DOI: 10.18721/JPM.12211
УДК 539.3

THE EXACT SOLUTION OF THE PROBLEM ON A CRACK EMERGING FROM THE TOP OF TWO DISSIMILAR WEDGES

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A closed connection of two different isotropic wedges has been considered within the scope of the anti-plane problem. A finite-length crack emerges from the top of this connection at an arbitrary angle to the symmetry axis of the structure. The exact solution of the problem was obtained through the problem's reducing to the Wiener – Hopf scalar equation. The dependence of the stress intensity factor (SIF) at the crack tip on the structural parameters was studied. The effects of an increase and a decrease in SIF were compared with those known for the case of a homogeneous medium. It was shown that the stress asymptotics near the junction vertex could have one or two singular terms determining both strong and weak singularities at this singular point.

Keywords: anti-plane crack, closed bimaterial wedge; strong singularity, weak singularity

Citation: V.V. Tikhomirov, The exact solution of the problem on a crack emerging from the top of two dissimilar wedges, St. Petersburg Polytechnical State University Journal. Physics and Mathematics. 12 (2) (2019) 126–134. DOI: 10.18721/JPM.12211

ТОЧНОЕ РЕШЕНИЕ ЗАДАЧИ ДЛЯ ТРЕЩИНЫ, ВЫХОДЯЩЕЙ ИЗ ВЕРШИНЫ ДВУХ РАЗНОРОДНЫХ КЛИНЬЕВ

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В рамках антиплоской задачи рассмотрено замкнутое соединение двух различных изотропных клиньев, из вершины которого выходит трещина конечной длины под произвольным углом к оси симметрии структуры. Путем сведения проблемы к скалярному уравнению Винера – Хопфа получено ее точное решение. Изучена зависимость коэффициента интенсивности напряжений (КИН) в вершине трещины от структурных параметров. Проанализированы эффекты увеличения и уменьшения КИН, по сравнению со случаем однородной среды. Показано, что асимптотика напряжений вблизи вершины соединения может иметь одно или два сингулярных слагаемых, определяющих как сильную, так и слабую особенности в этой особой точке.

Ключевые слова: антиплоская трещина, замкнутый биматериальный клин, сильная сингулярность, слабая сингулярность

Ссылка при цитировании: Тихомиров В.В. Точное решение задачи для трещины, выходящей из вершины двух разнородных клиньев // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2019. Т. 12. № 2. С. 130–139. DOI: 10.18721/JPM.12211

Introduction

Inhomogeneous structures often have singularities where geometric parameters and mechanical properties of the structure's components change radically. Such points are, for example, the edges of interfaces between several wedge-shaped structures consisting of different materials. These structures can be closed if all of their interfaces are connected, or open, if there are notches. The stresses determined by linear theory of elasticity grow without bound at corner points and, therefore, such singularity points are sources of crack propagation.

A large number of studies starting from Williams and Bogy [1, 2], focused mainly on plane problems, have considered elastic fields in bimaterial and multimaterial wedges. The class of anti-plane problems has been studied far less.

The characteristics of a stress singularity in a semi-infinite crack terminating at the interface between two or three isotropic wedges were discussed in [3–6] within the framework of the anti-plane problem. It was found that stress singularity has a power-law behavior at this point (symmetric problem), different from the classical case where the exponent is 0.5 [3, 6]. The asymptotic behavior of stresses near the vertex can contain two singular terms in the asymmetric case [5, 6].

The stress singularity at the tip of a semi-infinite longitudinal crack located in a three-component medium with functionally gradient properties was analyzed in [7], finding two real eigenvalues determining the properties of elastic fields.

A crack of finite length, emanating from the tip of a sharp notch located in a composite wedge-shaped structure, was considered in [8–11]. It was established in [8, 9] that the solution for anisotropic materials of the structure can be obtained based on the solution constructed for the isotropic case by using a linear coordinate transformation.

The critical loads at which cracks evolve at the tip of a sharp notch under anti-plane loading were estimated in [12, 13].

However, the problem of cracks emanating from a closed interface between wedge-shaped structures under anti-plane deformation still remains largely unexplored.

As a first step to solving this problem, this paper considers a crack of finite length, emanating from the top of the interface between two connected wedges composed of different materials. Its exact solution is constructed by reducing the problem to the scalar Riemann problem. An analytical representation is obtained for the stress intensity factor (SIF) at the crack tip, and its dependence on the structure parameters is studied.

Aside from its own significance, the obtained exact solution of the problem is one of the basic elements in analysis of brittle fracture based on the so-called finite fracture mechanics [14], using approximate analytical methods for plane problems due to lack of exact solutions.

Problem statement; reducing the problem to the Wiener–Hopf equation

Let us consider a rectilinear mode III crack of length ε, emanating from the top of the interface between two wedge-shaped regions (Fig. 1). The materials of the wedges are assumed to be isotropic, homogeneous, and with shear moduli μ_1 and μ_2 ($\mu_3 = \mu_2$) in the regions Ω_k ($k = 1, 2, 3$). The materials are assumed to have perfect contact. A self-balanced load $g(r)$ is applied to the edges of the crack (r is the polar radius).

The geometry of the given elastic composite can be conveniently described by two parameters: the vertex angle α ($0 < \alpha < 2\pi$) of the region Ω_1 and the angle β between the direction of the crack and the axis of symmetry of the region Ω_1. Evidently, $|\beta| \le \pi - \alpha/2$. Varying the angle β with a fixed α causes the region Ω_1 to rotate around its vertex. Thus, the angle β characterizes the mutual orientation of the crack and the region Ω_1. For example, the problem is symmetric for $\beta = 0$. An interfacial crack corresponds to the values $\beta = \pm(\pi - \alpha/2)$, and a crack emanating from the vertex along the interface to $\beta = \pm\alpha/2$.

It is known that the displacements w_k in the regions Ω_k are harmonic functions in this case:

$$\frac{\partial^2 w_k}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 w_k}{\partial \theta^2} + \frac{1}{r}\frac{\partial w_k}{\partial r} = 0 \quad (k = 1, 2, 3), \tag{1}$$

and the stresses in polar coordinates r and θ are found by the formulas

$$\tau_{\theta z k} = \frac{\mu_k}{r}\frac{\partial w_k}{\partial \theta}, \ \tau_{rzk} = \mu_k \frac{\partial w_k}{\partial r}.$$

Elastic fields at the interfaces should satisfy the conditions of perfect contact:

$$w_1 = w_2,\ \tau_{\theta z1} = \tau_{\theta z2} \text{ with } \theta = \beta + \alpha/2, \tag{2}$$
$$w_1 = w_3,\ \tau_{\theta z1} = \tau_{\theta z3} \text{ with } \theta = \beta - \alpha/2,$$

and the following mixed conditions along the crack line:

$$\tau_{\theta z2}(r, \pi) = \tau_{\theta z3}(r, -\pi) = g(r)\ (0 \le r \le \varepsilon), \tag{3}$$
$$\tau_{\theta z2}(r, \pi) = \tau_{\theta z3}(r, -\pi) = \tau(r),$$
$$w_2(r, \pi) = w_3(r, -\pi)\ (\varepsilon < r < \infty) \tag{4}$$

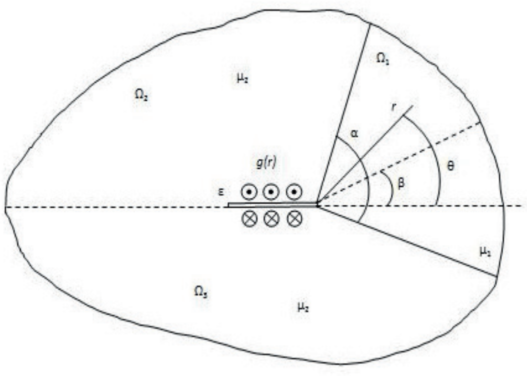

Fig. 1. Crack emanating from top of interface between two inhomogeneous wedges: $g(r)$ is the load on the edges of the crack; μ_1 and μ_2 are the shear moduli of materials; Ω_1, Ω_2, Ω_3 are the wedge-shaped regions; ε is the distance from the top of the interface to the tip of the crack; α and β are the vertex angles of the wedges; r and θ are polar coordinates

Here $\tau(r)$ is an unknown function.

As in [15], we search for the solution to the problem in the form of Mellin integrals:

$$w_k(r,\theta) = \frac{1}{2\pi i}\int_L W_k(p,\theta) r^{-p} dp,$$

$$\tau_{\theta z k}(r,\theta) = \frac{1}{2\pi i}\int_L T_{\theta z k}(p,\theta) r^{-p-1} dp \quad (5)$$

$$(k = 1,2,3),$$

where the transforms of displacements and stresses are found by the following formulas:

$$W_k(p,\theta) = A_k(p)\sin p\theta + B_k(p)\cos p\theta, \quad (6)$$

$$T_{\theta z k}(p,\theta) = \mu_k\, p[A_k(p)\cos p\theta - B_k(p)\sin p\theta] \quad (\mu_3 = \mu_2).$$

Since regularity conditions are imposed for the solution with $r \to 0$ and $r \to \infty$, the integration path L is parallel to the imaginary axis in the strip

$$-\delta_1 < \operatorname{Re} p < \delta_2 \quad (\delta_1, \delta_2 > 0).$$

Mixed conditions (3) and (4) lead to equalities

$$T_{\theta z2}(p,\pi) = T_{\theta z3}(p,-\pi) = [T_-(p) + G_+(p)]\varepsilon^{p+1}, \quad (7)$$

$$-p[W_2(p,\pi) - W_3(p,-\pi)] = U_+(p)\varepsilon^p,$$

where

$$T_-(p) = \int_1^\infty \tau(\varepsilon\rho)\rho^p d\rho, \quad G_+(p) = \int_0^1 g(\varepsilon\rho)\rho^p d\rho, \quad (8)$$

$$U_+(p) = \int_0^1 \frac{\partial}{\partial\rho}\left[w_2(\varepsilon\rho,\pi) - w_3(\varepsilon\rho,-\pi)\right]\rho^p d\rho.$$

The function $T_-(p)$ is regular and has no zeroes in the half-plane Ω_- to the left of the path L and the functions $G_+(p)$ and $U_+(p)$ have no zeroes in the right half-plane Ω_+ [16].

Substituting expressions (6) into the left-hand sides of equalities (7) and conditions (2) modified by the Melling transform, after eliminating the quantities $A_k(p)$ and $B_k(p)$, we obtain the scalar Wiener–Hopf equation:

$$F(p)[T_-(p) + G_+(p)] + \frac{\mu_2}{2\varepsilon}U_+(p) = 0 \; (p \in L). \quad (9)$$

Here, the imaginary axis can be taken as the path L, while the function $F(p)$ has the form

$$F(p) = f(p)/\Delta(p), \quad (10)$$

$$f(p) = 2[\sin^2\pi p - m^2\sin^2(\pi - \alpha)p], \quad (11)$$

$$\Delta(p) = \sin 2\pi p + 2m\sin\alpha p\cos 2\beta p - m^2\sin 2(\pi - \alpha)p. \quad (12)$$

The elastic properties of the composite are reproduced in these formulas through a single bielastic constant

$$m = (\mu_1 - \mu_2)/(\mu_1 + \mu_2) = (\mu - 1)/(\mu + 1),$$

where $\mu = \mu_1/\mu_2$ is the relative hardness of the inclusion ($0 \le \mu < \infty$).

This quantity satisfies the inequality $|m| \le 1$ for all combinations of shear moduli of materials. If the inclusion material is harder than the matrix material, then $0 < m < 1$; otherwise (for a soft inclusion), this parameter lies in the interval $-1 < m < 0$. The value $m = 0$ corresponds to a homogeneous medium, and the values $m = \pm 1$ correspond to an absolutely hard inclusion and a wedge-shaped notch.

Notice that the zeroes of function (11) are eigenvalues of the anti-plane problem for an interface between two wedge-shaped regions with symmetric and antisymmetric (with respect to the ray $\theta = 0$) stress distributions. The zeroes of function (12) determine the characteristics of the stress singularity at the tip of a semi-infinite crack terminating at an elastic wedge-shaped inclusion [6].

Solution of the Wiener–Hopf equation

Factorization of the coefficient of Eq. (9) is carried out similarly to the procedure in [15]:

$$F(p) = pF_+(p)F_-^{-1}(p), \quad (13)$$

$$F_{\pm}(p)=\Phi_{\pm}(p)X_{\pm}^{-1}(p),$$

$$X_{\pm}(p)=\left[\frac{\Gamma(1+p)}{\Gamma(1/2+p)}\right]^{\pm 1},$$

$$\Phi_{\pm}(p)=\exp\left[-\frac{1}{2\pi i}\int_{L}\frac{\ln\Phi(t)}{t-p}dt\right]$$

$$(p\notin L),$$

where

$$\Phi(p) = [1- m^2 \sin^2 (\pi - \alpha)\, p \sin^{-2}\pi p]\times$$
$$\times[2 + 1m \sin \alpha p \cos 2\beta p \sin^{-1} 2\pi p$$
$$- m^2 \sin 2(\pi - \alpha)p \sin^{-1} 2\pi p]^{-1},$$

and $\Gamma(p)$ is the gamma function.

From here, using formulas (13) and applying the Liouville theorem [16] from Eq. (9), taking into account the behavior of the terms at infinity, we obtain:

$$\Phi_{-}^{-1}(p)X_{-}(p)T_{-}(p)+Q_{=}(p)=$$
$$=-\frac{\mu_2}{2\varepsilon p}U_{+}(p)\Phi_{+}^{-1}(p)X_{+}(p)-Q_{+}(p)=0, \quad (14)$$

where

$$Q_{\pm}(p)=\mp\frac{1}{2\pi i}\int_{L}\frac{Q(t)}{t-p}dt,$$
$$Q(t)=\frac{1}{t}X_{+}(t)\Phi_{+}^{-1}(t)F(t)G_{+}(t). \quad (15)$$

Then we find from Eqs. (14)

$$T_{-}(p)=-\Phi_{-}(p)X_{-}^{-1}(p)Q_{-}(p). \quad (16)$$

Given that, with $p\to\infty$,

$$X_{-}(p)\sim\frac{1}{\sqrt{-p}},\ Q_{-}(p)\sim-\frac{C}{p},$$
$$C=\frac{1}{2\pi i}\int_{L}\frac{X_{+}(t)G_{+}(t)}{t\Phi_{+}(t)}F(t)dt, \quad (17)$$

we obtain the asymptote

$$T_{-}(p)\sim-C/(i\sqrt{p}).$$

Then, by the Abel-type theorem [16], we conclude that the stress asymptote at $r\to\varepsilon+0$ has the form

$$\tau(r)\sim-\frac{C}{i\sqrt{\pi(1-\rho)}}=C\sqrt{\frac{\varepsilon}{\pi}}\frac{1}{\sqrt{r-\varepsilon}}. \quad (18)$$

Stress intensity factor

Let us find the stress intensity factor (SIF) at the crack tip $r = \varepsilon$ by the formula

$$K_{\mathrm{III}} = \lim_{r\to\varepsilon+0}\sqrt{2\pi(r-\varepsilon)\tau(r)}.$$

Then, using asymptote (16), we obtain that

$$K_{\mathrm{III}}(\alpha,\beta,m,\varepsilon)=\sqrt{2\varepsilon}C. \quad (19)$$

Let self-balanced concentrated forces T_0 be applied to the edges of the crack at a distance r_0 from the top of the interface, i.e.,

$$g(r) = T_0\delta(r - r_0),$$

where $\delta(r)$ is the Dirac delta function, and $\varepsilon < r_0 < \infty$.

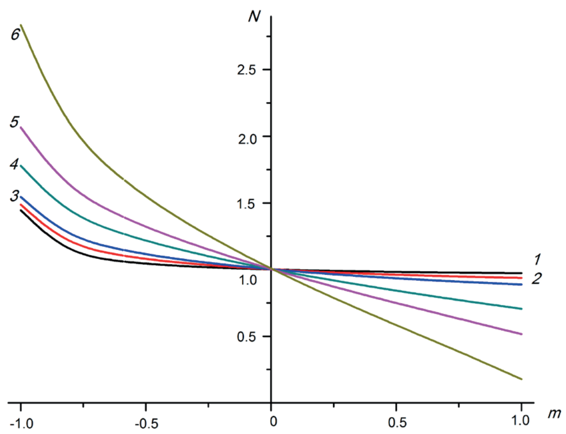

Fig. 2. Normalized stress intensity factor (NSIF) as function of parameter m at $\beta = 0$ and $r_0/\varepsilon = 0.5$ for different angles α: $\pi/4$ (*1*); $\pi/2$, $3\pi/2$ (*2*); $3\pi/4$, $5\pi/4$, $7\pi/4$ (*3*)

Then, calculating the function $G_+(t)$ by formula (8), combining the integration path with the imaginary axis in formula (17) and using the residue theorem in the region Ω_+, we obtain, according to equality (19):

$$K_{\text{III}} = -T_0\sqrt{\frac{2}{\varepsilon}}\sum_{k=1}^{n}\frac{X_+(p_k)f(p_k)}{p_k\Phi_+(p_k)\Delta'(p_k)}\left(\frac{r_0}{\varepsilon}\right)^{p_k}. \quad (20)$$

Here the prime denotes the derivative with respect to the variable p, and p_k are positive zeroes of function (12).

In case of a geometrically symmetric structure, with $\beta = 0$, series (20) is summed for some values of the bielastic constant m, and the expressions for SIF can be represented in a simple closed form. They have the following form for different cases:

$$K_{\text{III}}^0 = T_0\sqrt{\frac{2}{\pi\varepsilon}}\sqrt{\frac{r_0/\varepsilon}{1-r_0/\varepsilon}} \quad (21)$$

for a crack in a homogeneous medium ($m = 0$);

$$K_{\text{III}}^S = T_0\sqrt{\frac{2}{\pi\varepsilon a}}\frac{1}{\sqrt{1-(r_0/\varepsilon)^{1/a}}}$$

for a crack emanating from the tip of a notch ($\beta = 0$, $m = -1$);

$$K_{\text{III}}^H = T_0\sqrt{\frac{2}{\pi\varepsilon a}}\sqrt{\frac{(r_0/\varepsilon)^{1/a}}{1-(r_0/\varepsilon)^{1/a}}}$$

for a crack emanating from the vertex of an absolutely hard inclusion ($\beta = 0$, $m = 1$).

In these formulas,

$$a = 1 - \alpha/(2\pi),\ r_0/\varepsilon < 1.$$

Using formulas (20) and (21), let us introduce the normalized stress intensity factor (NSIF), which describes the variation of the SIF in a heterogeneous composite compared to the SIF at the tip of a similar crack in a homogeneous medium:

$$N = \frac{K_{\text{III}}}{K_{\text{III}}^0} = -\sqrt{\pi\left(1-\frac{r_0}{\varepsilon}\right)}\times \sum_{k=1}^{n}\frac{X_+(p_k)f(p_k)}{p_k\Phi_+(p_k)\Delta'(p_k)}\left(\frac{r_0}{\varepsilon}\right)^{p_k-0.5}. \quad (22)$$

Roots of the characteristic equation

$$\Delta(p) = 0, \quad (23)$$

located in the strip $0 < \text{Re}\, p < 1$, were analyzed in detail in [6]. It was established that, depending on the parameters of the composite α, β and m, Eq. (23) can either have one root $p_1 < 0.5$ or $p_1 > 0.5$ in this strip, as well as two roots:

$$0 < p_1 < 0.5 < p_2 < 1,$$

or

$$0.5 < p_1 < p_2 < 1.$$

The characteristic equation in case of a symmetrical structure ($\beta = 0$) takes the form

$$\Delta_*(p) = \cos \pi p + m \cos(\pi - \alpha)\, p = 0$$

and has a single root in the interval (0, 1).

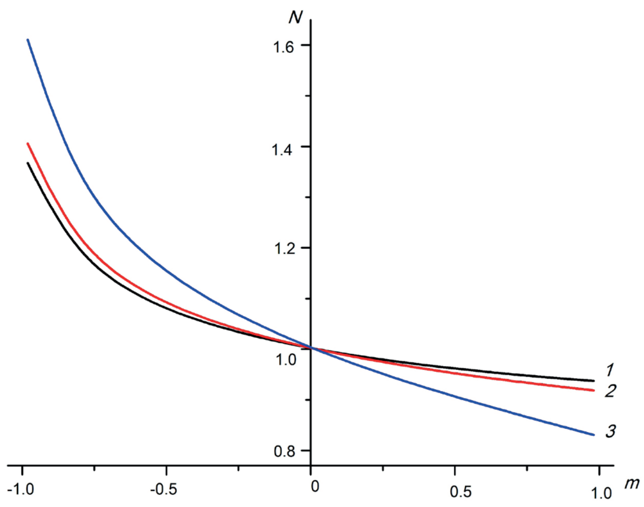

Fig. 3. NSIF as function of parameter m with $\alpha = \pi/2$ and $r_0/\varepsilon = 0.5$ for different angles β: 0 (*1*), $\pi/4$ (*2*), $\pi/2$ (*3*)

This root is larger than 0.5 with $m > 0$, and lies in the interval $0 < p_1 < 0.5$ with $m < 0$.

In this case, the NSIF exhibits a typical behavior depending on the parameter m at different values of the angle α and is a monotonically decreasing function over the entire variation range of the bielastic constant (Fig. 2). At the same time, an increase in NSIF compared with a homogeneous medium is observed if a crack is located in a harder material, when $\mu_2 > \mu_1$ and, therefore, $m < 0$. In contrast, there is a decrease in NSIF for a crack located in a relatively softer material ($m > 0$). These effects become more pronounced as the vertex angle α of the region Ω_1 increases. It follows then that a "symmetric" crack emanating from the top of the interface always propagates in a relatively harder medium.

The behavior of NSIF is less unambiguous in case of an asymmetric structure ($\beta \neq 0$). If the dimensionless parameter r_0/ε (characterizing how close to the top the load is applied) is not too small, then NSIF at the crack tip has a qualitatively similar behavior as in the symmetric case. Fig. 3 shows the variation of NSIF for the angle $\alpha = \pi/2$ and $r_0/\varepsilon = 0.5$ depending on m for different values of the asymmetry parameter β. These data indicate that the effects increasing and decreasing the NSIF intensify with increasing angle β.

However, when $r_0/\varepsilon \to 0$, the dominant term of series (22) is its first term. NSIF asymptote in this case takes the form

$$N \sim -\sqrt{\pi\left(1-\frac{r_0}{\varepsilon}\right)}\,\frac{X_+(p_1)f(p_1)}{p_1\Phi_+(p_{\cdot 1})\Delta'(p_1)}\left(\frac{r_0}{\varepsilon}\right)^{p_1-0.5}. \quad (24)$$

It follows then that if $p_1 > 0.5$ and $r_0/\varepsilon << 1$, then $(r_0/\varepsilon)^{p1-0.5} < 1$, $(r_0/\varepsilon)^{p1-0.5} > 1$, this leads to a decrease in NSIF. If p_1 <0.5, then, for small values of the relative distance r_0/ε in expression (24), the factor $(r_0/\varepsilon)^{p1-0.5} > 1$, which leads to a decrease in NSIF.

An example of increasing NCIN for small r_0/ε is shown in Fig. 4 for the case $\alpha = \beta = \pi/2$. The first root of Eq. (23) is less than 0.5 with these values of the angles, both if $m < 0$ and if $m > 0$ [6, 15]. Analysis of the behavior of the curves indicates that the dependence $N(m)$ is not monotonous. The increase in NSIF becomes more pronounced with a decrease in the distance r_0/ε. Besides, if the crack is located in a softer medium, the SIF values may exceed those for a similar crack located in a homogeneous medium with a sufficiently small ratio r_0/ε, due to the influence of heterogeneity and geometry of the structure (as opposed to the symmetric case).

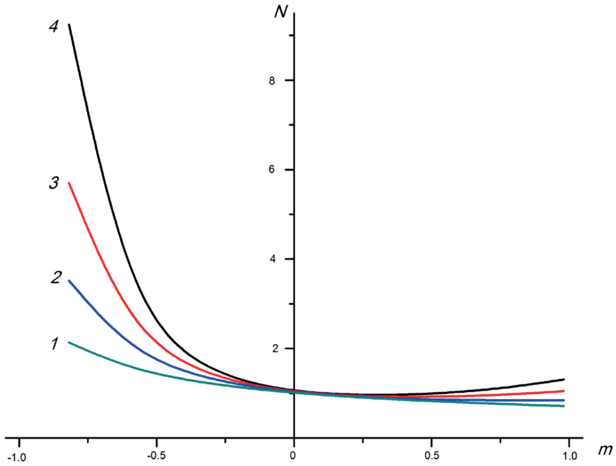

Fig. 4. NSIF as function of parameter m with $\alpha = \beta = \pi/2$
for small relative distance r_0/ε: 0.1 (*1*); 0.01 (*2*); 0.001 (*3*); 0.0001(*4*)

Stress singularity at the top of the interface between the wedges

Based on Eqs. (6)–(8), it is easy to obtain representations for the stresses in the regions Ω_j (j = 1, 2, 3):

$$\tau_{\theta zj}(r,\theta) = \frac{1}{\pi i}\int_L \frac{\tau_j(p,\theta)}{\Delta(p)}\left[T_-(p) + G_+(p)\right]\left(\frac{r}{\varepsilon}\right)^{-p-1} dp, \quad (25)$$

where

$$\tau_j(p,\theta) = a_j(p)\cos p\theta - b_j(p)\sin p\theta;$$

$$a_1(p) = (1+m)[\sin p\pi - m\cos 2p\beta \sin p(\pi-\alpha)];$$

$$b_1(p) = (1+m)m\sin 2p\beta \sin p(\pi-\alpha);$$

$$a_n(p) = \sin p\pi + m\sin p\alpha \cos[p(\pi + (-1)^n 2\beta)] - m^2\cos p\alpha \sin p(\pi-\alpha);$$

$$b_n = (-1)^n m\sin p\alpha\{\sin p(\pi-\alpha) - \sin[p(\pi+(-1)^n 2\beta]\} \ (n = 2, 3).$$

Substituting representations (15), (16) into integrand (25) and given that the transform of the concentrated load at the edges of the crack has the form

$$G_+(p) = T_0/\varepsilon(r_0/\varepsilon)^p,$$

we obtain the following expression for stresses:

$$\tau_{\theta zj}(r,\theta) = \frac{T_0}{\varepsilon\pi i}\int_L \frac{\tau_j(p,\theta)}{\Delta(p)}\left[\left(\frac{r_0}{\varepsilon}\right)^p - \frac{\Phi_-(p)}{X_-(p)}Q_-^*(p)\right]\left(\frac{r}{\varepsilon}\right)^{-p-1} dp, \quad (26)$$

where

$$Q_-^*(p) = \frac{1}{2\pi i}\int_L \frac{F(t)}{t(t-p)}\frac{X_+(p)}{\Phi_+(p)}\left(\frac{r_0}{\varepsilon}\right)^t dt.$$

By applying the theorem of the residues (calculated by the zero roots p_k of function (12) located in the left half-plane of the path L) to integral (26) we obtain with $r < r_0$:

$$\tau_{\theta zj}(r,\theta) = \frac{2T_0}{r_0}\left[\sum_{k=1}^{\infty}\frac{\tau_j(-p_k,\theta)}{\Delta'(-p_k)}\left(\frac{r}{r_0}\right)^{p_k-1} - \frac{r_0}{\varepsilon}\sum_{k=0}^{\infty}\frac{\tau_j(-p_k,\theta)}{\Delta'(-p_k)}\frac{\Phi_-(-p_k)}{X_-(-p_k)}\times Q_-^*(-p_k)\left(\frac{r}{\varepsilon}\right)^{p_k-1}\right]. \quad (27)$$

Notably, the first sum in this formula determines the stress distribution in case of a semi-infinite crack emanating from the top of a closed interface between the wedges [6]. The second sum is due to finite length of the given crack.

It follows from representation (27) that the stresses at the top of the interface have a singularity described by the power law. Depending on the structure parameters, the asymptotic behavior of stresses with $r \to 0$ can have one or two singular terms, determined by the roots of Eq. (23), located in the interval (0, 1). The singularity exponents $\lambda_k = 1 - p_k$ (k = 1, 2) can be both larger and smaller than 0.5 and, therefore, generate both strong and weak singularities at this point.

Conclusion

We have obtained an exact solution for the problem of an anti-plane crack emanating from a closed interface between two wedge-shaped regions based on the Mellin integral transform and the Wiener–Hopf method. We have analyzed the behavior of the stress intensity factor (SIF) at the tip of a crack upon variation of elastic properties and geometry of the structure, which can lead to an increase or decrease in SIF, compared with a homogeneous medium. We have observed a dependence of SIF on the relative hardness of the materials that is not characteristic for the symmetric case in the absence of geometric symmetry of the structure, for some values of the composite parameters and a concentrated load applied at a sufficiently small distance from the top of the interface. In particular, the SIF for a crack located in a relatively softer material may exceed the SIF for a similar crack in a homogeneous medium. Examining the stress singularity at the corner point of the interface, we have confirmed that this singularity can be both strong and weak.

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Received 15.03.2019, accepted 19.03.2019.

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Статья поступила в редакцию 15.03.2019, принята к публикации 19.03.2019.

СВЕДЕНИЯ ОБ АВТОРЕ

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