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IMPACT OF DIPOLE-DIPOLE INTERACTION ON THE CAVITY MODE EVOLUTION IN THE MODEL OF FEW EMITTERS LASER

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In the paper, the simple model of a two-atom laser is theoretically studied. The both dipole-dipole coupled atoms are under conditions of incoherent pump, are placed into the Fabry – Perot cavity and interact with a single damping field mode. In the switched-off pump position, the effect of dipole-dipole interaction on evolution of the damping mode has been considered. This evolution was shown to be strongly dependent on an initial atomic superposition state of atoms. In the switched-on pump position, the 'memory' for the initial atomic state should collapse with time.

Keywords: nanolaser, single-atom laser, dipole-dipole interaction, near-field effect

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ВЛИЯНИЕ ДИПОЛЬ-ДИПОЛЬНОГО ВЗАИМОДЕЙСТВИЯ НА ЭВОЛЮЦИЮ МОДЫ РЕЗОНАТОРА В МОДЕЛИ ЛАЗЕРА НА НЕСКОЛЬКИХ ИЗЛУЧАТЕЛЯХ

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В работе теоретически исследуется модель лазера на двух атомах. Оба двухуровневых атома находятся в условиях некогерентной накачки, помещены в резонатор Фабри – Перо и взаимодействуют с выделенной затухающей модой. В случае выключенного поля накачки рассматривается влияние межатомного диполь-дипольного взаимодействия на эволюцию затухающей моды резонатора. Показано, что эта эволюция существенно зависит от начального суперпозиционного состояния атомов. При включенной некогерентной накачке «память» о начальном состоянии атомов в резонаторе со временем исчезает.

Ключевые слова: нанолазер, одноатомный лазер, диполь-дипольное взаимодействие, ближнее поле

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Introduction

There is currently much interest in systems consisting of one or more quantum objects placed in certain conditions and interacting with an electromagnetic field modified by these conditions. Nanolasers [1–6], i.e., lasers whose working medium is either one or several quantum emitters (such as atoms [7], ions [8], quantum dots (QDs) [9]) are a striking example of these systems. Aside from the obvious fundamental interest in nanolasers as purely quantum systems, the devices have valuable practical applications. They can be used as sources of nonclassical field states or act as qubits in complex quantum networks [10, 11]. An example of this is a system consisting of several QTs placed in separate microcavities connected by fiber or field modes [12]. Another example is atoms embedded in optical fiber and interacting with the modes of this fiber [13]. Such a quasilinear system is used to create entangled atomic states and is in demand in quantum information science.

Collective effects associated with interatomic resonant dipole-dipole interaction appear if the emitters are placed at a distance of the order of emission wavelength relative to each other. It was found in [14, 15] for ensembles of impurity atoms embedded in a solid matrix in a Fabry–Perot cavity or for individual atoms located near a charged surface that dipole-dipole interaction substantially modifies the dynamics of spontaneous emission of atoms. It was established in [16, 17] that dipole-dipole interaction has to be taken into account to correctly describe the dynamics of entangled states of two or more atoms in a lossless cavity. Collective phenomena can also affect interference in light scattering in dense atomic clouds cooled in magneto-optical traps [18–20].

Several theoretical papers [21–23] considered a system consisting of one or two QDs associated with a single damped cavity mode, discussing the effect that frequency detuning of the cavity mode from QD resonances has on collective states of the emitters. Generation of an incoherently pumped laser was studied, and conditions for strong entanglement between the two emitters were found. The steady-state mode of such a laser was also investigated.

The specifics of a QD laser is that it has a valence band and a conduction band, and, consequently, a corresponding interaction operator; omitting these issues from consideration, we can compare the problems considered in [21–23] with the problem of a laser with two two-level atoms.

We have considered a model of a laser based on two two-level atoms with a closed incoherent pump circuit, interacting through a Fabry–Perot cavity with a single damped mode. The effect of interatomic dipole-dipole interaction on the evolution of the damped cavity mode was examined with the pump switched off. The evolution of the initial superposition states of atoms was studied under incoherent pumping. The main difference from the above-mentioned works was that we included the effects that the structure of cavity modes has on different atomic relaxation constants and took into account the finite lifetime of the selected cavity mode.

Nanolaser model

Let us describe a laser with two emitters using the following model. Two identical two-level atoms at rest are placed in a Fabry–Perot cavity. The dimensions of the mirrors are taken to be sufficiently large for diffraction at their edges to be neglected. Lossy cavity modes have a complex structure [24], so all atomic relaxation constants are calculated in the approximation of an ideal Fabry–Perot cavity [14].

Selecting a single damped cavity mode with which atoms interact is an approximation that has proven to be effective in some problems for conventional lasers (see, for example, [25, 26]). An emitter in a nanolaser interacts (in particular, due to fluctuations in the position of the mirrors) with a superposition of modes, characterized by a certain spread of wave vectors near the direction of the cavity axis. The components of this superposition significantly affect the behavior of the nanolaser. We have not considered this aspect in our study, focusing instead on a simplified situation when atoms interact with a single scalar damped mode.

The effect of incoherent pumping is assumed to be the same for both atoms, regardless of their spatial location.

Using the corresponding experiments as reference, we assume the intensity of the intracavity field to be small, so that the average number of photons in the cavity $\langle n \rangle << 1$. The magnitude of $\langle n \rangle$ can be estimated by the ratio of incoherent pumping rate of atoms to the decay rate of cavity mode.

The equation for the density operator of the given nanolaser model has the following form [27]:

$$\begin{aligned}\frac{\partial\hat{\rho}}{\partial t} = -\frac{i}{\hbar}\left[\hat{V},\hat{\rho}\right] + \frac{\kappa}{2}\left(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}\right) + \\ + \sum_{i=a,b}\frac{\gamma_i}{2}\left(2\hat{\sigma}_i\hat{\rho}\hat{\sigma}_i^{\dagger} - \hat{\sigma}_i^{\dagger}\hat{\sigma}_i\hat{\rho} - \hat{\rho}\hat{\sigma}_i^{\dagger}\hat{\sigma}_i\right) + \\ + \sum_{i=a,b}\frac{\Gamma}{2}\left(2\hat{\sigma}_i^{\dagger}\hat{\rho}\hat{\sigma}_i - \hat{\sigma}_i\hat{\sigma}_i^{\dagger}\hat{\rho} - \hat{\rho}\hat{\sigma}_i\hat{\sigma}_i^{\dagger}\right) + \\ + \sum_{i\neq j=a,b}\frac{\gamma_{ij}}{2}\left(2\hat{\sigma}_j\hat{\rho}\hat{\sigma}_i^{\dagger} - \hat{\sigma}_i^{\dagger}\hat{\sigma}_j\hat{\rho} - \hat{\rho}\hat{\sigma}_i^{\dagger}\hat{\sigma}_j\right) - \\ - i\sum_{i\neq j=a,b}\Omega_{ij}\left[\hat{\sigma}_i^{\dagger}\hat{\sigma}_j,\hat{\rho}\right].\end{aligned} \tag{1}$$

Here $\hat{a}$, $\hat{a}^{\dagger}$ are the photon production/annihilation operators in damped cavity mode; $\hat{\sigma}_i = |g_i\rangle\langle e_i|$, $\hat{\sigma}_i^{\dagger} = |e_i\rangle\langle g_i|$ are the projection operators for an *i*th atom ($i = a, b$), where $|g_i\rangle$ and $\langle e_i|$ are the vectors of the ground and the excited state of the *i*th atom, respectively.

The operator of interaction of atoms with the cavity mode has the form

$$\begin{cases}\hat{V} = i\hbar\sum_{i=a,b} g(i)\left(\hat{a}^{\dagger}\hat{\sigma}_i - \hat{\sigma}_i^{\dagger}\hat{a}\right), \\ g(i) = \sqrt{\dfrac{2\pi\omega_{n_0}}{\hbar L^2 a}}\, d_{21}\sin\left(k_{n_0}z_i\right),\end{cases} \tag{2}$$

where $g(i)$ is the coupling constant for the *i*th atom, depending on its spatial position; $\omega_{n0} = ck_{n0} = 2\pi c/\lambda_{n0}$ is the frequency of the atomic transition $|g_i\rangle \to |e_i\rangle$, coinciding with the frequency of the cavity mode.

As noted above, the dimensions of the mirrors are $L \times L$, such that $L >> a$ (a is the distance between the mirrors). The second term on the left-hand side of Eq. (1) describes the decay of the cavity mode at a rate κ. The third term in Eq. (1) is responsible for independent spontaneous decay of an *i*th atom at a rate γ_i due to interaction with reservoir cavity modes. Incoherent pumping of atoms at a rate Γ corresponds to the fourth term in Eq. (1). The last two terms in the equation result from resonant dipole-dipole interaction between atoms, where γ_{ij} is the relaxation constant describing dependent spontaneous decay of atoms, and Ω_{ij} is the corresponding shift.

Explicit expressions of the relaxation constants γ_i, γ_{ij}, Ω_{ij} for two-level atoms (transition $J_g = 0 \to J_e = 1$) were obtained in [14]. An example is the expression for the constant of independent spontaneous decay of an atom in a Fabry–Perot cavity (expressions for other constants can be found in [14]):

$$\begin{aligned}\frac{\gamma_i}{2} = \frac{\pi\gamma_0}{2ak_{n_0}} + \\ + \frac{\pi\gamma_0}{ak_{n_0}}\sum_{n=1}^{[[ak_{n_0}/\pi]]}\left(1 - \frac{k_n^2}{k_{n_0}^2}\cos^2\left(2k_n z_i\right)\right),\end{aligned} \tag{3}$$

where $\gamma_0 = \dfrac{2k_{n_0}^3 d_{eg}^2}{3\hbar}$, $k_n = \dfrac{\pi n}{a}$, $n = 1, 2, \ldots,$ and $[[x]]$ denotes the largest integer value of x less than x.

To solve operator equation (1), it was rewritten in terms of occupation numbers $|n\rangle$ with respect to the field mode and in projections to atomic states $|\alpha_i\rangle$ [3]:

$$\left\langle\alpha_i\right|\left\langle n\left|\hat{\rho}\right|m\right\rangle\left|\alpha_j\right\rangle \equiv \rho_{nm}^{\alpha_i\alpha_j},$$

where

$$\begin{aligned}&\begin{cases}\hat{a}\left|n\right\rangle = \sqrt{n}\left|n-1\right\rangle, \\ \hat{a}^{\dagger}\left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle;\end{cases} \\ &\begin{cases}\left|\alpha_1\right\rangle = \left|g_a\right\rangle\left|g_b\right\rangle, \left|\alpha_2\right\rangle = \left|e_a\right\rangle\left|e_b\right\rangle, \\ \left|\alpha_3\right\rangle = \left|e_a\right\rangle\left|g_b\right\rangle, \left|\alpha_4\right\rangle = \left|g_a\right\rangle\left|e_b\right\rangle.\end{cases}\end{aligned} \tag{4}$$

Calculation results and discussion

All calculations are carried out for the following geometry of the experiment. The distance between the mirrors is chosen equal to $a = 50\lambda_{n0}$, and the cavity axis is directed along the z axis. Atoms in the cavity are located at a distance

$$r_{ab} = \sqrt{\rho_{ab}^2 + \left(z_a - z_b\right)^2} \approx \lambda_{n_0}/2\pi$$

from each other, with their coordinates chosen so that each atom participates in the interaction with the damped mode:

$$\rho_{ab} = \sqrt{\left(x_a - x_b\right)^2 + \left(y_a - y_b\right)^2} = 0.2\lambda_{n_0},$$

$$z_a = 15.25\lambda_{n_0},\ z_b = 15.10\lambda_{n_0};$$

in this case, atomic relaxation constants and coupling constants (in units of κ) are equal to:

$$\gamma_a/\kappa = 19.7,\ \gamma_b/\kappa = 19.8,$$

$$\gamma_{ab}/\kappa = 6.0;\ \Omega_{ab}/\kappa = 0.6;$$

$$g(a)/\kappa = 5.0\ g(b)/\kappa = 2.7.$$

First let us consider the situation with incoherent pumping switched off. Fig. 1,*a* shows, for different ground states of the atom-field system

$$\left|\Psi_3\right\rangle = \left|\alpha_3\right\rangle\left|vac\right\rangle,\ \left|\Psi_4\right\rangle = \left|\alpha_4\right\rangle\left|vac\right\rangle,$$

the time dependence for the probability

$$p_{11}(t) = \mathrm{Sp}_{at}\left[\rho_{11}^{\alpha\alpha'}\right]$$

of detecting one photon in damped cavity mode. Here $Sp_{at}[...]$ denotes the trace with respect to atomic variables, and $|vac\rangle \equiv |n = 0\rangle$ the vacuum state of the damped cavity mode.

It follows from Fig. 1, *a* that including the dipole-dipole interaction substantially affects the result. For example, the probability $p_{11}(t)$ for the initial state $|\Psi_4\rangle$ (when the atom *b* is in the excited state, and the atom *a* in the ground state) gains an additional maximum if dipole-dipole interaction is taken into account. This behavior of the probability $p_{11}(t)$ can be explained by "transfer" of excitation from atom *b* to atom *a*, which interacts more strongly with the damped mode due to the spatial position chosen (i.e., $g(a) > g(b)$).

Fig. 1, *b* shows an example of initially entangled atomic states

$$\left|\Psi^{\pm}\right\rangle = \left(\left|\alpha_3\right\rangle \pm \left|\alpha_4\right\rangle\right)\left|vac\right\rangle / \sqrt{2}.$$

Different decay rates are observed for these states, the same as for free space. Unlike the previous case, no additional maxima appear on the time dependence of $p_{11}(t)$ if dipole-dipole interaction is taken into account, due to symmetric distribution of atomic excitations at the initial time.

Notably, including dipole-dipole interaction increases the probability of finding a photon in the damped mode for state $|\Psi^-\rangle$, while the situation is reversed for all other initial states considered. The reason for this is that atomic excitation can decay into two channels: into the damped cavity mode and into the mode reservoir determining the atomic relaxation constants. By taking dipole-dipole interaction into account we enhance the coupling between the atomic subsystem and the reservoir. Consequently, the probability of spontaneous decay outside the damped mode increases. However, if the atoms are in the state $|\Psi^-\rangle$, decay into the reservoir is suppressed and atomic αexcitation decays into another channel.

Fig. 2 shows the time evolution of the average number of photons in the cavity and their statistics, the Mandel *Q*-parameter:

$$Q = \frac{\langle n(n-1)\rangle - \langle n\rangle^2}{\langle n\rangle}.$$

The pumping rate was chosen such that the average number of photons in the cavity was less than unity ($\Gamma/\kappa < 1$). It can be seen from the curves in Fig. 2 that steady state for the given parameters of the system is established at times of the order of the photon lifetime $\tau_{ph} \sim 1/\kappa$ in the cavity. Information about the initial states of the atomic subsystem vanishes due to dissipative processes and incoherent pumping.

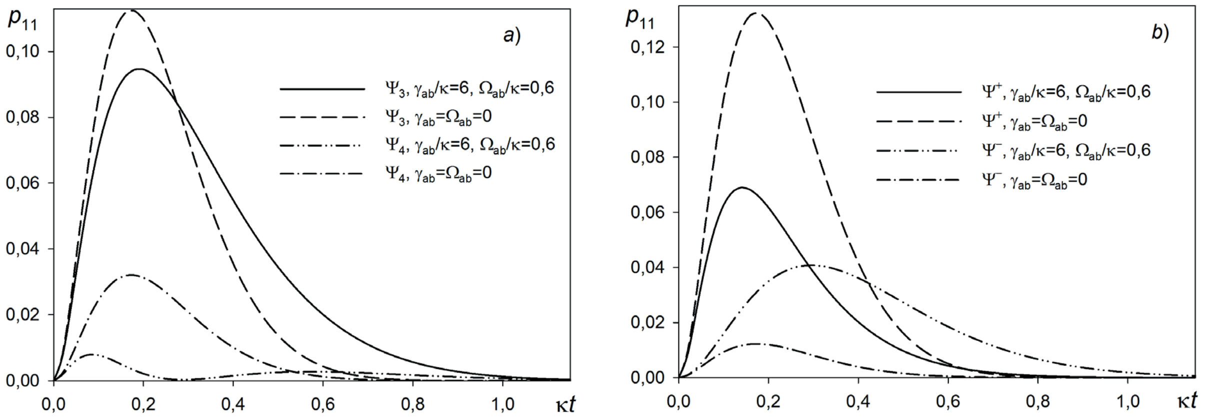

Fig. 1. Time dependence for probability of detecting one photon in cavity mode for four different ground states: $|\Psi_3\rangle$, $|\Psi_4\rangle$ (*a*); $|\Psi^+\rangle$, $|\Psi^-\rangle$ (*b*).
The calculations were carried out excluding (dashed line, dash-dotted line) and including (solid line, dash-double-dotted line) dipole-dipole interaction

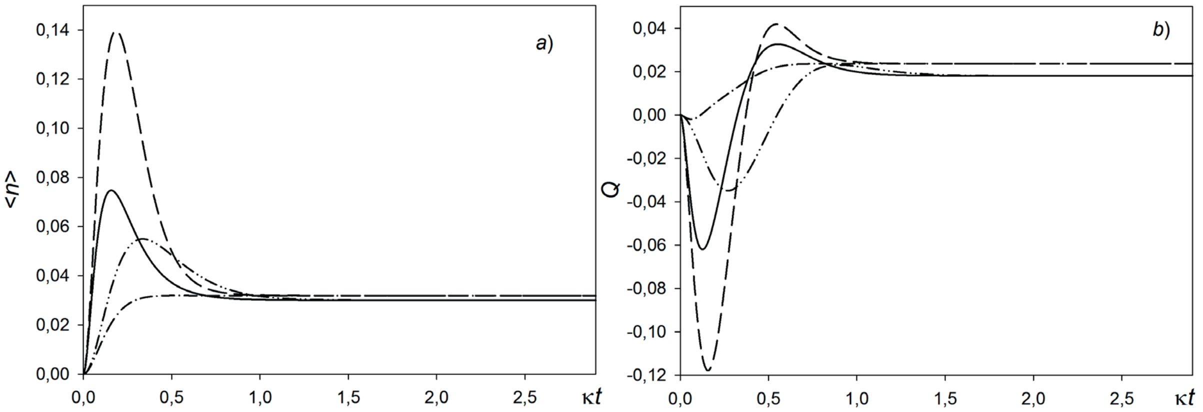

Fig. 2. Time dependences of average number of photons in cavity (*a*) and Mandel *Q*-parameter (*b*). System parameters and notations are the same as in Fig. 1. Pumping rate $\Gamma/\kappa = 0.75$

Fig. 3 shows the time evolution of the atomic subsystem. Evidently, for the given rate of incoherent pumping and the selected parameters of the nanolaser, the probability

$$p^{--}(t) = \mathrm{Sp}_f\left[\rho_{nn}^{\alpha^-\alpha^-}\right]$$

of detecting the atomic subsystem in state

$$|\alpha^-\rangle = (|\alpha^3\rangle - |\alpha^4\rangle)/\sqrt{2}$$

is higher than the probability

$$p^{++}(t) = \mathrm{Sp}_f\left[\rho_{nn}^{\alpha^+\alpha^+}\right]$$

of detecting the same subsystem in a state

$$|\alpha^+\rangle = (|\alpha^3\rangle + |\alpha^4\rangle)/\sqrt{2}$$

(Sp_f[...] denotes the trace with respect to the field variables).

Most likely, the subsystem is detected in the ground state $|\alpha_1\rangle$, indicating the absence of population inversion in this case. The reason for the difference between the probabilities $p^{--}(t)$ and $p^{++}(t)$, observed only if dipole-dipole interaction is taken into account, can be explained, the same as above, by atomic excitation decaying into two channels that are different from each other (see the explanations to Fig. 1, *b*).

Conclusion

We have theoretically studied the effect of interatomic resonant dipole-dipole interaction on the behavior of a laser with several emitters. We have proved that this interatomic interaction substantially affects both the relaxation of the intracavity field and the time evolution of atomic superposition states.

As noted above, the single-mode approximation has to be abandoned to obtain a more rigorous description of a laser with several emitters. This problem may possibly be solved using the approach developed in [28], considering the spatial behavior of a field with nonclassical statistics.

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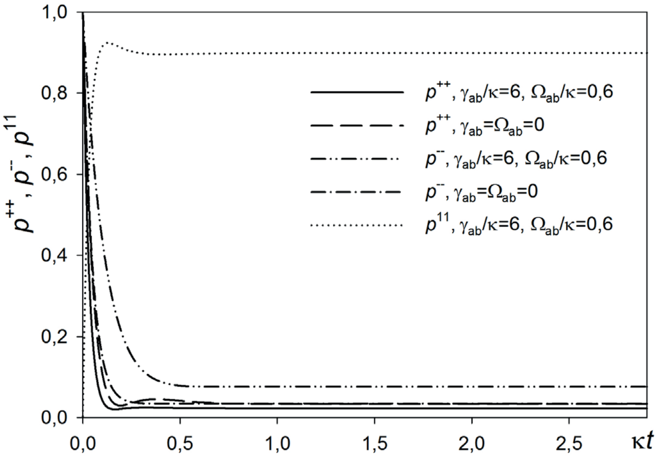

Fig. 3. Time evolution of different atomic states (with and without dipole-dipole interaction); $p^{11}(t) = \mathrm{Sp}_f\left[\rho_{nn}^{\alpha_1\alpha_1}\right]$ is the probability of finding an atomic subsystem in state $|\alpha_1\rangle$. System parameters and notations are the same as in Figs. 1 and 2.

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