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A QUASISTATIC APPROACH TO THE THERMOELASTICITY PROBLEM OF ROTATING BODIES

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The non-stationary problem of thermoelasticity for rotating bodies has been solved through determining the optimal temperature and stress fields in the rolling mills of hot rolling systems, this determination being an issue of the day. The Eulerian approach was applied, it allowed us to reduce the number of independent variables and consider these fields as quasistatic ones. The heavy temperature gradients and stresses bound up with them, as well as the rotating nature of these fields are typical for the processes taking place in the roll core. To solve the problem of simulation of these processes, we proposed to use Fourier series, which allowed us to obtain a solution with a sufficient accuracy for the large number of terms of the series being considered. The peculiarity of the solution obtained is that the stress maximum locates at an insignificant depth beneath the roll surface.

Keywords: temperature stress, mill roll, rotating system, Fourier series

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КВАЗИСТАТИЧЕСКИЙ ПОДХОД К РЕШЕНИЮ ЗАДАЧИ ТЕРМОУПРУГОСТИ ВРАЩАЮЩИХСЯ ТЕЛ

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Представлено решение нестационарной задачи термоупругости вращающихся тел на примере определения оптимальных температурных полей и полей напряжений в прокатных валках систем горячего проката, что представляет собой актуальную проблему. Используется пространственное описание, позволяющее уменьшить число независимых переменных и рассматривать поля температуры и напряжений как квазистатические. Для процессов, происходящих в теле валка, характерны большие градиенты температуры и связанные с ними напряжения, а также вращающийся характер полей. Для решения задачи моделирования указанных процессов предлагается использовать ряды Фурье, которые позволяют при довольно большом количестве рассматриваемых членов ряда получать решение с достаточной точностью. Особенностью полученного решения является локализация максимальных напряжений на незначительной глубине от поверхности валка.

Ключевые слова: температурное напряжение, прокатный валок, вращающаяся система, ряд Фурье

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Introduction

Problems of thermoelasticity in rotating bodies subjected to thermal shocks remain relevant today, with rational solutions obtained for different practical applications.

Simulation of hot rolling of sheet metal, with rotating rolls of the rolling mill operating under heavy mechanical and thermal stress, is a vivid example of such an application. The rolls are affected by a combination of cyclic, mechanical and thermal stresses; characteristic failure of the orange peel type, associated with blister delamination, is observed in the surface layer of the rolls as a result.

The goal of this study has consisted in developing a method for effectively solving thermoelastic problems in rotating bodies exposed to thermal shocks.

The uncoupled thermoelastic problem is considered in several stages. The first stage involves solving a non-stationary boundary problem of thermal conductivity, generating a temperature field that depends on both time and space coordinates. The second stage is aimed at determining the stress-strain state of a rotating body. We have taken a mathematical approach reducing partial differential equations with three independent variables to ordinary differential equations, which makes it possible to obtain a solution in the form of a single series.

The approach we have proposed is universally applicable, as evidenced by a complete solution to the problem of finding thermal stresses in rotating rolls during hot rolling of sheet metal. It was observed that surface fracture of mill rolls in operation has a lamellar structure and cracks form at a relatively small depth (1–2 mm) from the surface rather than on it. The nature of cracks suggests that one of the key factors causing them are non-stationary thermal stresses occurring as a result of sudden changes in temperature. These effects are explained by significant temperature gradients induced by thermal shocks in physics of metals.

The developed approach for analytical calculation of localized temperature fields is preferable to numerical methods, since it does not involve iterative selection of the size of the grids, which is unknown in advance.

The given problem is particularly important for industry, as confirmed by numerous studies considering both temperature fields as a separate issue and temperature fields and mechanical stresses arising during rolling.

Three main approaches are used in study of temperature fields:

direct experimental measurement of surface temperature fields [1];

calculation of temperature fields by the finite element method (FEM) or by the grid method taking into account roll rotation or boundary conditions [2–10];

calculation of temperature fields by the Fourier method in the form of a sum of a series with respect to eigenfunctions [11–14].

A notable group of studies [15–17] developed the theory of harmonic finite elements. In this case, a sequence of one-dimensional FEM problems is solved separately for each harmonic amplitude in a Fourier series and the resulting amplitudes are then multiplied by the corresponding harmonic function and summed.

Study of mechanical stress fields has only been carried out by finite element methods [3, 5, 6, 8–10, 16–18]. The temperature field is taken as the load and the plane problem of elasticity theory is then solved. Practically all solutions, except those given in [17], yield the maximum stresses (by the von Mises criteria) located on the surface of the rolls, in the area of contact with the rolled metal. This result is generally recognized as valid, so in [14], after the temperature was calculated by the Fourier method, the modulus of surface kinematic hardening was calculated, instead of the stresses, using the temperature potential.

However, the nature of failure in rolls indicates that the maximum mechanical stresses are located at small depths within the roll core. The standard FEM approach is inadequate for describing this situation. The same applies to “harmonic” finite elements [15–17]: it was found that these elements cannot be used to calculate stresses, since they yield stresses that are 1.5 times higher than those obtained by the “standard” FEM approach.

In this study, we suggest a spatial approach reducing the non-stationary thermal conductivity problem to a quasistationary one, which makes it possible to develop a mathematical model of thermal conductivity in a rotating elastic body. The temperature distribution field can then be found, and thermal stresses which are particular solutions to the thermoelastic problem are obtained using the thermoelastic potential. The boundary conditions are to be satisfied via the Airy stress function using the example of trivial loading conditions.

Determining the temperature field in the rolls

Fig. 1 shows the heating and cooling patterns of the mill roll. The roll contacts a hot slab in a narrow contact sector, and then undergoes a complex cooling cycle in different media. In our case, we consider alternate cooling with water and air; other possible schemes involve cooling with a water-vapor mixture, with only water, or only air. The sectors indicate areas with different types of cooling.

Assuming that the temperature field does not depend on the axial coordinate, the non-stationary thermal conductivity equation has the form

$$\lambda\Delta T - \gamma\dot{T} = 0, \quad (1)$$

where T is the temperature; λ and γ are the thermal conductivity and heat capacity of the roll material, respectively; Δ is the two-dimensional Laplace operator; the dot indicates the material derivative with respect to time.

We have used the spatial (Eulerian) approach, reducing the number of independent variables. Provided that the roll rotates at a constant angular velocity ω, the expression for the material derivative of the temperature field is simplified:

$$\dot{T}(r,\varphi,t) = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial r}\frac{dr}{dt} + \frac{\partial T}{\partial \varphi}\frac{d\varphi}{dt} = \omega\frac{\partial T}{\partial \varphi}, \quad (2)$$

where r and φ are the radius and the angle in the polar coordinate system.

Since the problem is stationary,

$$\frac{\partial T}{\partial t} = 0,\ \frac{dr}{dt} = 0,\ \frac{d\varphi}{dt} = \omega.$$

As a result, we obtain the equation of stationary thermal conductivity with two independent variables:

$$\lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial \varphi} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}\right) - \gamma\omega\frac{\partial T}{\partial \varphi} = 0. \quad (3)$$

The surface temperature of the roll in the narrow zone of direct contact between the roll and the sheet, where $0 \le \varphi \le \varphi_0$ (see Fig. 1) is taken equal to T_c = 600 °C as the average between the temperature of the hot sheet and the temperature of the roll. A boundary condition of the third kind is imposed for the rest of the zone ($\varphi_0 < \varphi \le 2\pi$):

$$r = R, \varphi_0 < \varphi \le 2\pi,$$

$$\lambda\frac{\partial T}{\partial r} + \beta(T - T_m) = 0,$$

where R is the radius of the roll; $T = T_m(\varphi)$ is the temperature of the air/water mixture cooling the surface of the roll.

These two conditions can be written as a single boundary condition of the third kind:

$$r = R,\ \eta\frac{\partial T}{\partial r} + T - T_e = 0, \quad (4)$$

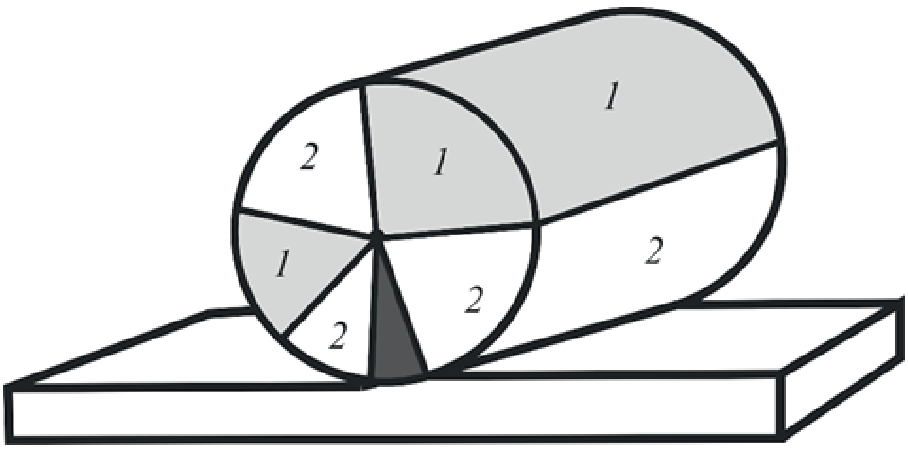

Fig. 1. Rolling pattern (only upper roll is shown) and pattern for cooling roll with water (*1*) and air (*2*); the black sector corresponds to the contact area

where

$$T_e = T_e(\varphi) = \begin{cases} T_c, & 0 \le \varphi \le \varphi_0, \\ T_m(\varphi), \varphi_0 < \varphi \le 2\pi; \end{cases}$$
$$\eta = \frac{\lambda}{\beta} = \begin{cases} 0, & 0 \le \varphi \le \varphi_0, \\ \eta(\varphi), & \varphi_0 < \varphi \le 2\pi; \end{cases} \qquad (5)$$

where the parameters $\lambda(\varphi)$, $\beta(\varphi)$, $T_m(\varphi)$ are piecewise constant functions of the angle φ.

To solve boundary problem (3)–(5), we apply the method of integral transforms:

$$\frac{1}{2\pi}\int_0^{2\pi} T(r,\varphi)e^{-in\varphi}d\varphi = T_n(r),$$

and then seek a solution in the form of a bilateral series:

$$T(r,\varphi) = \sum_{n=-N}^{N} T_n(r)e^{in\varphi}. \qquad (6)$$

After integral transformation, Eq. (3) takes the form

$$\frac{d^2T_n}{dr^2} + \frac{1}{r}\frac{dT_n}{dr} + \left(-\frac{n^2}{r^2} - \frac{in\gamma\omega}{\lambda}\right)T_n = 0. \qquad (7)$$

The solution of Eq. (7) for $n \ne 0$ is expressed in terms of Bessel functions J_n and Y_n:

$$T_n(r) = A_n J_n\left(\sqrt{\frac{-in\omega\gamma}{\lambda}}r\right) + B_n Y_n\left(\sqrt{\frac{-in\omega\gamma}{\lambda}}r\right). \qquad (8)$$

Since $Y_n(0)=\infty$, $B_n=0$ due to limited temperature in the center of the roll. Then,

$$T_n(r) = A_n J_n\left(\sqrt{\frac{-in\omega\gamma}{\lambda}}r\right) = A_n J_n\left(-\frac{1-i}{\sqrt{2}}a\sqrt{n}\xi\right), \qquad (9)$$

where $a = \sqrt{\frac{\omega\gamma}{\lambda}}R$ and a dimensionless radius $\xi = r/R$, $0 \le \xi \le 1$ is introduced.

Let us consider the case $n = 0$ separately. In this case, Eq. (7) has the form

$$\frac{d^2T_0}{\partial r^2} + \frac{1}{r}\frac{dT_0}{\partial r} = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_0}{\partial r}\right) = 0.$$

Then, $T_0 = C\ln r + A_0$, and it follows that $C = 0$, i.e., $T_0 =$ const, from the boundedness condition for $r = 0$.

Now let us carry out integral transformation of boundary condition (4):

$$\frac{1}{2\pi}\int_0^{2\pi}\left[\eta(\varphi)\frac{\partial T}{\partial r} + T - T_e(\varphi)\right]e^{-in\varphi}d\varphi = 0. \qquad (10)$$

Representing $\eta(\varphi)$ in the form of a Fourier series, we introduce the following notations:

$$\eta(\varphi) = \sum_{k=0}^{k}\eta_k e^{ik\varphi},$$
$$\eta_k = \frac{1}{2\pi}\int_0^{2\pi}\eta(\varphi)e^{-ik\varphi}d\varphi, \qquad (11)$$
$$\tau_n = \frac{1}{2\pi}\int_0^{2\pi}T_e(\varphi)e^{-in\varphi}d\varphi.$$

Let us now separately calculate the first term in Eq. (10):

$$\frac{1}{2\pi}\int_0^{2\pi}\eta(\varphi)\frac{\partial T}{\partial r}e^{-in\varphi}d\varphi = \frac{1}{2\pi}\sum_{k=0}^{k}\eta_k\int_0^{2\pi}\frac{\partial T}{\partial r}e^{-i(n-k)\varphi}d\varphi = \sum_{k=0}^{k}\eta_k\frac{d}{dr}\left(\frac{1}{2\pi}\int_0^{2\pi}Te^{-i(n-k)\varphi}d\varphi\right) = \sum_{k=0}^{k}\eta_k\frac{d}{dr}T_{n-k}(r) = \sum_{k=0}^{k}\eta_k T'_{n-k}.$$

Substituting the explicit expression for T_n and calculating the derivative of the Bessel function by the rule

$$J_n'(z) = J_{n-1}(z) - \frac{n}{z}J_n(z) = 0,$$

we obtain the following formulation for the boundary condition:

$$\begin{cases} A_0 - \tau_0, n = 0; \\ \sum_{k=0}^{K}\left(\frac{\eta_k}{r}A_{n-k}\left[\frac{1-i}{\sqrt{2}}a\sqrt{n-k}J_{n-k-1}(\chi_{n-k}) - (n-k)J_{n-k}(\chi_{n-k})\right] + A_nJ_n(\chi_{n-k})\right) - \tau_n = 0, \ n \ne 0, \end{cases} \qquad (12)$$

where $\chi_{n-k} = \frac{1-i}{\sqrt{2}}a\sqrt{n-k}$.

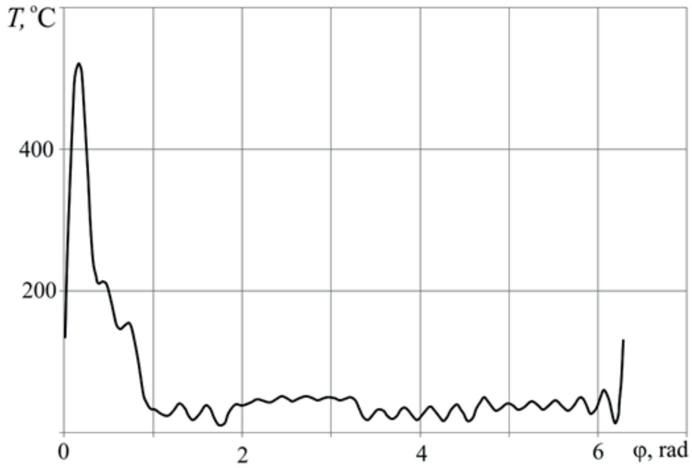

Fig. 2. Calculated temperature on surface of roll as function of angular coordinate ($r = R$); calculation parameters are given in the text

As a result, we obtain a system of n equations with unknown coefficients A_n. To solve it, we use the asymptotic behavior of the Bessel function for large values of the argument:

$$J_n(z) = \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{n\pi}{2} - \frac{\pi}{4}\right),$$

since the argument of Bessel functions for real parameters of hot rolling (a = 201.6) takes the values

$$\left|\frac{1-i}{\sqrt{2}}\right| a\sqrt{n} \approx 201.6\sqrt{n}, \; n = 1, 2, \ldots .$$

Substituting these values of the argument, we obtain:

$$J_n(z) = \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{n\pi}{2} - \frac{\pi}{4}\right) \approx \approx \frac{1}{2}\sqrt{\frac{2\sqrt{2}}{\pi(1-i)a\sqrt{n}}} \cdot e^{\frac{a\sqrt{n}}{\sqrt{2}}} \cdot e^{i\left[\frac{a\sqrt{n}}{\sqrt{2}} - \frac{n\pi}{2} - \frac{\pi}{4}\right]}. \quad (13)$$

Thus, by solving the system of equations for the coefficients A_n and substituting them into Eqs. (9) and (6) for $T(r,\varphi)$, we obtain the temperature distribution in the roll.

Since the coefficients in Eq. (12) contain exponential factors, the system's matrix is ill-conditioned. For this reason, we need to estimate the required number of series terms for each function and use a special algorithm for the solution.

The nature of Eqs. (12) allows using an algorithm with choice of the pivot element. If the matrix (η_k) is given asymetrically (see expression (11)), the system matrix becomes triangular, thus allowing to successively calculate all the coefficients A_n. A graph of the temperature field on the surface of the roll as a function of the angular coordinate is given below (Fig. 2) as an example of the calculations for the following numerical values of the system parameters:

1. T_c = 600 °C, φ_0 = 12.68° = 0.2213 rad (in the contact zone);

2. T_e = 25 °C, β = 41700 W/°C m^2 (for water);

T_0= 25 °C, β = 1500 W/°C m^2 (for air);

3. λ = 31 W/°C m^2, γ = 0.673 kJ/°C kg (for rolls)

Calculating thermal stresses

To calculate the thermal stress field, we use the thermoelastic displacement potential Φ which is introduced by the equality $\underline{u} = \nabla\Phi$. In the problem of plane deformation, The thermoelastic potential **Φ** is introduced in the problem of plane deformation, satisfying the equation [19]:

$$\Delta\Phi = \frac{1+\nu}{1-\nu}\alpha T. \quad (14)$$

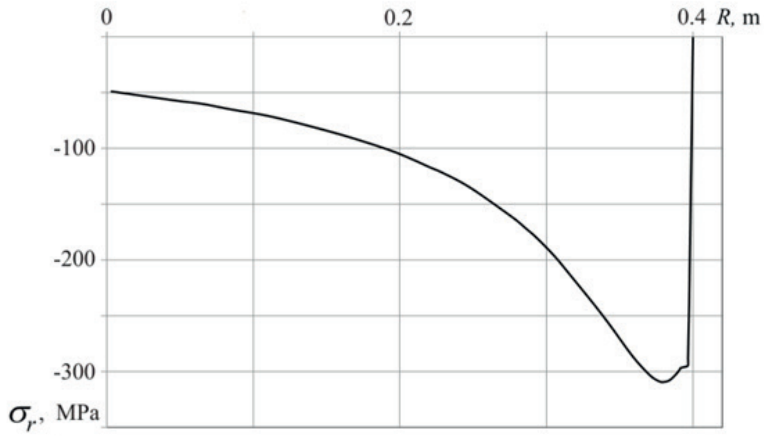

Fig. 3. Radial distribution of stresses σ_r in roll core with φ = 0.1 rad

Differentiating both sides of this equation over time and substituting dT/dt from Eq. (1), we obtain:

$$\frac{d}{dt}\Delta\Phi = \frac{1+\nu}{1-\nu}\alpha\frac{\lambda}{\beta}\Delta T. \quad (15)$$

Since the thermoelastic potential provides only a partial solution of the thermoelastic equation, the sign of the operator Δ can be omitted in both sides of Eq. (15). As a result, we have:

$$\frac{d}{dt}\Phi = \frac{1+\nu}{1-\nu}\alpha\frac{\lambda}{\beta}T. \quad (16)$$

The material derivative of the thermoelastic potential in a rotating coordinate system is calculated similarly to calculation by expression (2):

$$\frac{d}{dt}\Phi = \frac{\partial\Phi}{\partial\varphi}\frac{\partial\varphi}{\partial t} = \omega\frac{\partial\Phi}{\partial\varphi} = \frac{1+\nu}{1-\nu}\alpha\frac{\lambda}{\beta}T, \quad (17)$$

from which we obtain the explicit expression for the thermoelastic potential:

$$\Phi = \frac{1}{\omega}\frac{1+\nu}{1-\nu}\alpha\frac{\lambda}{\beta}\int T d\varphi = \\ = \frac{1}{\omega}\frac{1+\nu}{1-\nu}\alpha\frac{\lambda}{\gamma}\left[\sum_{n=-N}^{N} T_n(r)\frac{1}{in}e^{in\varphi}\right]. \quad (18)$$

The thermal stresses corresponding to the obtained thermoelastic potential are found by forward differentiation (see [19]):

$$\sigma_r^{\Phi} = -2G\left[\frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\varphi^2}\right],$$
$$\sigma_\varphi^{\Phi} = -2G\frac{\partial^2\Phi}{\partial r^2},$$
$$\tau_{r\varphi}^{\Phi} = 2G\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\right], \quad (19)$$
$$\sigma_z^{\Phi} = -2G\Delta\Phi = -2G\frac{1+\nu}{1-\nu}\alpha T,$$
$$\tau_{zr}^{\Phi} = \tau_{z\varphi}^{\Phi} = 0.$$

Airy function for satisfying boundary conditions

The given thermal stresses are particular solutions, and in the general case they do not satisfy the boundary conditions. Below we demonstrate how to fulfill trivial force conditions on the entire surface of the roll. To adjust the solution so that it satisfies the trivial boundary conditions, let us introduce the total stresses:

$$\sigma_r = \sigma_r^{\Phi} + \sigma_r^{U},$$
$$\sigma_\varphi = \sigma_\varphi^{\Phi} + \sigma_\varphi^{U},$$
$$\tau_{r\varphi} = \tau_{r\varphi}^{\Phi} + \tau_{r\varphi}^{U}$$

and so on.

The superscript here indicates the stresses determined by the Airy function in the plane

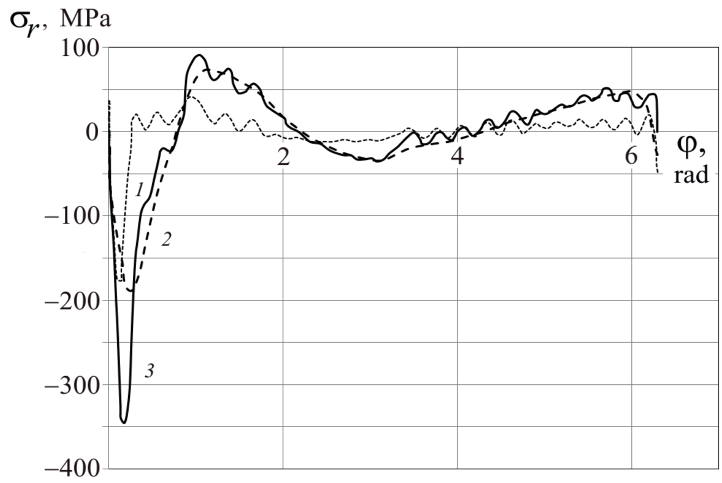

Fig. 4. Radial distribution of stresses σ_r along angular coordinate φ at different depths h, mm, from roll surface: 0.5 (*1*), 4 (*2*), 100 (*3*)

problem $U(r\ \varphi)$.

It satisfies the biharmonic equation $\Delta\Delta U = 0$ and allows to find the stresses:

$$\sigma_r^U = \frac{1}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2}\frac{\partial U^2}{\partial \varphi^2},$$

$$\sigma_\varphi^U = \frac{\partial^2 U}{\partial r^2},$$

$$\tau_{r\varphi}^U = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial U}{\partial \varphi}\right), \quad (20)$$

$$\sigma_z^U = \nu\left(\sigma_r^U + \sigma_\varphi^U\right),$$

$$\tau_{zr}^U = \tau_{z\varphi}^U = 0.$$

The Airy function as a Fourier series has the following general form (see [19, 20]):

$$U(\xi,\varphi) = \left[\alpha\xi^3 + \beta\xi^{-1} + \lambda\xi + \gamma\xi\ln\xi + \chi\varphi\xi\right]e^{i\varphi} + \sum_{n=2}^{\infty}\left(P_n\xi^n + Q_n\xi^{n+2}\right)e^{in\varphi}, \quad (21)$$

and the coefficients α, β, λ, γ, χ, P_n, Q_n can be found from the no-stress boundary condition imposed for the entire surface of the roll:

$$r = R,\ \sigma_r^\Phi + \sigma_r^U = 0,\ \tau_{r\varphi}^\Phi + \tau_{r\varphi}^U = 0. \quad (22)$$

Fig. 3 shows the stress distribution along the roll radius with the angle φ = 0.1 rad.

Fig. 4 shows a family of radial stress distributions along the angular coordinate in the roll core with different values of its radius.

Rapid growth of stresses at a small depth from the surface and a smooth decrease towards the center of the roll can be clearly observed from Fig. 3. The stress maximum is reached at a depth *h of* about 4 mm from the surface (see Fig. 4).

Discussion of results

The distribution of radial stresses in the roll core that we have obtained differs considerably from the known results calculated using FEM. The maximum radial stresses of 350 MPa are close to the yield strength, with peculiar drops observed on the curves. The zone of these stresses lies in the roll core at a depth of 2–4 mm from the surface, which explains well blister fatigue failure (with blisters 1–4 mm thick falling off from the roll surface during rolling).

The stress maximum below the surface was obtained in [9] by simulating a multilayer system with different mechanical characteristics near the roll surface.

Sharp drops on the stress curves were obtained in [17] by taking into account contact stresses (in addition to thermal ones) in the rolling zone. Unlike the solution we obtained, the depth of the stress maximum by the von Mises criterion was 5% of the radius, which did not help explain blister delamination. In our case, this depth was about 3 cm.

Thus, the approach we proposed and used made it possible to obtain new results for explaining the experimentally observed blister failure of the surface by “plunging” cyclic radial stresses.

The given problem cannot be solved by the FEM method due to huge radial stress gradients evolving in the subsurface layer, which is about 0.1% of the roll radius. The stress averaging, which automatically occurs inside finite elements, does not allow to adequately model these gradients.

Notably, the Bessel functions in the Fourier series with asymptotic representation contain factors that are exponents of the form $\exp(\lambda r/R)$, where factors λ have a characteristic value of about 1000. Therefore, there is no sense in taking more than forty terms of the Fourier series, since the coefficients of the Fourier series cannot be determined with a given double precision even if special normalization is applied. The "fluctuations" of all functions, seen on the graphs of angular distributions, are related to this circumstance. Naturally, the numerical FEM solution runs into a similar problem for the case with ill conditions, as discussed in the above-cited studies.

Conclusion

We have introduced a spatial approach to the thermoelastic problem in rotating bodies, which made it possible to reduce the number of independent variables and obtain exact solutions for the temperature and stress fields in the form of a single Fourier series. We have formulated the equations for thermal stresses through the thermoelastic potential that we have then used to calculate thermal stresses on the surface and in the core of the roll. We have confirmed that the Airy function could be tailored to satisfy the boundary conditions on the roll surface. While this approach allows to satisfy any boundary conditions set in advance, we have confined ourselves to the case of trivial force boundary conditions imposed on the entire surface of the roll.

As an example of practical application, we have found a thermal stress field for the mill rolls during hot rolling of sheet metal.

Based on the investigation we have carried out, we were able to conclude that thermal stresses make a significant contribution to the stress state of mill rolls, and the magnitude of the temperature component of the stresses and the cyclic pattern with which they occur indicate that roll surface failure can evolve solely due to thermal "shock" induced by heating and cooling of the roll surface.

The effect of thermal stresses dropping to a small depth beneath the roll surface has been obtained for the first time. Importantly, this effect adequately explains the blister nature of failure in the roll. The object chosen for study appeared to be a good model for describing the processes occurring during operation of rotating systems subjected to complex thermal loading.

Thus, the proposed approach to solving the problem may have other important practical applications for analysis of this type of systems, in particular, in metallurgy and mechanical engineering.

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