on the reasoning similar to that used to prove Theorem 1, it is possible to confirm that the relationship

$$\lim_{t \rightarrow +\infty} N(t, X_0) = 0$$

holds true for the component  $N(t, X_0)$  of the vector  $X(t, X_0)$  only if  $X_0$  belongs to this subset.

In other words, the trajectory of system for the final time interval falls into the sufficiently small neighborhood of the plane  $N = 0$ . However, there are no solutions which become infinite on this plane. Therefore, the theorem about the continuous dependence on the initial data (12) guarantees that the trajectory on this subset also cannot extend into infinity. Consequently, if we assume the existence of this trajectory, then it must fall in  $G_1$  via the plane  $A = \beta/\gamma$ .

*Stage 3.* The reasoning of the previous stages makes it possible to draw the conclusion that if a trajectory which becomes infinite is located in the set  $G$ , then it must intersect the plane  $A = \beta/\gamma$  an infinite number of times. Suppose that such a trajectory exists. Let us note in this case that in view of expression (18), the intersection of plane  $A = \beta/\gamma$  towards the decrease in  $A(t)$  occurs with

$$N > \frac{\rho C}{\eta\beta} - \frac{\lambda}{\eta\gamma},$$

because the straight line

$$N = \frac{\rho C}{\eta \beta} - \frac{\lambda}{\eta \gamma}$$

is the intersection of surface (18) with the plane  $A = \beta/\gamma$  (see Fig. 2)) and with  $N < p(C - C_*)$ . Because  $p < \rho/\eta\beta$ , the straight lines

$$N = \frac{\rho C}{\eta \beta} - \frac{\lambda}{\eta \gamma}, N = p(C - C_*)$$

intersect at the endpoint of plane  $A = \beta/\gamma$ . In this case a precompact set

$$S = \left\{ (C, A, N) \in G : A = \frac{\beta}{\gamma}, \right. \\ \left. \frac{\rho C}{\eta \beta} - \frac{\lambda}{\eta \gamma} < N < p(C - C_*) \right\}$$

is formed in the set  $G$ .

Let us compose a sequence  $\{x_k\}$  of such intersection points of the trajectory of plane  $A = \beta/\gamma$ , from which let us isolate a converging subsequence. Let us denote its limit as  $x^*$ . Then the trajectory falls from the point  $x^*$  into the subset  $G_1$ , where  $A < \beta/\gamma$ , from where, according to what was proved during the first stage, it intersects the plane  $A = \beta/\gamma$  towards an increase in  $A(t)$  at the endpoint.

As a result, there is contradiction with the assumption that the trajectory becomes infinite. Consequently, all given trajectories of the system are limited in the set  $G$ .

Statement 6 is proved.

**Theorem 3.** *If condition (19) is satisfied for system (10), (11) in the parameter space  $\Omega_2$ , the set  $G$ , determined by expression (20), is the estimate of the domain of attraction of the asymptotically stationary solution  $X_{2st}$ .*

**Proof.** Let us give a Lyapunov function on the set  $G$  (recall that it is invariant according to Statement 5):

$$V(X, t) = \gamma AN - \beta N - \gamma \int_0^t \dot{A} N d\tau.$$

By virtue of system (10), (11), its derivative has the following form:

$$\dot{V}(X, t) = \gamma \dot{N} A + \gamma \dot{A} N - \beta \dot{N} - \gamma \dot{A} N = \\ = \dot{N}(\gamma A - \beta) = -(\beta - \gamma A)^2 N \leq 0.$$

Let us prove that the function  $V(X, t)$  is bounded from below. The term  $\gamma AN - \beta N$  is bounded from below because the trajectories

of the system are limited on the set  $G$ . The last term of function  $V(X, t)$  on a set where  $\dot{A} < 0$  is positive. On a set where  $\dot{A} > 0$ , the term can be estimated thus:

$$-\gamma \int_0^t \dot{A} N d\tau \geq -\gamma \int_0^t \dot{A} N_{\max} d\tau = \\ = -\gamma N_{\max} [A(t) - A(0)] \geq \\ \geq -\gamma N_{\max} A_{\max} + \gamma N_{\max} A(0).$$

Therefore, the function  $V(X, t)$  is bounded from below. Obviously, the derivative  $\dot{V}(X, t)$  is also bounded from below. Thus, according to Statement VIII.4.7, given in (13), it is possible to claim that

$$V(X, t) \text{ with } t \rightarrow +\infty.$$

This means that the trajectory of the system tends to its  $\omega$ -limit set

$$M_0 \in M = \left\{ (C, A, N) \in G : A = \frac{\beta}{\gamma} \right\}.$$

According to the property of  $\omega$ -limit sets for autonomous systems,  $M_0$  is invariant by virtue of system (10), (11). However,  $M_0$  is only invariant on the plane  $A = \beta/\gamma$  when  $M_0 = \{X_{2st}\}$ . Therefore,

$$X(t, X_0) \rightarrow X_{2st} \text{ with } t \rightarrow +\infty.$$

Thus,  $G$  is the estimate of the domain of attraction for  $X_{2st}$ .

Theorem 3 is proved.

**Theorem 4.** *Let the condition (19) and the relationship*

$$\rho\alpha - \mu\eta\gamma > \beta\eta\gamma. \quad (25)$$

*be satisfied for system (10), (11) in the parameter space  $\Omega_2$ .*

Then the entire space

$$R^+ = \{(C, A, N) \in R^3 : C \geq 0, A \geq 0, N > 0\}$$

*is part of the domain of attraction of the asymptotically steady stationary solution  $X_{2st}$ .*

**Proof.** Let us confirm that all trajectories which originate in  $R^+$ , fall into the set  $G$ , from where, according to theorem 3, they tend to the stationary solution  $X_{2st}$  with  $t \rightarrow +\infty$ .

The right-hand side of the equation for  $C(t)$  of system (10) guarantees that a trajectory with the initial data from  $R^+$  falls into the invariant subspace, where  $C(t) \geq C_*$ . Therefore, let us consider in this subspace. Let us break it into two subsets:



$$T_1 = \{(C, A, N) \in R^+ : C \geq C_*, A \geq A_{1st}\},$$

$$T_2 = \{(C, A, N) \in R^+ : C \geq C_*, 0 \leq A \leq A_{1st}\}.$$

Let a point in the trajectory be located in  $T_1 \setminus G$ . Then, for a certain number  $q > 0$ , this point, according to relation (21), lies on the plane

$$N = p(C - C_*) + q.$$

Consequently, trajectories from this plane fall into the set  $G$ , or into the subset  $T_2$ . Let us prove now that all trajectories from  $T_2$  fall into the region where

$$N < \frac{\rho C}{\eta \beta} - \frac{\lambda}{\eta \gamma},$$

(see Fig. 2) and, therefore,  $\dot{A} > 0$ .

Let us give a function  $V_{T_2} = \dot{A}$  in the subset  $T_2$  and consider the sign of its derivative, taking into account the properties of system (10) on the surface  $V_{T_2}(X) = 0$ . In view of expression (18), we have:

$$\begin{aligned} \dot{V}_{T_2} &= \ddot{A} = \rho \dot{C} - \eta \gamma \dot{N} A = \\ &= \left( \frac{\rho C - \lambda A}{\eta \gamma} \right) (\rho \alpha - \mu \eta \gamma - \beta \eta \gamma + \eta \gamma^2 A) + \\ &\quad + \rho \mu C_* - \lambda \mu A. \end{aligned}$$

If relationship (25) is fulfilled in the subset  $T_2$ ,

$$\dot{V}_{T_2}(X) \Big|_{X: V_{T_2}(X)=0} > 0.$$

If  $V_{T_2}(X) < 0$  in a certain part of the space  $T_2$ , the study of the sign of the derivative of  $V_{T_2}(X)$  is reduced by virtue of system (10) to calculating the sign of the derivative on the surface  $V_{T_2}(X) = 0$ , since

$$\begin{aligned} \dot{V}_{T_2}(X) \Big|_{X: V_{T_2}(X) \leq 0} &= \ddot{A} = \rho \dot{C} - \lambda \dot{A} - \\ &\quad - \eta \gamma \dot{N} A - \eta \gamma \dot{A} N \geq \\ &\geq \rho \dot{C} - \eta \gamma \dot{N} A = \dot{V}_{T_2}(X) \Big|_{X: V_{T_2}(X)=0} > 0. \end{aligned}$$

This means that all trajectories from  $T_2$  fall into the region where  $\dot{A} > 0$  and, therefore, also into the region  $G$ , from which they tend to the asymptotically stationary solution  $X_{2st}$ .

Theorem 4 is proved.

*Interpretation.* In this section we have obtained the relations for the parameters of the systems characterizing the readiness of society together with the existing concept to accept new ideas. Therefore, any new idea which was appeared in the media finds response. Over time, old and new ideas come to co-exist with their shares of acceptance in the society.

## Conclusion

The study carried out allows to formulate the following main results.

1. We have found the generalized factors and patterns describing how new information propagates in society, constructing a basic mathematical model for the dissemination of new information. The obtained model is the system of four ordinary differential equations with quadratic nonlinearity in the right-hand sides.

2. Using methods of qualitative analysis, we considered the global properties of the phase portrait of the constructed dynamic system.

3. We have given an interpretation of the key findings, allowing to isolate several possible scenarios of the course of events and to influence them.

The results obtained in this study continue the systemic studies started in [14] and further developed in [15–18]. Our focus was the study of media systems as one of the most important and high-speed dynamic systems. Mathematical methods are fundamental tools making it possible to carry out in-depth media studies with novel scientific value. Methods of nonlinear dynamics provide the means for comprehensive analysis of the structure and properties of processes in the mass media system.

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