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WAVE SCATTERING BY AN ANISOTROPIC TWO-SCALE ROUGH SURFACE

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The scalar-wave scattering by a rough anisotropic surface has been considered in the paper. In order to solve the problem, a two-scale model taking into account the wave scattering by coarse irregularities under the Kirchhoff's approximation and determining that one by fine roughnesses through the Rayleigh method was used. The scattering cross section was averaged over the anisotropic distribution of the surface slopes. The results of numerical calculations of the total scattering cross section were presented. They demonstrate the anisotropy impact of the roughness distribution on the wave scattering indicatrix.

Keywords: scattering, rough surface, anisotropy, Kirchhoff's approximation, two-scale model

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РАССЕЯНИЕ ВОЛН АНИЗОТРОПНОЙ ДВУХМАСШТАБНОЙ ШЕРОХОВАТОЙ ПОВЕРХНОСТЬЮ

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Рассмотрено рассеяние скалярных волн шероховатой анизотропной поверхностью. Для решения задачи использована двухмасштабная модель, в которой рассеяние на крупных неоднородностях учитывается в приближении Кирхгофа, а рассеяние на мелких шероховатостях определяется методом Рэлея. Проведено усреднение сечения рассеяния по анизотропному распределению наклонов поверхности. Представлены результаты численных расчетов полного сечения рассеяния, демонстрирующие влияние анизотропии распределения шероховатостей на индикатрису рассеяния волны.

Ключевые слова: рассеяние, шероховатая поверхность, анизотропия, приближение Кирхгофа, двухмасштабная модель

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Introduction

Real rough surfaces typically have a complex random structure with both large and small (compared to wavelength) inhomogeneities. Natural and artificial objects have these similar characteristics, observed in scattering of acoustic and electromagnetic waves [1–5]. Rough sea surface [6, 7], which can be represented as a result of superposition of small ripples on a large wave is an example of such a natural surface combining irregularities of different scales.

A two-scale model first developed by Kuryanov [8], considering isotropic rough surfaces, is used to describe scattering of waves by a rough surface with multiscale irregularities. In this case, the calculations can be carried out to completion and the answer can be expressed in terms of the error function. In particular, the calculations given are for scattering by a perfectly smooth and a perfectly rigid surface. In the acoustic case, the two-scale model is applicable, for example, to calculations of scattering from vegetation on a randomly inhomogeneous surface of the Earth, on the seabed, or on air bubbles on the surface of the water. Subsequent refinement of the model allowed to obtain a number of quantitative corrections, retaining both the basic assumptions underlying the model's approach and the main qualitative conclusions on the angular dependence of the total scattering cross-section for such a surface [9–14]. All known results obtained earlier within the framework of the two-scale model are for isotropic rough surfaces.

The goal of this study has been to demonstrate the set of tools that the two-scale model offers for describing wave scattering by rough surfaces with anisotropic characteristics.

For the purposes of our study, we shall assume the distribution of fine roughnesses to be uniform and isotropic, and the distribution of coarse roughnesses to be uniform and anisotropic. We have used the expression for the scattering cross-section for fine irregularities taking into account its dependence on the correlation function and calculated the integrals with anisotropic Gaussian distribution for the surface slopes of coarse irregularities. We have used the two-scale model to construct scattering diagrams for an anisotropic rough surface and compared them with the results for the isotropic model. Based on the calculations, we can conclude that anisotropy of the rough surface affects scattering of waves.

Model description and governing equations

Let us consider a rough surface that can be represented as small displacements $\zeta(\mathbf{r})$ against the background of coarse irregularities $\eta(\mathbf{r})$. Roughnesses $\zeta(\mathbf{r})$ and $\eta(\mathbf{r})$ should satisfy the following conditions:

$$2k\sigma_\zeta \sin\alpha << 1,\ \left\langle (\nabla_\eta \zeta)^2 \right\rangle << 1,$$
$$\sigma_\zeta^2 = \left\langle \zeta^2 \right\rangle, \tag{1}$$
$$2k\tilde{R}\sin\alpha << 1,$$

where k is the wavenumber; $\tilde{R}$ is the characteristic curvature radius of the large-scale surface; α is the local grazing angle measured from the plane tangential to the large-scale surface; $\nabla_\eta\zeta$ is the displacement gradient in the tangent plane.

The scattering cross-section can be represented as

$$\sigma = \sigma_1 + \sigma_2, \tag{2}$$

where σ_1 is the cross-section of scattering by the surface $\eta(\mathbf{r})$, calculated by the tangent plane method [5, 6] in the Kirchhoff approximation [15, 16]; σ_2 is the cross-section of scattering by fine roughnesses $\zeta(\mathbf{r})$, calculated by the Rayleigh method over coarse irregularities $\eta(\mathbf{r})$.

For a normal distribution of the slopes of coarse roughness [6], expressed as

$$p(\eta_x,\eta_y) = (\pi\delta_x\delta_y)^{-1}\exp\left(-\frac{{\eta_x}^2}{{\delta_x}^2} - \frac{{\eta_y}^2}{{\delta_y}^2}\right), \tag{3}$$

the scattering cross-section σ_1 has the form

$$\sigma_1 = \frac{(VF)^2}{\pi\delta_x\delta_y}\exp\left(-\frac{1}{K^2}\left(\frac{{Q_x}^2}{{\delta_x}^2} + \frac{{Q_y}^2}{{\delta_y}^2}\right)\right), \tag{4}$$

where $F = 1/2(1 + Q^2/K^2)$; $\mathbf{Q} = \mathbf{q} - \mathbf{q}_i$ is the projection of the vector $\mathbf{k} - \mathbf{k}_i$ on the surface plane ($\mathbf{k}$, $\mathbf{k}_i$ are the wave vectors of the incident and scattered waves, respectively); $K = k_{iz} - k_z$ is the sum of vertical projections of the wave vector of the incident and scattered waves. The parameters δ_x and δ_y are found from the variance tangents of the surface slope along the coordinate axes:

$$\delta_x^2 = 2\left\langle \eta_x^2 \right\rangle, \delta_y^2 = 2\left\langle \eta_y^2 \right\rangle.$$

To formulate the expression for σ_2, let us assume that the role of the underlying surface

$\eta(\mathbf{r})$ can be taken into account using the tangent plane method, and scattering by fine irregularities $\zeta(\mathbf{r})$ by the Rayleigh method. Let us assume that a sufficiently large element of the surface $\eta(\mathbf{r})$ can be regarded as flat, with dimensions much smaller than the curvature radius but much larger than the correlation length of fine roughness, which, in turn, is much smaller than the wavelength. Within this region, the normal $\mathbf{n}$ to the surface can be assumed to be a constant vector, and then the cross-section of scattering by fine roughnesses can be calculated for a flat surface.

Let us denote this local cross-section $\sigma_{loc}(\mathbf{Q}_\eta)$ where $\mathbf{Q}_\eta$ is the projection of the vector $\mathbf{k} - \mathbf{k}_i$ on the plane tangent to the surface $\eta(\mathbf{r})$. To find σ_2, the value of the local scattering cross-section $\sigma_{loc}(\mathbf{Q}_\eta)$ should be averaged over all possible slopes of the coarse roughness, i.e.,

$$\sigma_2 = \int \sigma_{loc}(\mathbf{Q}_\eta) p(\gamma)\, d\gamma \qquad (5)$$

where $\gamma = \{\eta_x(\mathbf{r}),\ \eta_y(\mathbf{r})\} = \nabla_\perp \eta(\mathbf{r})$; $p(\gamma)$ is the two-dimensional distribution (3) of these slope coefficients.

The local scattering cross-section by perfectly soft fine roughnesses (in the Dirichlet problem) can be written in the form [17]

$$\sigma_{loc}(\mathbf{Q}_\eta) = \frac{4}{n_z}(\mathbf{kn})^2(\mathbf{k}_i\mathbf{n})^2 c(\mathbf{Q}_\eta), \qquad (6)$$

where

$$c(\mathbf{Q}_\eta) = \frac{1}{2\pi}\int c(\mathbf{r}) \exp(-i\mathbf{Q}_\eta \mathbf{r})\, d\mathbf{r}, \qquad (7)$$

Here $c(\mathbf{Q}_\eta)$ is the characteristic function; $c(\mathbf{r}) = \langle \zeta(\mathbf{r})\zeta(0)\rangle$ is the correlation function of the displacements $\zeta(\mathbf{r})$.

Eq. (6) generalizes the expressions for the scattering cross-section to the case when the underlying surface is not a horizontal plane. Next, we need to calculate the integral in Eq. (5) for the anisotropic distribution.

Analyzing scattering, Kuryanov [8] used the Bessel function to describe the roughness correlations and obtained the result for the case $k_l << 1$ where l is the correlation length. In fact, for the Gaussian correlation function, the result will be the same, since

$$c(\mathbf{q}) \sim exp(-Q_\eta^2 l^2/2) \sim 1 \qquad (8)$$

with $Q_\eta l << 1$, i.e., the effect of the correlation function is reduced to multiplication by a constant.

Thus, Kuryanov's approximation is fully justified for this problem, and anisotropy of fine roughnesses is really insignificant.

We shall confine ourselves to the case of back-scattering that is the most important for practical applications. In these conditions

$$(\mathbf{kn})^2(\mathbf{k}_i\mathbf{n})^2 = (\mathbf{k}_i\mathbf{n})^4 = k^4\cos^4\theta$$

(θ is the angle of wave incidence), and Eq. (6) takes the form

$$\sigma_{loc}(\mathbf{Q}_\eta) = \frac{4k^4}{n_z}\cos^4\theta\, c(\mathbf{Q}_\eta). \qquad (9)$$

The multiplier $Q_\eta l << 1$ fully determines the angular dependence of scattering cross-section (9) for $\cos^4\theta$. We are going to use this circumstance in averaging over the slopes of the coarse roughness.

Formulae for calculations; numerical results

For further calculations, let us choose the direction of the x, z coordinate axes in the plane of beam incidence, so that the z axis is perpendicular to the smooth underlying plane. Since the ellipsoid of Gaussian anisotropic distribution can be oriented arbitrarily with respect to the x axis , it should be written in new coordinates, taking into account the rotation of the ellipse axis through an angle β, set by the conditions of wave incidence.

The coordinate transformation for such rotation is

$$x' = x\cos\beta + y\sin\beta,$$
$$y' = y\cos\beta - x\sin\beta$$

with the Jacobian of the transformation $J = 1$.

We obtain the distribution of the slopes in terms of the new coordinates given the equalities

$$\eta_{x'} = \eta_x\cos\beta + \eta_y\sin\beta,$$
$$\eta_{y'} = \eta_y\cos\beta - \eta_x\sin\beta$$

in the form

$$p_2(\eta_x,\eta_y) = (\pi\delta_x\delta_y)^{-1}\times$$
$$\times\exp\left(-\frac{(\eta_x\cos\beta+\eta_y\sin\beta)^2}{\delta_x^2} - \frac{(\eta_y\cos\beta-\eta_x\sin\beta)^2}{\delta_y^2}\right). \qquad (10)$$

The intensity of the scattered wave due to the presence of fine irregularities on a coarse irregular surface differs from the intensity for scattering by fine irregularities on a plane by a multiplier

$$D = \iint \frac{(1-u\,\mathrm{tg}\theta)^4}{(1+u^2+v^2)^{3/2}} \times \\ \times \exp\left(-\frac{(u\cos\beta + v\sin\beta)^2}{\delta_x^2} - \frac{(v\cos\beta - u\sin\beta)^2}{\delta_y^2} \right) \frac{dudv}{\pi\delta_x\delta_y}, \quad (11)$$

where $\mathrm{tg}\theta = k_x/k_z$.

With $\delta x = \delta y$, Eq. (11) corresponds to isotropic roughness [8]. Next, we need to calculate the resulting double integral (11), which can be represented as the sum

$$D = A_0 + 6A_1\mathrm{tg}^2\theta + A_2\mathrm{tg}^4\theta \quad (12)$$

where

$$A_n = \iint \frac{u^{2n}}{(1+u^2+v^2)^{3/2}} \times \\ \times \exp\left(-\frac{(u\cos\beta + v\sin\beta)^2}{\delta_x^2} - \frac{(v\cos\beta - u\sin\beta)^2}{\delta_y^2} \right) \frac{dudv}{\pi\delta_x\delta_y}, \quad (13)$$

$$n = 0,1,2$$

Integrals over odd degrees of u are equal to zero because the integrand is an odd function.

Let us use polar coordinates

$$u = r\cos\phi, v = r\sin\phi.$$

The integrals (13) are then transformed to the form

$$A_n = \frac{1}{\pi\delta_x\delta_y}\int_0^{2\pi} \cos^{2n}(\phi)\, h_n(g)\,\phi d\phi,$$

$$h_n(g) = \int_0^{\infty} \frac{r^{2n}}{(1+r^2)^{3/2}} \exp\left(-r^2g^2\right) rdr, \quad (14)$$

$$p^2 = \frac{\cos^2(\phi-\beta)}{\delta_x^2} + \frac{\sin^2(\phi-\beta)}{\delta_y^2}.$$

With $\delta_x = \delta_y = \delta$, $p^2 = \delta^{-2}$, the integral over the angular variable in Eq. (14) is also factorized, so that

$$A_n = \frac{h_n(p)}{\pi\delta^2}\alpha_n, \alpha_n = \int_0^{2\pi} \cos^{2n}(\phi)\, d\phi. \quad (15)$$

Since

$$\alpha_0 = 2\pi, n = 0,$$

$$\alpha_n = \int_0^{2\pi} \cos^{2n}\phi\, d\phi = \\ = 2\pi\frac{1\cdot 3\cdot 5...(2n-1)}{2\cdot 4\cdot 6...2n}, n > 0, \quad (16)$$

we obtain that $\alpha_0 = 2\varpi$, $\alpha_1 = \varpi$, $\alpha_2 = 3\varpi/4$.

The coefficients A_n for the isotropic distribution are expressed in terms of the error function [8, 18] in the form

$$\Phi(p) = \frac{2}{\sqrt{\pi}}\int_p^{\infty} e^{-z^2} dz. \quad (17)$$

The formulae for the coefficients A_n are the following in this case:

$$h_0 = 1 - K(p),$$

$$h_1 = \frac{K(p)}{2p^2} - (1-K(p)), \\ = \frac{1}{p^2}\left[\frac{1}{2} - K(p)\left(1 - \frac{1}{4p^2}\right)\right] + (1-K(p) \quad (18)$$

where

$$K(p) = p\sqrt{\pi}\Phi(p)\exp(p^2), \quad (19)$$

with $p >> 2$, $\Phi(p) \approx 0$.

Asymptotic calculation of integral (15) with $\infty \to 0$ gives the expression

$$h_0 = 1/2p^2, h_1 = 1/2p^4, h_2 = 1/p^6.$$

Respectively,

$$A_0 = 1, A_1 = 1/2p^2, A_2 = 3/4p^4$$

in the isotropic case, and the factor D increases monotonically with increasing angle of incidence.

The result of calculating the factor D for the wave vertically incident on an isotropic rough surface as a function of slope variance δ is shown in Fig.1. The graph shows that the correction increasingly deviates from unity as the roughness increases.

Taking into account expressions (19) for the functions $h_n(p)$, it is fairly simple to calculate the coefficients A_n and the sought-for factor D in the anisotropic case by single numerical integration over the angular variable ϕ in Eq. (14). Thus, we obtain the following expression for back-scattering

$$\sigma_2(\mathbf{Q}) = 4k^4 \cos^4\theta\, c(0) D. \quad (20)$$

Notably, the cross-sections of scattering by the surface are dimensionless quantities. The correlation function of fine roughnesses with a Gaussian distribution has a spectral form [17]

$$\tilde{n}(Q) = \frac{\sigma_\zeta^2 l^2}{\pi} \exp(-Q^2 l^2), \quad (21)$$

so that $c(0) = \sigma^2_\zeta l^2/\varpi$.

The final scattering cross-section can be written as

$$\sigma_2 = \frac{4}{\pi}(k^2\sigma_\zeta l)^2 (A_0 \cos^4\theta + \\ + A_1 \sin^2\theta\cos^2\theta + A_2 \sin^4\theta). \quad (22)$$

Now it remains to express the cross-section σ_1 for scattering by the underlying anisotropic coarse surface through the angle of incidence, based on expression (4). In this case, the projection $Q = 2k\cos\theta$, and $K = 2k\sin\theta$ is the length of the projection of wave vector variation on the x, y plane. Since $K_x = K\cos\beta$, $K_y = K\sin\beta$, let us write the final expression for the cross-section of back-scattering by coarse roughnesses:

$$\sigma_1 = \frac{V^2}{4\pi\delta_x\delta_y \cos^4\theta} \times \\ \times \exp\left[-\mathrm{tg}^2\theta\left(\frac{\cos^2\beta}{\delta_x^2} + \frac{\sin^2\beta}{\delta_y^2}\right)\right] \quad (23)$$

We need the value of the back-scattering coefficient and the dimensionless parameters of the surface slope distribution to carry out specific numerical calculations. Let us set $V = -1$ for further calculations. We shall calculate the scattering cross-section by taking

$$\delta_x = 0.1,\ \delta_y = 0.4;\ (k\sigma_\zeta)^2 = 0.05, (kl)^2 = 0.1.$$

Fig. 2 shows the factor D calculated as a function of the angle of wave incidence, measured in radians. Evidently, the distribution of roughnesses in the plane of incidence has a predominant effect on the angular characteristics of scattering intensity. Rotation of the plane of incidence from a finer roughness to a coarser one leads to a marked increase in scattering. With a normal wave incidence, the ratio of the cross-sections is

$$\sigma_2(0)/\sigma_1(0) = 16\delta_x\delta_y(k^2\sigma_\zeta l)^2 A_0^2. \quad (24)$$

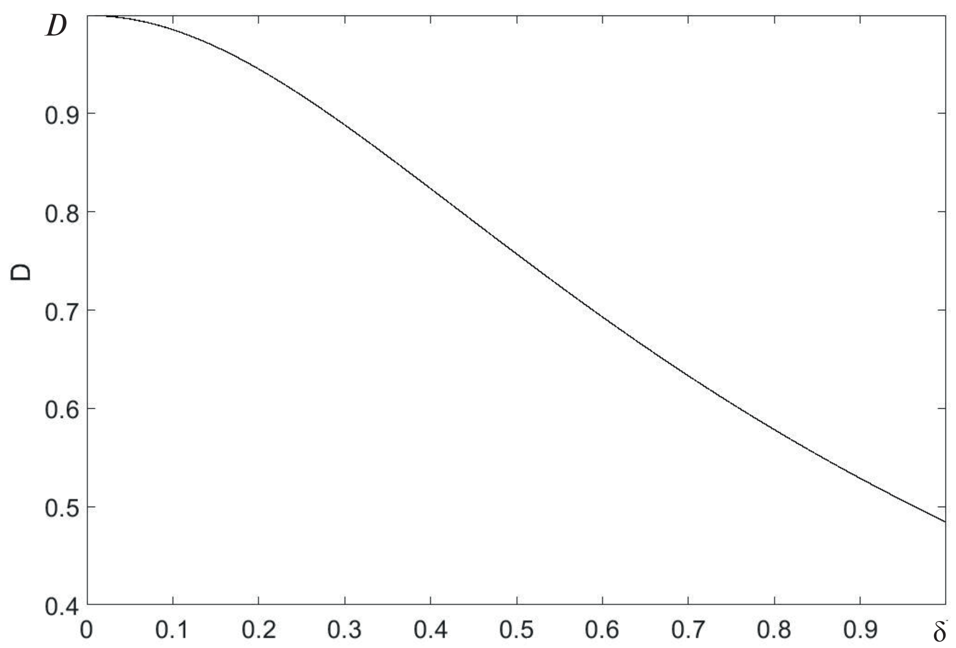

Fig. 1. Factor D for a vertically incident wave as a function of slope variance δ

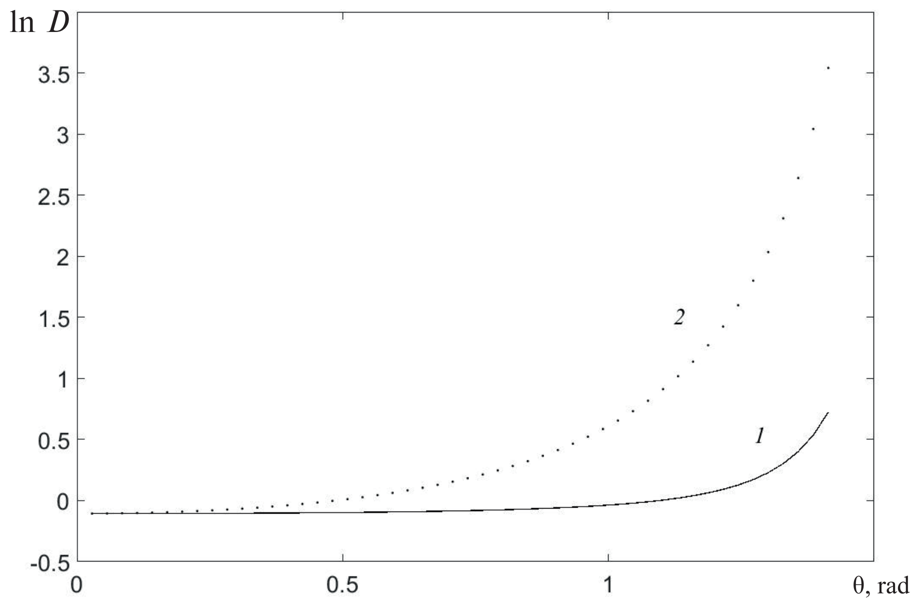

Fig. 2. Logarithmic anisotropy factor D as a function of incident angle θ for two cases: plane of incidence passes through the minor (*1*) and through the major (*2*) axis of the anisotropy ellipse

This ratio is ~$3\cdot10^{-3}$ for the given parameters, i.e., the cross-section σ_2 in the region of small scattering angles is a small correction to the main cross-section σ_1 of scattering by coarse roughnesses. Taking it into account is only important at large angles for which the main component decreases sharply, and the contribution to scattering by fine roughnesses is manifested.

The functional dependence of the back-scattering cross-section by coarse anisotropic roughnesses on the angle of incidence is clear evidence that the variation in scattering depends on the position of the plane of incidence. The cross-sections of back-scattering by coarse roughnesses depending on the angle of incidence for the plane of incidence oriented

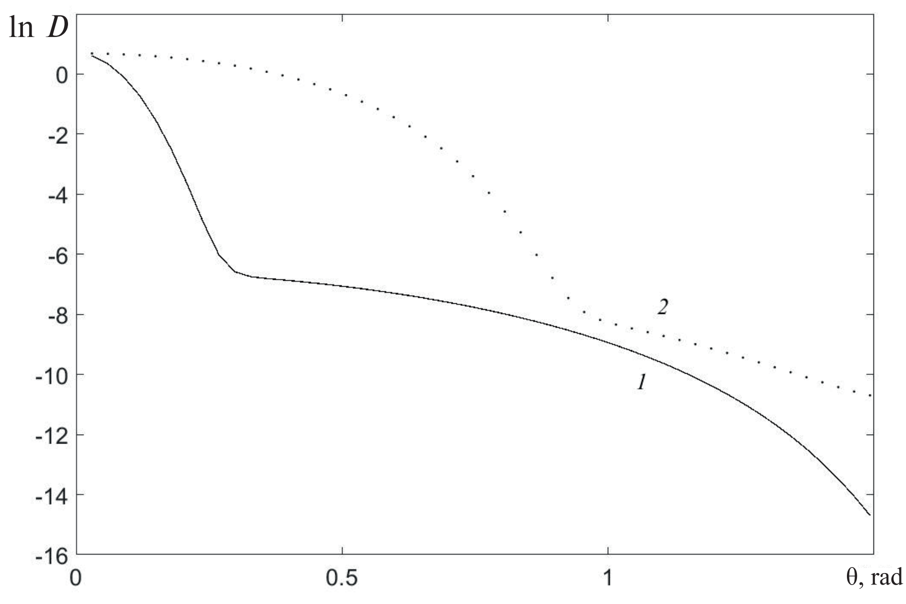

Fig. 3. Logarithm of total scattering cross-section for a two-scale rough surface as a function of incidence angle with different orientations of the plane of wave incidence: along smaller irregularities (*1*) and along irregularities with large slopes (*2*)

along the minor axis of the anisotropy ellipse and along the major axis of anisotropy differ by a factor

$$\exp\left[\mathrm{tg}^2\theta\left(\frac{1}{\delta_x^2}-\frac{1}{\delta_y^2}\right)\right],$$

which increases with increasing angle of incidence for $\delta_x < \delta_y$.

Scattering by fine irregularities against the background of coarse roughnesses as a function of the angle of incidence with a different orientations of the plane of incidence is a smooth curve.

Fig. 3 shows the logarithm of the total cross-section of scattering by a two-scale rough surface for two different orientations of the plane of incidence. The lower curve with less scattering corresponds to the plane of incidence oriented along the minor axis of the ellipse. The upper curve corresponds to the plane of incidence oriented along the major axis of the ellipse, which corresponds to coarser roughnesses. The kink on the curve is associated with a rapid transition from the distribution in the form (23) at small angles of incidence to distribution in the form (22) with increasing angle. This behavior is also characteristic for an isotropic scattering surface [8], but in this case, a new dependence on the position of the plane of incidence is observed.

Conclusion

The analysis we have carried out using the two-scale model for scattering of waves by an anisotropic rough surface revealed that the role of anisotropy of fine roughnesses is insignificant, in the sense that it does not contribute to the angular dependence of backscattering. As for anisotropy of coarse irregularities with dimensions greater than the wavelength, it is very pronounced both in the part actually generated by these smooth surface structures, and in the scattering by random fine roughnesses. These fine irregularities with sizes much smaller than the wavelength govern the low-intensity wing in the cross-section at large scattering angles. The intensity of the back-scattered wave varying as the plane of wave incidence is rotated can serve as an indicator of the degree of anisotropy of random coarse roughnesses.

The effect that fluctuations of the actual parameters of anisotropic roughness along the surface, including on scales much larger than the correlation length of coarse roughnesses, have on the scattering is an interesting subject for further study.

REFERENCES

[1] **T.M. Elfouhaily, C.-A. Guйrin,** A critical survey of approximate scattering wave theories from random rough surfaces, Waves in Random Media. 14 (4) (2004) R1—R40.

[2] **A.A. Maradudin,** editor, Light scattering and nanoscale surface roughness, Springer, New York (2007).

[3] **N. Pinel, C. Bourlier,** Electromagnetic wave scattering from random rough surfaces, asymptotic models, John Wiley & Sons, Hoboken (2013).

[4] **A.K. Morozov, J.A. Colosi,** Equations for normal-mode statistics of sound scattering by a rough elastic boundary in an underwater waveguide, including backscattering, J. Acoust. Soc. Am. 142 (3) (2017) EL292— EL298.

[5] **A. Ishimaru,** Electromagnetic wave propagation, radiation and scattering, John Wiley & Sons, Hoboken (2017).

[6] **L.M. Brekhovskikh, Yu.P. Lysanov,** Fundamentals of ocean acoustics, Springer-Verlag, Berlin (1991).

[7] **K.V. Horoshenkov, T. Van Renterghem, A. Nichols, A. Krynkin,** Finite difference time domain modeling of sound scattering by the dynamically rough surface of a turbulent open channel flow, Applied Acoustics. 110 (September) (2016) 13—22.

[8] **B.F. Kur'yanov,** Acoustic scattering by roughness with two types of irregularities, Acoustical Journal. 8(3) (1962) 325—333.

[9] **S.A. Boukabara, L. Eymard, C. Guillou, et al.,** Development of a modified two-scale electromagnetic model simulating both active and passive microwave measurements: Comparison to data remotely sensed over the ocean, Radio Science. 37 (4) (2002) 16-1—16-11.

[10] **D. Lemaire, P. Sobieski, C. Craeye, A. Guissard,** Two-scale models for rough surface scattering: Comparison between the boundary perturbation method and the integral equation method, Radio Science. 37 (1) (2002) 1-1—1-16.

[11] **F. Nunziata, P. Sobieski, M. Migliaccio,** The two-scale BPM scattering model for sea biogenic slicks contrast, IEEE Trans. on Geoscience and Remote Sensing. 47 (7-1) (2009) 1949—1956.

[12] **A.A. Mouche, B. Chapron, N. Reul,** A simplified asymptotic theory for ocean surface electromagnetic wave scattering, Waves in Random and Complex Media. 17 (3) (2007) 321–341.

[13] **M.Y. Ayari, A. Khenchaf, A. Coatanhay,** Simulations of the bistatic scattering using two-scale model and the unified sea spectrum, Journal of Applied Remote Sensing. 1 (1) (2007) 013532.

[14] **G. Soriano, C.-A. Guйrin,** A cutoff invariant two-scale model in electromagnetic scattering from sea surface, IEEE Geoscience and Remote Sensing Letters. 5 (2) (2008) 199–203.

[15] **F. Shi, W. Choi, M.J.S. Lowe, et al.,** The validity of Kirchhoff theory for scattering of elastic waves from rough surfaces, Proceedings of the Royal Society of London. A. 471 (2178) (2015) 20140977.

[16] **A.V. Osipov, S.A. Tretyakov,** Modern electromagnetic scattering theory with application. Wiley & Sons, Chichester (2017).

[17] **S.M. Rytov, Yu.A. Kravtsov, V.I. Tatarskiy**, Vvedeniye v statisticheskuyu radiofiziku. Sluchaynyye polya. Ch. 2. Introduction to statistical radiophysics. Random fields, Part 2., Moscow, Nauka (1978).

[18] **K. Cahil,** Physical Mathematics, Cambridge University Press, Cambridge (2013).

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СПИСОК ЛИТЕРАТУРЫ

1. **Elfouhaily T.M., Guйrin C.-A.** A critical survey of approximate scattering wave theories from random rough surfaces // Waves in Random Media. 2004. Vol. 14. No. 4. Pp. R1–R40.

2. **Maradudin A.A.** (editor). Light scattering and nanoscale surface roughness. New York: Springer, 2007. 496 p.

3. **Pinel, N., Bourlier C.** Electromagnetic wave scattering from random rough surfaces. Asymptotic models. Hoboken: John Wiley & Sons, 2013. 162 p.

4. **Morozov A.K., Colosi J.A.** Equations for normal-mode statistics of sound scattering by a rough elastic boundary in an underwater waveguide, including backscattering // The Journal of the Acoustic Society of America. 2017. Vol. 142. No. 3. Pp. EL292–EL298.

5. **Ishimaru A.** Electromagnetic wave propagation, radiation and scattering. Hoboken: John Wiley & Sons, 2017. 944 p.

6. **Бреховских Л.М., Лысанов Ю.П.** Теоретические основы акустики океана. М.: Наука, 2007. 370 с.

7. **Horoshenkov K.V., Van Renterghem T., Nichols A., Krynkin A.** Finite difference time domain modelling of sound scattering by the dynamically rough surface of a turbulent open channel flow // Applied Acoustics. 2016. Vol. 110. September. Pp. 13–22.

8. **Курьянов Б.Ф.** Рассеяние звука на шероховатостях с двумя типами неровностей // Акустический журнал. 1962. Т. 8. Вып. 3. С. 325–333.

9. **Boukabara S.A., Eymard L., Guillou C., Lemaire D., Sobieski P., Guissard A.** Development of a modified two-scale electromagnetic model simulating both active and passive microwave measurements: Comparison to data remotely sensed over the ocean // Radio Science. 2002. Vol. 37. No. 4. Pp. 16-1–16-11.

10. **Lemaire D., Sobieski P., Craeye C., Guissard A.** Two-scale models for rough surface scattering: Comparison between the boundary perturbation method and the integral equation method // Radio Science. 2002. Vol. 37. No. 1. Pp. 1-1–1-16.

11. **Nunziata F., Sobieski, P., Migliaccio M.** The two-scale BPM scattering model for sea biogenic slicks contrast // IEEE Transactions on Geoscience and Remote Sensing. 2009. Vol. 47. No. 7. Part 1. Pp. 1949–1956.

12. **Mouche A.A., Chapron B., Reul N.** A simplified asymptotic theory for ocean surface electromagnetic wave scattering // Waves in Random and Complex Media. 2007. Vol. 17. No. 3. Pp. 321–341.

13. **Ayari M.Y., Khenchaf A., Coatanhay A.** Simulations of the bistatic scattering using two-scale model and the unified sea spectrum // Journal of Applied Remote Sensing. 2007. Vol. 1. No. 1. P. 013532.

14. **Soriano G., Guйrin C.-A.** A cutoff invariant two-scale model in electromagnetic scattering from sea surface // IEEE Geoscience and Remote Sensing Letters. 2008. Vol. 5. No. 2. Pp. 199–203.

15. **Shi F., Choi W., Lowe M.J.S., Skelton E.A., Craster R.V.** The validity of Kirchhoff theory for scattering of elastic waves from rough surfaces // Proceedings of the Royal Society of London. A. 2015. Vol. 471. No. 2178. P. 20140977.

16. **Osipov A.V., Tretyakov S.A.** Modern

electromagnetic scattering theory with application. Chichester: Wiley & Sons, 2017. 806 p.

17. **Рытов С.М., Кравцов Ю.А., Татарский В.И.** Введение в статистическую радиофизику. Случайные поля. Ч. 2. М.: Наука, 1978. 463 с.

18. **Cahil K.** Physical mathematics. Cambridge: Cambridge University Press, 2013. 666 p.

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