DOI: 10.18721/JPM.12210
УДК 539.12

# ASYMPTOTIC EFFECTS IN DIJET PRODUCTION IN PROTON-PROTON COLLISIONS AT EXTREMELY HIGH ENERGIES

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In the paper, the scope for the search of the Balitsky–Fadin–Kuraev–Lipatov (BFKL) evolution effects at future proton-proton colliders at center-of-mass energies of 14, 27 and 100 TeV has been analyzed for processes of dijets production with a large jet separation in rapidity at a dijet. Simulation of proton-proton collisions using Monte Carlo calculations performed with generator packages PYTHIA8 and HERWIG++ based on Dokshitzer–Gribov–Lipatov–Altarelli–Parisi evolution and with generator package HEJ+ARIADNE based on BFKL approach was carried out. The simulation observations pointed to a promise to reveal the BFKL effects experimentally under conditions established at future proton-proton colliders.

**Keywords:** quantum chromodynamics, BFKL approach, dijet production; large rapidity

**Citation:** Egorov A.Yu., Berdnikov Ya.A., Asymptotic effects in dijet production in proton-proton collisions at extremely high energies, St. Petersburg Polytechnical State University Journal. Physics and Mathematics. 12 (2) (2019) 118–125. DOI: 10.18721/JPM.12210

# АСИМПТОТИЧЕСКИЕ ЭФФЕКТЫ ПРИ РОЖДЕНИИ ПАР АДРОННЫХ СТРУЙ В ПРОТОН-ПРОТОННЫХ СТОЛКНОВЕНИЯХ ПРИ СВЕРХВЫСОКИХ ЭНЕРГИЯХ

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В работе проанализированы возможности поиска эффектов эволюции Балицкого – Фадина – Кураева – Липатова (БФКЛ) на будущих протон-протонных коллайдерах при максимальных энергиях в системе центра масс протонов 27 ,14 и 100 ТэВ в процессах рождения пар адронных струй с большим разделением по быстроте между струями в паре. Выполнено моделирование протон-протонных столкновений в программных пакетах Монте-Карло, основанных на эволюции Докшицера – Грибова – Липатова – Альтарелли – Паризи PYTHIA8 и HERWIG++, а также в программном пакете HEJ+ARIADNE, основанном на приближении БФКЛ. Результаты моделирования указали на перспективность экспериментального обнаружения эффектов БФКЛ при условиях, созданных на будущих протон-протонных коллайдерах.

**Ключевые слова:** квантовая хромодинамика, приближение БФКЛ, рождение пар адронных струй, большая быстрота

**Ссылка при цитировании:** Егоров А.Ю., Бердников Я.А. Асимптотические эффекты при рождении пар адронных струй в протон-протонных столкновениях при сверхвысоких энергиях // Научно-технические ведомости СПбГПУ. Физико-математические науки. 2019. Т. 12. № 2. С. 121–129. DOI: 10.18721/JPM.12210

## Introduction

Hard collisions of partons at extremely high center-of-mass energies ($\sqrt{s} \to \infty$), large momentum transfer ($Q \to \infty$) and a fixed ratio $Q/\sqrt{s} \sim x$ (this limit is called the Bjorken limit, and $x$ is the scaling variable) are described in terms of perturbative quantum chromodynamics within collinear factorization. This provides factorization of the hadron-hadron cross section into a hard subprocess and parton distribution functions. This kinematic mode implies summation of the diagrams amplified by large logarithms of momentum transfer, namely, the terms of the perturbative series proportional to

$$(\alpha_s(Q^2)\ln Q^2)^n,$$

where $\alpha_s(Q^2)$ is the running coupling constant of strong interaction, $n$ is the order of the term in the perturbative series.

Such summation in all orders of perturbation theory is achieved in the leading logarithmic approximation (LLA) in the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [1–5] describing the evolution of parton distribution functions with the scale $Q$. Summing the terms proportional to $[\alpha_s(Q^2)]^n[\ln Q^2]^{n-1}$ leads to DGLAP equations in the logarithmic approximation next to LLA (NLLA).

The parton scattered in a hard subprocess emits bremsstrahlung, forming a partonic shower. The parton shower can also be described in different logarithmic approximations of DGLAP splitting functions. The emitting partons are ordered by the transverse momentum $p_\perp$, while preserving the same order of rapidity $y$:

$$y = 1/2\ln[(E + p_z)/(E - p_z)],$$

where $E$ is the parton energy, $p_z$ is the longitudinal momentum (momentum along the beam of colliding hadrons).

Parton shower and hadronization lead to production of jets. Inclusive production of jets agrees well with calculations in the framework of NLLA DGLAP approach in a wide range of transverse momenta, for all experimentally available energies currently obtained in the HERA (DESY, Germany) and Tevatron (Fermilab, USA) accelerators and in the Large Hadron Collider (LHC, CERN, Switzerland). However, agreement with the experiment deteriorates if jets are largely separated in rapidity [6].

When the collision energy $\sqrt{s}$ of the center-of-mass system tends to infinity, i.e., greatly exceeds the finite scale $Q$ of hard interaction, such that $Q/\sqrt{s} \sim x \to 0$ (the Regge–Gribov limit), hard partons scatter at large rapidities $y$, while parton emission increases rapidly with an increase in the phase space accessible in rapidity. Such dynamics is due to diagrams enhanced by large logarithms:

$$\ln s \sim \ln(1/x).$$

Summation of these logarithms was obtained in LLA and NLLA in the Balitsky–Fadin–Kuraev–Lipatov (BFKL) evolution equations [7–9].

Searches for BFKL evolution in production of jets in proton-proton collisions were previously carried out at the Tevatron in the D0 experiment [10–12] for the energies $\sqrt{s}$ = 630 and 1800 GeV, and also at the LHC in the ATLAS [13, 14] and CMS [ 6, 15 – 17] experiments for the energy $\sqrt{s}$ = 7 TeV, reached in LHC Run I. While DGLAP evolution was well confirmed in the experiment in the Bjorken limit (high center-of-mass energies and large momentum transfers), experiments on searches for BFKL evolution which should dominate in semi-hard processes (high center-of-mass energies and moderate momentum transfers) have not yielded any definitive results. There are several reasons for this. For one, no Monte Carlo generator can currently simulate BFKL evolution in NLLA and no generator can run pure simulation in the DGLAP approximation. Existing Monte Carlo generators based on DGLAP equations include phenomenological model corrections partially simulating the BFKL effects such as color coherence phenomena, angular ordering in parton cascades and dipole parton showers. Additional uncertainty in searches for BFKL evolution is the theoretical uncertainty regarding the energy scale $\sqrt{s_0}$ on which the BFKL effects become dominant. Therefore, energies that are the most readily accessible experimentally are required to search for such effects.

In this study, we used Monte Carlo simulation to consider possible searches for BFKL effects at future hadron colliders at maximum center-of-mass energies, namely, $\sqrt{s}$ = 14 TeV. This is the nominal energy of the Large Hadron Collider (LHC), which it should reach in Run III. Energies $\sqrt{s}$ = 27 and 100 TeV, which are, respectively, the energy of the planned HE-LHC (High-Energy Large Hadron Collider) [18] and the planned FCC (Future Circular Collider), are also of interest [19].

### Monte Carlo generators used

We used Monte Carlo generators based on DGLAP evolution and BFKL evolution to simulate proton collisions. The first type includes the PYTHIA8 (8153) generator [20] with Tune 4C [21] and the HERWIG++ (2.7.1) generator [22] with Tune UE-EE-3C [23]. The generators compute matrix elements in leading order of perturbation theory, refined by taking into account the parton shower in LLA DGLAP. The difference between the PYTHIA8 and HERWIG++ generators is that they use different phenomenological models for simulating parton showers and hadronization.

What is important for the purposes of this study is that these generators use different methods for accounting for color coherence effects in parton cascades, which partially emulate BFKL evolution:

PYTHIA8 uses a *dipole* cascade, ordered by *transverse momentum*;

HERWIG++ uses a *parton* cascade ordered by *angle*.

These effects only partially account for the dynamics of BFKL. The Monte Carlo simulation based on LLA BFKL was performed with the HEJ generator (1.4.0) [24] at the parton level. Hadronization at the parton level was performed with the ARIADNE (4.12J01) generator [25]. Predictions based on LLA BFKL are referred to as HEJ+ARIADNE below.

Simulations with Monte Carlo generators give predictions at the hadron level. Hadrons in the final state can produce jets carrying information about the parton subprocess. Infrared and collinear-safe cluster algorithms, including the anti-$kT$ algorithm are considered to be the best for reconstructing jets [26]. The anti-$kT$ algorithm used in our study was implemented in the FASTJET software package [27]. The value of the jet size parameter was chosen to be 0.5 in the space with pseudorapidty $\eta$ and azimuthal angle $\phi$. Pseudorapidity $\eta$ is a dimensionless physical quantity:

$$\eta = -\ln[\mathrm{tg}(\theta/2)],$$

where $\theta$ is the azimuthal angle.

The selected parameter value corresponds to that used in measurements at the LHC with the center-of-mass energy $\sqrt{s} = 7$ TeV [15].

### Observables sensitive to BFKL effects

One of the main difficulties in detecting BFKL effects is in selecting a value that can be actually measured conveniently. It was found in [28] for proton collisions that measurement of the cross section for production of dijets with large separation in rapidity is sensitive to BFKL effects. In this case, a pair of jets with the highest and lowest rapidities ($y_{\max}$ and $y_{\min}$) among the jets produced in a proton-proton collision, with transverse momenta above a certain threshold ($p_{\wedge} \geq p_{\wedge\min}$), is called a Mueller–Navelet (MN) dijet. The BFKL approximation was used in the study to calculate the ratio of the production cross section for an MN dijet to the Born cross section (MN $K$ factor).

Notably, the $K$ factor is defined as the ratio of the cross section calculated in higher orders of perturbation theory to the Born cross section. However, it is virtually impossible to measure the Born cross section, since it is impossible to kinematically forbid the virtual corrections. Nevertheless, an “exclusive” dijet cross section, i.e., the cross section of the process producing strictly two jets with transverse momenta above the threshold $p_{\perp\min}$, can be measured instead of the Born cross section.

The inclusive $K$ factor, that is, the ratio of inclusive cross section for production of a dijet to the Born cross section, was calculated in [29] in framework of BFKL theory. All jets with transverse momenta above the threshold $p_{\perp\min}$ make pairwise contributions to the inclusive cross section for production of dijets. It seems preferable to measure the inclusive cross section for production of dijets rather than the MN cross section, since the rapidity of an MN dijet may fall beyond the detector’s acceptance at high center-of-mass energies [29].

Notably, searches for BFKL effects should be performed at the highest possible center-of-mass energy and, at the same time, the lowest possible threshold $p_{\perp\min}$ for the transverse momentum. A lower cutoff is imposed on the transverse momentum of the jets in experimental measurements. The detector should be capable of detecting jets with large rapidities, because colliding beams cross at small angles. For example, the ATLAS measurements [13, 14] detected dijet observables sensitive to BFKL effects for the average pair transverse momentum

$$\langle p_\perp \rangle = (p_{\perp 1} + p_{\perp 2})/2 > 50(60) \text{ GeV}$$

with rapidity separation up to

$$\Delta y = |y_1 - y_2| = 6(8),$$

where $y_1$, $y_2$ are the dijet rapidities.

On the other hand, the CMS experiment [15–16] measured dijets with transverse momenta $p_\perp \geq 35$ GeV and rapidity separation

$\Delta y = 9.4$. Thus, CMS measurements are more sensitive to possible BFKL effects.

Ref. [16] reported on using the CMS detector to measure the quantities $R^{incl}$ and $R^{\text{MN}}$, the ratios of cross sections for dijet production in proton-proton collisions at $\sqrt{s} = 7$ TeV as functions of separation in rapidity $\Delta y$:

$$R^{incl} = \sigma^{incl}/\sigma^{excl}, \quad R^{\text{MN}} = \sigma^{\text{MN}}/\sigma^{excl}, \qquad (1)$$

where $\sigma^{incl}$ is the inclusive cross section for dijet production with the transverse momentum $p_\perp \geq$ 35 GeV; $\sigma^{excl}$ is the "exclusive" section for dijet production; $\sigma^{\text{MN}}$ is the MN cross section for dijet production (MN dijet is a pair of jets with maximum separation in rapidity among the jets with transverse momenta $p_\perp \geq 35$ GeV produced in the event).

Events producing a single dijet with the transverse momentum $p_\perp \geq 35$ GeV contribute to the "exclusive" cross section. Measurement results were compared with Monte Carlo predictions in [15]. The predictions were obtained with the same generators that we used in this study. It was established in [15] that the PYTHIA8 (4C) generator adequately describes the experimental data, while HERWIG++ (UE-EE-3C) overestimates them in the region of large rapidities. The HEJ+ARIADNE generator considerably overestimates the experimental data. However, since LLA BFKL predicts a stronger rise in cross sections that actually observed, it is important to take into account the contribution of NLLA BFKL, which is known to predict a slower rise in cross sections.

## Computational results and discussion

We carried out predictive calculations of quantities (1) by the Monte Carlo method as functions of rapidity separation $\Delta y = |y_1 - y_2|$, ($y_1$ and $y_2$ are the rapidities of the first and second jets in the dijet) with different generators (see the section "Monte Carlo generators used" for the description) for proton-proton collisions with energies $\sqrt{s} = 14$, 27 and 100 TeV, accessible for future colliders. We used the PYTHIA8 (4C) and HERWIG++ (UE-EE-3C) models, calculating matrix elements in leading order of perturbation theory, matched to LLA DGLAP parton showers, and the HEJ+ARIADNE model, based on LLA BFKL. The jets were reconstructed using an anti-$kT$ algorithm with the jet size of 0.5. The calculation results are shown in Fig. 1.

The dependences obtained for the ratios of cross sections for dijet production $R^{incl}$ and $R^{\text{MN}}$ versus rapidity separation $\Delta y = |y_1 - y_2|$ in the dijets have the predicted form. $R^{incl}$ and $R^{\text{MN}}$ were observed to increase with increasing rapidity separation $\Delta y$, which is associated with the phase space extending for emission of additional jets and with dynamic effects. The ratios decrease at the largest rapidity intervals, which is associated with kinematic restrictions imposed on production of jets with transverse momenta above the threshold $p_{\perp\min} = 35$ GeV, in addition to "exclusive" dijets. The ratios should equal unity with the maximum value of $\Delta y$, when all center-of-mass energy has been spent on production of an "exclusive" dijet.

The phase space available with respect to $\Delta y$ extends with increasing center-of-mass energy. The maximum of the ratios is shifted towards large rapidity intervals.

Calculations with the HEJ+ARIADNE generator (based on LLA BFKL) predict considerably stronger rise of the rations with rapidity separation $\Delta y$, than calculations with the PYTHIA8 and HERWIG++ generators. However, LLA BFKL calculations can produce overestimated values for cross section rise.

Analyzing the results obtained using HEJ+ARIADNE, we concluded that a faster rise of ratios with the center-of-mass energy $\sqrt{s}$ is predicted in this case than when using the PYTHIA8 and HERWIG ++ generators. Consequently, an increase in the interaction energy makes the measurements more sensitive to BFKL effects.

The dynamics of DGLAP equations has no evolution in rapidity. Emission of partons (hadron jets) should be equally probable over the entire range of rapidities. Therefore, the cross section ratios should remain constant over the entire range of rapidities. The observed increase in the values calculated using the PYTHIA8 and HERWIG ++ generators (based on DGLAP) may be due to the phase space extending and to the phenomena partially emulating the BFKL effects, such as color coherence, angular ordering in the parton cascade and dipole cascade. The difference in the predictions obtained using the PYTHIA8 and HERWIG++ generators is because they are based on different color coherence models, in the first case, a dipole cascade ordered by transverse momentum, and in the second case, a parton cascade ordered by angle.

The obtained results indicate that these models predict different behavior of the calculated values with increasing center-of-mass energies. A stronger rise of ratios is predicted in the first case than in the second. Notably, the models taking into account color coherence were introduced into the calculations in DGLAP-based

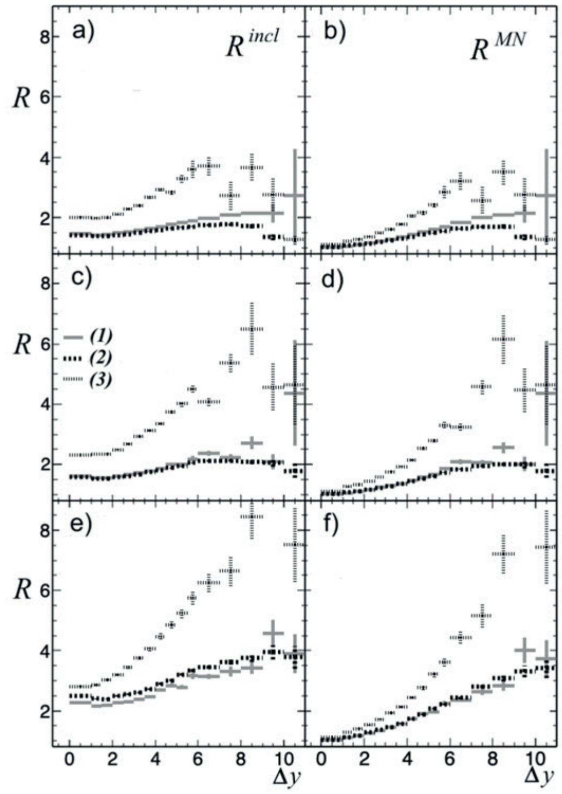

Fig. 1. Calculated $R^{incl}$ (*a*, *c*, *e*) and $R^{\text{MN}}$ (*b*, *d*, *f*) as functions of rapidity separation $\Delta y$ in dijet events, with different energies $\sqrt{s_{NN}}$, TeV: 14 (*a*, *b*), 27 (*c*, *d*), 100 (*e*, *f*). Transverse momentum of jets: $p_\perp \geq 35$ GeV.
The generators used were HERWIG++ (*1*), PYTHIA8 (*2*) and HEJ+ARIADNE (*3*).

generators in order to improve agreement with the experiment in central rapidity regions, i.e., for calculations with small corrections. The simulation results demonstrated the unstable behavior of these corrections at large rapidities and high center-of-mass energies.

Comparing the ratios of $R^{incl}$ and $R^{\text{MN}},$ we can conclude that the first of these quantities always exceeds the second, lying well above the second for small rapidity intervals and becoming comparable for large ones. A possible explanation for this is that MN dijets constitute a subset of inclusive pairs. Both MN dijets and pairwise combinations of jets lying in the rapidity interval between MN jets contribute to the inclusive cross section for dijet production. As follows from the results obtained, the rapidity interval in an MN pair can reach $\Delta y > 11$ at extremely high energies. These events are the most sensitive to BFKL effects. However, they are rather difficult to detect experimentally. At the same time, these events can contribute to the inclusive cross section due to jets produced together with an MN dijet, ordered by rapidity with smaller rapidity intervals.

Thus, inclusive cross sections for dijet production should be preferred in searches for BFKL effects at extremely high energies.

## Conclusion

The simulation results obtained indicate that experimental searches for BFKL effects at future proton-proton colliders may be a promising endeavor.

More definitive and clear conclusions regarding the manifestation of BFKL effects can be drawn by obtaining pure predictions based on evolution of DGLAP without corrections partially emulating the BFKL effects. Furthermore, both analytical computations and Monte Carlo estimates should be developed based on BFKL evolution in the logarithmic approximation next to LLA (NLLA).

The results of this study were obtained within the framework of State Task of the Ministry of Education and Science of Russian Federation 3.1498.2017/4.6.

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*Received 20.03.2019, accepted 01.04.2019.*

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*Статья поступила в редакцию 20.03.2019, принята к публикации 01.04.2019.*

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