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## METHOD OF CALCULATION OF TURBULENT PRANDTL NUMBER FOR THE SST TURBULENCE MODEL

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We present a new model for turbulent Prandtl number that provides an improvement of prediction capabilities of the SST turbulence model in application to wall heat transfer problems. The model was calibrated using Kader's empirical correlation for near-wall temperature profile. To get an initial assessment of the model we performed computations of the fully developed flow in a round tube and a flat channel with Prandtl number varying from 0.004 to 95; the simulation results were validated against benchmark DNS data and empirical correlations for the Nusselt number. According to the tests, applying the new model resulted in considerable reduction of the Nusselt number prediction error (by factor two and more) in the whole range of Prandtl number considered; the most pronounced effect was observed at Prandtl number values below 0.1.

**Keywords:** turbulent flow, wall heat transfer, numerical simulation, turbulent Prandtl number

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## МЕТОД РАСЧЕТА ТУРБУЛЕНТНОГО ЧИСЛА ПРАНДТЛЯ ДЛЯ SST-МОДЕЛИ ТУРБУЛЕНТНОСТИ

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В работе представлена новая модель для турбулентного числа Прандтля, обеспечивающая улучшение предсказательных возможностей популярной полуэмпирической модели турбулентности SST (Shear Stress Transport – модель переноса сдвиговых напряжений) при ее использовании для расчетов пристенного теплообмена. Начальное тестирование разработанной модели проведено на задачах установившегося течения и теплообмена в круглой трубе и плоскопараллельном канале при варьировании числа Прандтля в широких пределах: от 0,004 до 95. По результатам тестов погрешность расчета теплоотдачи во всем диапазоне значений числа Прандтля снизилась в два раза и более. Наибольший положительный эффект от использования разработанной модели наблюдается при числах Прандтля, меньших 0,1.

**Ключевые слова:** турбулентное течение, пристенный теплообмен, численное моделирование, турбулентное число Прандтля

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### Introduction

Numerical simulation of turbulent heat transfer based on Reynolds-averaged Navier–Stokes (RANS) equations commonly involves some sort of semi-empirical model of turbulent viscosity [1, 2] for most practical applications, with the relationship between the coefficients of turbulent heat transfer  $\lambda_t$  and turbulent viscosity  $\mu_t$  given by the turbulent Prandtl number  $Pr_t = \mu_t C_p / \lambda_t$ . Models which entail solving additional differential equations to determine the coefficient of turbulent heat transfer (for example, the transfer equations for the “energy” of temperature fluctuations and dissipation rate [3]) are often seen as overly complex and have not as yet gained wide acceptance.

Heat transfer models using the turbulent Prandtl number typically take it for a constant value (as a rule,  $Pr_t = 0.85$  or  $0.90$ , depending on the turbulence model used), which does not actually happen in most cases. The general understanding of the mechanism of turbulent heat transfer, supported by many computational and experimental studies (see, for example, reviews [4–6]) is that the turbulent Prandtl number varies depending on the contribution of molecular conduction to the scale of turbulent fluctuations. Specifically, if molecular conduction can be neglected (at a distance from the walls bounding the flow with sufficiently high Reynolds numbers), the turbulent Prandtl number should have a certain boundary value  $Pr_{t, \infty} < 1$ ; as the relative role of molecular conduction increases (for example, upon approaching a wall, or with decreasing Reynolds number and/or Prandtl molecular number  $Pr = \mu C_p / \lambda$ ),  $Pr_t$  should increase.

Numerous models have been proposed in literature for turbulent Prandtl number, aimed to gain better agreement between the computed and experimental data on heat transfer for some classes of flows than that obtained using the “standard” value  $Pr_t \approx 0.85$ . We can mention, for example, the following formulations from [7, 4, 8, 9, 5], respectively:

$$Pr_t = \Gamma / (1 - e^{-\Gamma}), \quad (1)$$

$$\Gamma = (0.014 Re^{0.45} Pr^{0.2})^{-1};$$

$$Pr_t = \left(1 + \frac{100}{\sqrt{Re Pr}}\right) \times \left(\frac{1}{1 + 120/\sqrt{Re}} - 0.15\right); \quad (2)$$

$$Pr_t = 1.855 - \text{th}(0.2y^+ - 1.5);$$

$$y^+ = \frac{\sqrt{\rho \tau_w} d}{\mu}; \quad (3)$$

$$\frac{1}{Pr_t} = \frac{0.5}{Pr_{t, \infty}} + \frac{\alpha Pe_t}{Pr_{t, \infty}^{0.5}} - (\alpha Pe_t)^2 \left[1 - \exp\left(\frac{-1}{\alpha Pe_t Pr_{t, \infty}^{0.5}}\right)\right]; \quad (4)$$

$$Pr_t = 0.85 + \frac{f}{Pe_t};$$

$$Pe_t = Pr_t \frac{\mu_t}{\mu}. \quad (5)$$

Models (1) and (2) were constructed for specific cases of fully developed flow in cylindrical tubes. The turbulent Prandtl number is assumed constant over the cross-section of the tube in these models but its value depends on the global regime parameter, the Reynolds number  $Re$ . For this reason, even though the temperature profile is not predicted completely correctly, the computation gives the correct value of the heat transfer coefficient.

Models (3)–(5), where the value of  $Pr_t$  varies in space, increasing as it approaches the wall, seem more physically justified. In particular, the distance to the wall  $d$  is explicitly included in the definition of the universal near-wall coordinate  $y^+$  in formulation (3). On the other hand, however, model (3) is very inconvenient from the standpoint of modern hydrodynamic CFD (Computational Fluid Dynamics) codes, since the argument  $y^+$  used in expression (3) is not a purely local variable: the shear stress  $\tau_w$  on the wall, at a point closest to the given point of the flow, has to be found to compute it.

It follows then that models without any non-local computations are more attractive. In particular, the argument in formulations (4) and (5) with the parameters  $Pr_{t, \infty} = 0.85$ ,  $\alpha = 0.3$ ,  $f = 2$  is the purely local parameter  $Pe_t$ , often called the turbulent Peclet number.

Notice that “non-local” modifications (of little use for general-purpose CFD codes) were proposed in literature for both formulations. In particular, it was pointed out in [5] that it would be practical to “switch” from correlation (5) to a fixed value  $Pr_t = 1.07$  for  $y^+ < 10$ ; it was proposed in [10] to compute the boundary value of  $Pr_{t, \infty}$  in expression (4) with the Reynolds number taken into account to achieve better agreement with experimental data on heat transfer in tubes with small Prandtl numbers (liquid metals)



$$\text{Pr}_{t,\infty} = 0.85 + 100\text{Pr}^{-1}\text{Re}^{-0.888}.$$

Additionally, it should be borne in mind that the effect from applying a particular method for computing the turbulent Prandtl number may depend on the turbulence model used, since different models predict different distributions of turbulent viscosity. We can mention [11] as an example, where the accuracy of  $k-\varepsilon$  [12] (realizable) and  $k-\omega$  SST (Shear Stress Transport) [13] models was assessed as applied to heat transfer computations; the assessment was carried out for the problem of liquid metal flow ( $\text{Pr} = 0.025$ ) in a flat channel. The computations in [11] indicate that correlation (2) yielded the best agreement with the data of reference computations [14], performed by direct numerical simulation (DNS) in case of the  $k-\varepsilon$  model, while in case of the SST model, formulation (5) with the parameter  $f = 0.7$  produced the best results.

Thus, the model for the turbulent Prandtl number should be constructed for a specific semi-empirical turbulence model.

In this study, we have proposed a new method for computing the turbulent Prandtl number, aimed primarily at simulation of near-wall heat transfer at low and moderate Prandtl numbers using the popular  $k-\omega$  SST turbulence model [13].

We have introduced the concept of the calibration method, presenting its final mathematical formulation and providing the results of initial testing.

The computations were carried out using the SINF/Flag-S in-house CFD code, focused on numerically solving hydrodynamics and heat transfer problems with structured and unstructured computational grids embedded in the flow. Some applications of the code and the details of the numerical schemes used in it are given in [15–19].

### Mathematical model

The standard formulation of the SST turbulence model [13] assumes the value of the turbulent Prandtl number to be  $\text{Pr}_t = 0.85$ , which usually provides acceptable accuracy for simulation of turbulent heat transfer for media with the Prandtl number of the order of unity. However, in case of liquid metals whose Prandtl number is smaller by two orders of magnitude, the computations with the “standard”  $\text{Pr}_t$  value significantly overestimate heat transfer on the wall because, as already noted, the turbulent Prandtl number should

increase as the relative role of molecular conduction increases; this is especially important near the wall with low Prandtl numbers.

Considering the relations given in literature for computing the turbulent Prandtl number, we have chosen formulation (5) [5] as a basis for refining the model of near-wall heat transfer, as it is simple and satisfies the above general requirements. We should also emphasize that the turbulent Prandtl number far from the wall (where  $\mu_t \gg \mu$  and, respectively,  $\text{Pe}_t \gg 1$ ) tends to the “standard” value  $\text{Pr}_t = 0.85$  within this formulation, which is why the model of turbulent heat transfer has to be adjusted only in the near-wall region, without changing the properties of the standard model for the rest of the flow.

The proposed model was tailored for the turbulent Prandtl number on the problem of fully developed plane flow of incompressible fluid in the gap between differentially heated walls, one of which moved in the axial direction (Couette flow). The Reynolds number  $\text{Re}$ , constructed from the height of the channel and the velocity of the wall, was taken to be  $10^7$ ; the Prandtl numbers  $\text{Pr}$  ranged from 0.001 to 95.

Although this problem is essentially one-dimensional, the computations were carried out in a fully three-dimensional formulation, with periodic (i.e., in fact, homogeneous) conditions imposed in the axial direction. The computational grid across the channel was taken fine enough to obtain a grid-independent solution (the value of the normalized near-wall coordinate in the computational point closest to the wall was  $y^+ < 0.03$ ). A series of computations was carried out for each selected Prandtl number with different values of the coefficient  $f$  in expression (5) (the total number of variants was 15). The results of the computations were used to determine the “optimal” value of the coefficient  $f$  providing the closest match between the computed temperature profile  $T^+(y^+)$  and the following approximation

$$\begin{aligned} T^+ &= e^{-G} \text{Pr} y^+ + e^{-1/G} T_{\log}^+; \\ G &= \frac{0.01(\text{Pr} y^+)^4}{1 + 5 \text{Pr}^3 y^+}; \\ T_{\log}^+ &= 2.12 \ln(\text{Pr}(1 + y^+)) + \\ &+ \max(0; 3.85 \text{Pr}^{1/3} - 1.3)^2, \end{aligned} \quad (6)$$

where, as usual,

$$T^+ = \frac{(T - T_w) \rho C_p u_\tau}{q_w};$$

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}; y^+ = \frac{\rho du_\tau}{\mu}.$$

This approximation is based on Kader's well-known semi-empirical correlation [20], which successfully approximates extensive experimental data with Prandtl numbers ranging from 0.025 to 95 and, therefore, can serve as their fairly reliable analytical "equivalent". Since the Reynolds number used was rather, we removed the corrections taking into account the influence of the channel height from the original Kader correlation [20]. Additionally, a limiter was introduced in approximation (6), eliminating non-monotonicity with respect to the Prandtl number in the region of its very low values (the second term in the expression for  $T_{log}^+$ ).

Aside from selecting the coefficient  $f$ , we introduced some modifications to relations (5) in order to obtain the best agreement of the computed temperature profile with approximation (6). In particular, we tested different limiters preventing unbounded growth of  $Pr_i$  with  $Pe_i \rightarrow 0$ . The final formulation of the developed model for the turbulent Prandtl number is described by the following relations:

$$Pr_t = 0.85 \cdot \begin{cases} (1 + f^*/Pe_i^*) & , f^* > 0 \\ (1 - f^*/Pe_i^*)^{-1} & , f^* < 0 \end{cases};$$

$$Pe_i^* = \sqrt{0.01 + (Pr \mu_i / \mu)^2};$$

$$f^*(Pr) = (\psi_1 - 1) \cdot (\psi_2^2 + \psi_3^2)^{1/2}; \quad (7)$$

$$\psi_1 = \left[ (0.68 \ln(1 + 50/Pr) + 0.46)^{-4} + 2.2^{-4} \right]^{-1/4};$$

$$\psi_2 = 0.25 + \frac{0.75}{1 + 500 Pr^2};$$

$$\psi_3 = 0.11 + \frac{0.89}{1 + 5 Pr^{-2}}.$$

Here the function  $f^*(Pr)$  is given by the expressions obtained as a result of approximating discrete "optimal" values of the coefficient  $f^*$ . The quality of this approximation is illustrated in Fig. 1.

Fig. 2 compares the near-wall temperature profiles for Couette flow, computed with  $Pr = 0.7$  and  $0.025$ , with the corresponding experimental data given in [20]. It can be seen that using the developed model (7) provided a

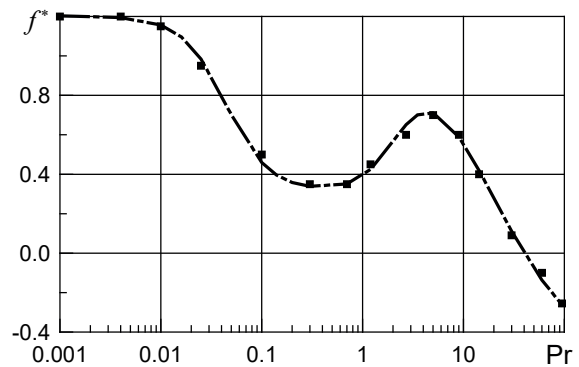


Fig. 1. Form of function  $f^*(Pr)$  in relations (7); discrete "optimal" values of  $f^*(Pr)$  (symbols) and approximating function (line) are shown

much better agreement between the computed and measured temperature profiles, as compared with the case of the standard value  $Pr_t = 0.85$ . This means that we can also expect the prediction for the heat transfer coefficient to be more accurate.

### Test simulations

The above configuration of the heat transfer model (7) was carried out for the Couette model problem with a rather large Reynolds number (uncommon for practical applications), which was dictated by the desire to minimize the influence of channel height on the near-wall flow and obtain a pronounced "logarithmic" segment in the temperature profile. The predictive capabilities of the developed model in the conditions closer to real configurations were assessed by running RANS computations for fluid flow in a round tube and in a flat channel with moderate Reynolds numbers (about  $10^4 - 10^5$ ). We considered fully developed flow (under the action of a given pressure difference) with a volumetric heat source in both cases. The same as for the Couette flow, the problem was solved in a three-dimensional statement with periodic conditions imposed in the axial direction; the computational grid was sufficiently fine ( $y^+ < 0.03$ ) to obtain a grid-independent solution.

Let us consider the problem of flow in a round tube in more detail. Within the adopted formulation, motion is given by superposition of the axial pressure gradient  $dp/dx$  (equivalent body force), and the mean flow rate  $U$  is found from the computation results. The pressure gradient is related to the shear stress  $\tau_w$  on the wall and the diameter  $D$  of the tube by the balance ratio

$$\frac{dp}{dx} = -\frac{4\tau_w}{D}.$$

If we introduce dimensionless quantities, we can formulate the expression, relating the Reynolds number  $Re$ , the “dynamic” Reynolds number  $Re_\tau$  and the friction factor  $\xi$ :

$$\frac{Re_\tau}{Re} = \frac{u_\tau}{U} = \sqrt{\frac{\xi}{8}},$$

where the dimensionless parameters are defined as follows:

$$\begin{aligned} Re &= \frac{\rho UD}{\mu}; \\ Re_\tau &= \frac{\rho u_\tau D}{\mu}; \\ \xi &= \left| \frac{dp}{dx} \right| \cdot \frac{2D}{\rho U^2}. \end{aligned} \quad (8)$$

Thus, the parameter set in the problem is actually the “dynamic” Reynolds number  $Re_\tau$ , and the values of  $Re$  and  $\xi$  are found by computing the velocity profile.

The fluid is heated relative to a fixed wall temperature  $T_w$  by a uniform volumetric heat source  $Q$ , which actually sets the heat flux  $q_w$  on the wall:  $q_w = QD/4$ . This model formulation of the problem with a volumetric heat source approximately corresponds to the conditions of physical experiments with a constant heat flux on the wall. The bulk temperature  $\bar{T}$  and the dimensionless heat transfer coefficient corresponding to it, that is, the Nusselt number are determined from the computed fields of flow velocity and temperature:

$$Nu = \frac{q_w D}{(\bar{T} - T_w) \lambda}. \quad (9)$$

The computations were carried out using the  $k-\omega$  SST turbulence model [13] for two

values of the “dynamic” Reynolds number:  $Re_\tau = 10^3$  and  $5 \cdot 10^3$ ; the values obtained for  $Re$  and for the friction factor  $\xi$  (8) were, respectively,  $Re = 1.671 \cdot 10^4$  and  $1.045 \cdot 10^5$ ,  $\xi = 0.0286$  and  $0.01883$ . Notably, the deviation of the computed friction factors from the values given by the well-known Blasius formula [21]

$$\xi = \frac{0.3164}{Re^{0.25}}$$

does not exceed 4%.

The computations were performed with the Prandtl numbers varying from 0.004 to 95. Along with the developed model (7), we also used the standard approach setting a fixed value of the turbulent Prandtl number  $Pr_t = 0.85$ .

Fig. 3 shows the summary results of test computations, namely, comparison of the computed Nusselt numbers (points) with known empirical correlations for smooth tubes (lines):

$$Nu = \frac{(\xi/8) Pr Re}{1.07 + 12.7 \sqrt{\xi/8} (Pr^{2/3} - 1)}; \quad (10)$$

$$Nu = 6.3 + 0.0167 Re^{0.85} Pr^{0.93}. \quad (11)$$

Correlation (10) [22, 23] for moderate Prandtl numbers ( $0.5 < Pr < 200$ ) relates the Nusselt number to the friction factor  $\xi$  (see formula (8)); the error of this correlation in the range of Reynolds numbers  $10^4 < Re < 5 \cdot 10^6$  does not exceed 6%.

Correlation (11) [24] is considered to be one of the best for liquid metals in the range of Reynolds numbers  $10^4 < Re < 10^6$ .

It can be seen from Fig. 3 that the deviation of the computed Nusselt numbers from empirical correlation (10) does not exceed 10%

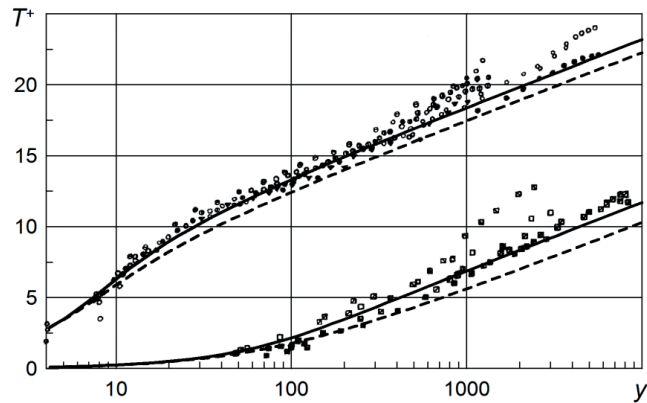


Fig. 2. Computed (lines) and measured (symbols) temperature profiles in near-wall region of quasi-steady turbulent flow for Prandtl numbers  $Pr = 0.7$  and  $0.025$  (upper and lower profiles, respectively); the figure shows the results computed by model (7) (solid lines) and with  $Pr_t = 0.85$  (dashed lines) are given; symbols correspond to the experimental data obtained by different authors cited in [20]

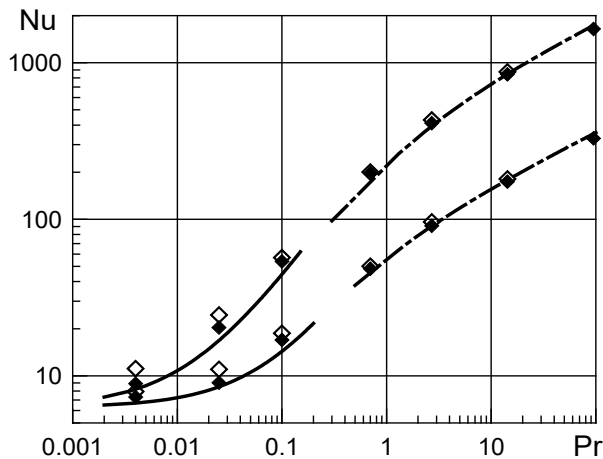


Fig. 3. Comparison of computed (symbols) and experimental (lines) data for heat transfer in round tube with “dynamic” Reynolds numbers  $Re_\tau = 10^3$  and  $5 \cdot 10^3$  (lower and upper curve, respectively); the figure shows the results computed by model (7), with  $Pr_i = 0.85$  (shaded and empty symbols, respectively), and empirical correlations using formulae (10) and (11) (solid and dashed lines, respectively)

for Prandtl numbers  $Pr \geq 0.7$ , and the positive effect from using model (7) is relatively small, that is, less than 5%. In case of low Prandtl numbers ( $Pr \leq 0.1$ ), the computation with the “standard” value  $Pr_i = 0.85$  gives a significantly overestimated heat transfer rate (by almost one and a half times at the maximum); using model (7) significantly improves the situation, as the deviation from correlation (11) does not exceed 20%. In general, considering the wide scatter of experimental data on heat transfer for liquid metals, we can assume that these

computations using model (7) have fairly satisfactory accuracy.

We used the open database of DNS computations [25] carried out at Prandtl numbers from 0.025 to 10 for the second test verifying the performance of the proposed model (flow in a flat channel). The statement of the problem is completely identical to the one considered above for the case of a round tube.

First, following the conditions of numerical experiments [25], we carried out computations for flow in the channel with “dynamic” Reynolds numbers  $Re_\tau = 360$  and  $790$  (the height  $D$  of the channel was taken as the length scale). However, the obtained values of  $Re$  (8) turned out to be 4% lower than the corresponding values given in [25]; this is equivalent to overestimation of the friction factor by about 7%. Since the Nusselt number is customarily related to the Reynolds number  $Re$  (and not to  $Re_\tau$ ) in thermohydraulic analysis, further computations were performed with the corrected values  $Re_\tau = 373$  and  $814$ . In this case, the deviation of the obtained values  $Re = 5.70 \cdot 10^3$  and  $1.41 \cdot 10^4$  from the “reference” values given in [25] did not exceed 0.5%.

The main source of error in determining the friction factor in our computations is obviously the SST model itself, as it is not particularly accurate at predicting turbulent viscosity values in near-wall flow. In particular, as seen in Fig. 4, the value obtained for turbulent viscosity at the boundary of the viscous sublayer ( $y^+ \approx 10$ ) with  $Re = 1.41 \cdot 10^4$ , computed according to the given model, is overestimated by a factor of 1.5, which noticeably distorts the flow velocity profile  $u^+ = u/u_\tau$ . Naturally, this also affects the

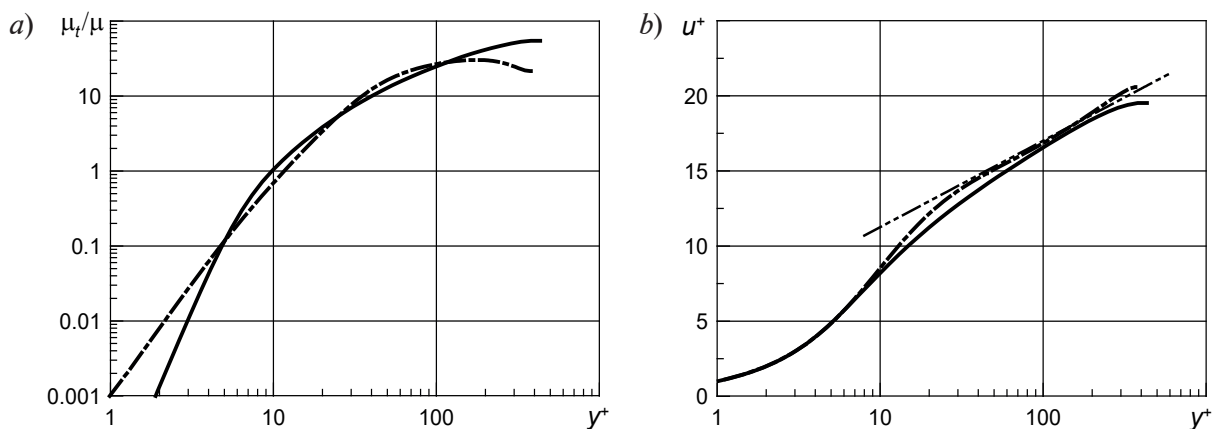


Fig. 4. Normalized profiles of turbulent viscosity (a) and velocity (b) in flat channel with  $Re = 1.41 \cdot 10^4$ ;

the figure shows the results computed by the SST model (solid lines), by DNS [25] (dash-dotted lines), and analytical solution (12) (double dash-dotted lines)

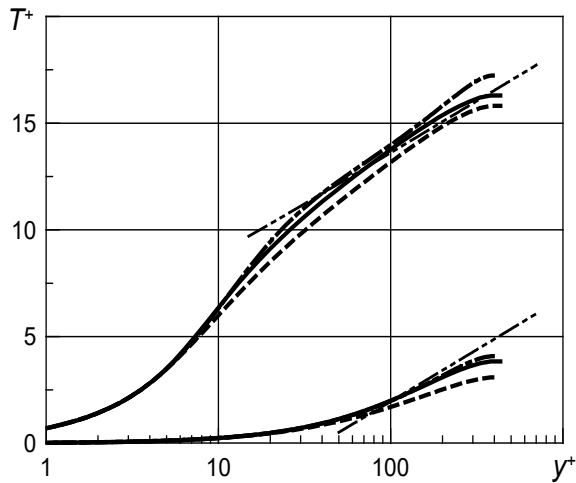


Fig. 5. Profiles of temperature  $T^+(y^+)$  in flat channel with  $Re = 1.41 \cdot 10^4$  for  $Pr = 0.71$  and  $0.025$  (upper and lower family of curves, respectively); figure shows the results computed by model (7) (solid lines), with  $Pr_t = 0.85$  (dashed lines), by DNS [25] (dash-dotted lines), and “logarithmic” profiles  $T_{log}^+$  (6) (double dash-dotted lines)

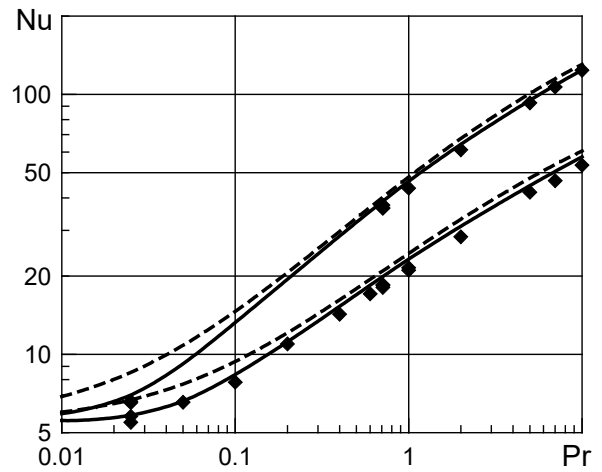


Fig. 6. Computed heat transfer in flat channel with  $Re = 5.7 \cdot 10^3$  and  $1.41 \cdot 10^4$  (lower and upper family of curves, respectively); figure shows the results computed by model (7) (solid lines), with  $Pr_t = 0.85$  (dotted lines), by DNS[25] (symbols)

magnitude of shear stress on the wall.

Fig. 4, *b* also shows the well-known analytical solution

$$u_{log}^+ = 2.5 \ln(y^+) + 5.5 \quad (12)$$

for the velocity distribution in the equilibrium “logarithmic” region of near-wall flow (see, for example, [21]). Experience of computations by the SST turbulence model indicates that the “logarithmic” region in the near-wall velocity profile tends to start with a delay in this model. It can be seen from Fig. 4, *b* that DNS computations reveal a fairly extended “logarithmic” region in the near-wall velocity profile, while the corresponding region is virtually absent in the SST model.

Fig. 5 shows the normalized temperature profiles  $T^+(y^+)$  obtained in this study and by reference DNS computations [25] with the Prandtl numbers  $Pr = 0.71$  and  $0.025$ . Evidently, the same as in Fig. 2, using model (7) yields much better agreement of the computed temperature profiles with the reference data.

Fig. 6 shows summary results of thermal computations, namely, comparison of the Nusselt numbers (9) obtained in this study (lines) and in DNS computations [25] (points). Similar to the previous test (see Fig. 3), using the developed model (7) significantly improves the accuracy of heat transfer computations for all values of the Prandtl number. The maximum deviation of the Nusselt number from the reference values [25]

decreased from 15 to 8% in the region with  $Pr \geq 0.2$ , and from 24 to 6% for  $Pr \leq 0.1$ .

We should also note that the final error in computing heat transfer is of the same order of magnitude as the error in computing friction (about 7%). This leads us to conclude that the reason for both errors is that the  $k-\omega$  SST turbulence model is not quite accurate at predicting the behavior of turbulent viscosity.

### Conclusion

We have obtained the following main results.

We have developed a new model for computing the local turbulent Prandtl number, with improved prediction of heat transfer characteristics in fluid flows with small and moderate Prandtl numbers as applied to the popular  $k-\omega$  SST turbulence model.

We have carried out initial testing of the developed model for problems of fully developed flow and heat transfer in a round tube and a flat channel by varying the Prandtl number from 0.004 to 95. We have confirmed that using the proposed model helps substantially decrease the error in computing heat transfer (by about two or more times). The greatest positive effect is achieved with the Prandtl numbers less than 1/10.

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