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SINGLE-ATOM LASER WITH A LOW-FINESSE CAVITY OPERATING IN THE STRONG-COUPLING REGIME

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In this work, transition processes and a stationary regime of a single-atom laser with incoherent pumping under conditions of the strong-coupling and bad-cavity limit have been studied. In the thresholdless regime, our numerical results were compared with corresponding experimental data. The amplitude and the frequency of the relaxation oscillations as the functions of the incoherent pumping value were also analyzed. The pumping strength was established to have a very significant effect on the amplitude of these oscillations and no appreciable effect on their frequency. The possibility of the noise reduction by means of increasing of the number of atoms in the cavity was discussed.

Keywords: single-atom laser, strong-coupling regime, thresholdless lasing, relaxation oscillations

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ОДНОАТОМНЫЙ ЛАЗЕР, РАБОТАЮЩИЙ В РЕЖИМЕ СИЛЬНОЙ СВЯЗИ И НИЗКОДОБРОТНОГО РЕЗОНАТОРА

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Численно исследованы эволюция и стационарный режим работы одноатомного лазера с некогерентной накачкой в условиях сильной связи и низкодобротного резонатора. В режиме беспороговой генерации проведено сравнение полученных численных результатов с соответствующими экспериментальными. Проанализированы зависимости амплитуды и частоты релаксационных колебаний от величины некогерентной накачки. Установлено, что величина накачки оказывает существенное влияние на амплитуду этих колебаний и практически не влияет на их частоту. Обсуждается возможность уменьшения шумов лазера посредством увеличения числа атомов в резонаторе.

Ключевые слова: одноатомный лазер, режим сильной связи, беспороговый режим генерации, релаксационные колебания

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Introduction

The single-atom laser is one of the fundamental models in quantum optics. It is a two-level system with incoherent pumping, interacting with one damped field mode [1]. This model was considered in many studies (see [1–8] and references therein), discovering the properties of the single-atom laser, such as the "self-quenching" effect [1], entangled states between the atomic and the field subsystems [2], thresholdless lasing and lasing without inversion [3, 6], multiple squeezed photon number fluctuations [4] and sub-Poisson statistics of the photon number in the cavity [4, 6], deviation from the Schawlow–Townes linewidth for a conventional incoherently pumped laser [1, 6].

Interest in this problem was stimulated, in addition to its fundamental properties, by successful experiments in this field, began in 2003 by Kimble's group [9]. Experiments were conducted not only for a single atom as the gain medium [9] but for single ions [10] and quantum dots [11].

Single-atom lasers show promise for practical application in metrology and frequency standards [12–16], where they can be used as sources of "non-classical" light states, and in quantum computing, where they can serve as "qubits" in complex quantum networks [17].

Most of the theoretical studies on the subject considered different operation modes of a single-atom laser; however, some aspects were not examined in sufficient detail. For example, distributions over coherent states, based on the approximate expression obtained for the Glauber P function, were used to investigate the properties of a laser in [6]. While an attempt to compare the results of calculations for a theoretical model with the parameters of a real experiment was made in [10], the comparison was incomplete, lacking data for photon statistics in the cavity. This was because, firstly, the selected parameter values did not fully satisfy the conditions necessary for the approximate solution for the P function to work, and, secondly, because the numerical solution of the corresponding equations encounters difficulties in describing generalized functions [7].

The spectrum of amplitude fluctuations for strong coupling and a low-finesse cavity was analyzed in [6] by linearizing the Heisenberg–Langevin equations around a semi-classical solution. While relaxation oscillations were discovered for this case, the results were not numerically verified; in particular, the dependence of frequency and amplitude of these oscillations on the pumping rate was not considered.

Notably, controlling these transient processes is important for lasers with a high coupling efficiency □ of spontaneous emission [1–20], i.e., for a class of lasers with a large contribution of spontaneous emission to the lasing mode that includes the single-atom laser under consideration.

Thus, our goals have been conducting numerical study of the parameters of relaxation oscillations and the statistics of photon numbers in a single-atom laser with strong coupling and a low-finesse cavity.

The structure of the paper is as follows. The first section describes the model of the single-atom laser and its modes of operation, with the governing equations used for numerical simulation formulated. Next, calculation results are presented and discussed. Finally, conclusions are drawn; increasing the number of emitters in the cavity is considered as an option for reducing noise.

Model of a single-atom laser. Governing equation

The simplest model of a single-atom laser is represented by a two-level system with incoherent pumping from the lower to the upper energy level, interacting with a single lossy cavity mode. This model is characterized by only four parameters:

γ is the spontaneous decay rate of the excited atom;

Γ is the incoherent pumping rate from the lower level $|a\rangle$ to the higher level $|b\rangle$;

κ is the field decay rate in the cavity with a partially reflecting mirror;

g is the coupling between the atom and the field.

The equation for the density operator $\hat{\rho}$ of the given laser has the following form:

$$\begin{aligned}\frac{\partial\hat{\rho}}{\partial t} = -\frac{i}{\hbar}\left[\hat{V},\hat{\rho}\right] + \frac{\kappa}{2}\left(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}\right) + \\ + \frac{\gamma}{2}\left(2\hat{\sigma}\hat{\rho}\hat{\sigma}^{\dagger} - \hat{\sigma}^{\dagger}\hat{\sigma}\hat{\rho} - \hat{\rho}\hat{\sigma}^{\dagger}\hat{\sigma}\right) + \\ + \frac{\Gamma}{2}\left(2\hat{\sigma}^{\dagger}\hat{\rho}\hat{\sigma} - \hat{\sigma}\hat{\sigma}^{\dagger}\hat{\rho} - \hat{\rho}\hat{\sigma}\hat{\sigma}^{\dagger}\right), \\ \hat{V} = i\hbar g\left(\hat{a}^{\dagger}\hat{\sigma} - \hat{\sigma}^{\dagger}\hat{a}\right),\end{aligned} \quad (1)$$

where $\hat{a}$, $\hat{a}^{\dagger}$ are the photon annihilation/creation operators in the cavity mode; $\hat{\sigma} = |a\rangle\langle b|$, $\hat{\sigma}^{+} =$

$|b\rangle\langle a|$ are the atomic projection operators; $\hat{V}$ is the operator of atom interaction with the cavity mode.

The second term in Eq. (1) describes the decay of the cavity mode, the third and the fourth terms describe spontaneous decay of the excited state of the atom and its incoherent pumping, respectively.

The states of the atom satisfy the completeness relation

$$|a\rangle\langle a|+|b\rangle\langle b|=\hat{1},$$

where $\hat{1}$ is the unity operator.

In accordance with [6], we are going to characterize our laser with the following three dimensionless parameters:

$c = 4g^2/\kappa\gamma$ is the dimensionless coupling (cooperativity parameter);

$I_s = \gamma/\kappa$, $r = \Gamma/\gamma$ are the dimensionless saturation intensity and pumping rate, respectively.

Let us discuss some of the laser's operation modes in more detail. Formally, the case $I_s >> 1$ is called the good cavity mode, and the case $I_s << 1$ the bad cavity mode. The inequality $cI_s >> 1$ ($g >> k$) corresponds to the case when the photon repeatedly interacts with the atom during its lifetime. The same condition makes it possible to accumulate a large number of coherent photons in the resonator at certain pumping rates. The following inequality $c >> I_s$ ($g >> \gamma$) represents the so-called strong coupling condition, when the coupling of an atom with the field is stronger than with the thermostat ensuring its spontaneous decay outside the cavity mode. Thresholdless lasing and generation of "non-classical" light states can be observed if this condition is satisfied.

Provided that $c >> 1$, there is a semi-classical solution for intracavity intensity [6]:

$$I(r)=\frac{I_s}{2}\left[(r-1)-\frac{(r+1)^2}{c}\right]. \tag{2}$$

The roots of the equation $I(r) = 0$ determine the semi-classical threshold of the single-atom laser r_{th} and the pumping rate r_q for which the laser's self-quenching occurs:

$$r_{th}=r_m-\frac{c}{2}\sqrt{1-8/c};$$

$$r_q=r_m+\frac{c}{2}\sqrt{1-8/c},$$

where $r_m = c/2 - 1$ is the pumping rate at which intensity (2) takes its maximum value,

$$I(r_m)=\frac{cI_s}{8}(1-8/c).$$

Operator equation (1) can be written in different representations. For numerical formulation, it is best rewritten in terms of occupation numbers:

$$\begin{aligned}
\frac{\partial\rho_{nm}^{aa}}{\partial t}&=2\kappa\sqrt{(n+1)(m+1)}\rho_{n+1,m+1}^{aa}-\\
&-\kappa(n+m)\rho_{nm}^{aa}+2\gamma\rho_{nm}^{bb}-2\Gamma\rho_{nm}^{aa}+\\
&+2g\left(\sqrt{n}\rho_{n-1,m}^{ba}+\sqrt{m}\rho_{n,m-1}^{ab}\right);\\
\frac{\partial\rho_{nm}^{bb}}{\partial t}&=2\kappa\sqrt{(n+1)(m+1)}\rho_{n+1,m+1}^{bb}-\\
&-\kappa(n+m)\rho_{nm}^{bb}-2\gamma\rho_{nm}^{bb}+2\Gamma\rho_{nm}^{aa}-\\
&-2g\left(\sqrt{m+1}\rho_{n,m+1}^{ba}+\sqrt{n+1}\rho_{n+1,m}^{ab}\right);\\
\frac{\partial\rho_{nm}^{ab}}{\partial t}&=2\kappa\sqrt{(n+1)(m+1)}\rho_{n+1,m+1}^{ab}-\\
&-\kappa(n+m)\rho_{nm}^{ab}-(\Gamma+\gamma)\rho_{nm}^{ab}+\\
&+2g\left(\sqrt{n}\rho_{n-1,m}^{bb}-\sqrt{m+1}\rho_{n,m+1}^{aa}\right);\\
\frac{\partial\rho_{nm}^{ba}}{\partial t}&=2\kappa\sqrt{(n+1)(m+1)}\rho_{n+1,m+1}^{ba}-\\
&-\kappa(n+m)\rho_{nm}^{ba}-(\Gamma+\gamma)\rho_{nm}^{ba}+\\
&+2g\left(\sqrt{m}\rho_{n,m-1}^{bb}-\sqrt{n+1}\rho_{n+1,m}^{aa}\right).
\end{aligned} \tag{3}$$

Here $\rho_{nm}^{\alpha\beta}=\rho_{nm}^{\alpha\beta}(t)$ is the element of the atom-field density matrix; the subscripts run through the photon occupation numbers $n, m = 0, 1, 2, ..., \infty$; the superscripts correspond to the atomic states $\alpha, \beta = a, b$.

System (3) is an infinite system of equations. Because of this, our numerical simulation involved analyzing the convergence of the mean photon number $\langle n\rangle$ and the photon variance

$$\langle\Delta n^2\rangle=\langle n^2\rangle-\langle n\rangle^2,$$

depending on the number of occupation numbers k taken into account in the calculation. In other words, system of equations (3) "stopped" at a certain k, initially chosen from the obvious condition $k > [\Gamma/2k]$, with $\langle n\rangle$ and $\langle\Delta n^2\rangle$ calculated.

The next calculation was carried out for larger values of k and compared with the previous result, and so on.

The mean photon number and the Mandel Q parameter were calculated by the formulae:

$$\langle n^s \rangle = \sum_{n=0}^{k} n^s \rho_{nn},$$
$$Q = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} - 1. \quad (4)$$

where $\rho_{nn} = \rho_{nn}^{aa} + \rho_{nn}^{bb}$ is the field density matrix.

Calculation results and discussion

The mean photon number and the Mandel Q parameter are plotted in Fig. 1 as functions of the pumping rate r for two values of the coupling constant c and several values of the saturation intensity I_s; the curves were obtained by numerical simulation of system (3). The points on the curves represent the experimental data taken from [10] and corresponding to thresholdless operation of the single-atom laser. Since the experimental points were plotted so as to correspond to a certain photocell efficiency, scaling was introduced: the values along the horizontal axis were multiplied by 1.30, and the values along the vertical axis by 7.22.

The following values of the parameters and were found in [6] in an attempt to compare the theoretical model with the experiment in [10]. It can be seen from the figure that such values are completely unsuitable for describing the given experiment. Noise increases with increasing pumping rate, and no characteristic minimum is observed for the Q parameter.

In view of this, we carried out numerical analysis of the dependences $\langle n(r) \rangle$ and $Q(r)$ in a wide range of c, I_s values and selected those for which our model qualitatively describes the experiment [10]: $c = 10$, $I_s = 0.3$. The corresponding graphs as well as graphs for other values of I_s are shown in Fig. 1. Evidently, enhancing the coupling by decreasing the saturation parameter and switching to bad cavity mode allows to reduce the fluctuation of the photon number in the cavity and to provide sub-Poissonian statistics of the field. The greatest noise reduction is observed for $cI_s \approx 1$, that is, provided that a photon interacts with an atom only once before leaving the cavity (the antibunching effect).

Importantly, the given experimental scenario [10] corresponds to a small photon number in the cavity ($cI_s \approx 1$), while the approximate expression for the P function found in [6] can only be used provided that $cI_s >> 1$, i.e., if a macroscopic photon number is accumulated in the mode.

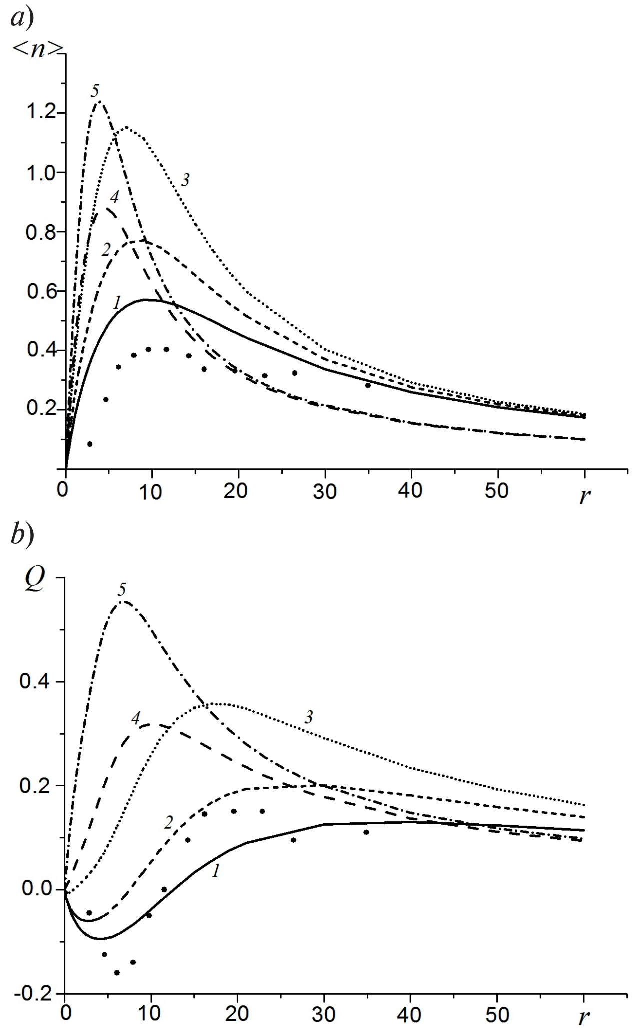

Fig. 1. Experimental ([10], points) and calculated (lines) dependences of the mean photon number in the cavity (*a*) and the Mandel *Q* parameter (*b*) on the pumping rate r for different values of the parameters I_s (curves *1*–*5*) and c.
c = 10.00 (*1*–*3*) and 5.75 (*4*, *5*); Is = 0.3 (*1*), 0.5 (*2*); 1.0 (*3*), 1.6 (*4*), 3.5 (*5*)

Fig. 2 shows time evolution of the mean photon number in the cavity and its variance for the case with strong coupling and bad cavity: c = 400, I_s = 0.32 (where $cI_s >> 1$). We selected several pumping rates near the classical threshold $r_{th} \approx 1$ (see formula (2) for intracavity intensity), and several values that substantially exceeded it but did not fall into the region of the laser's

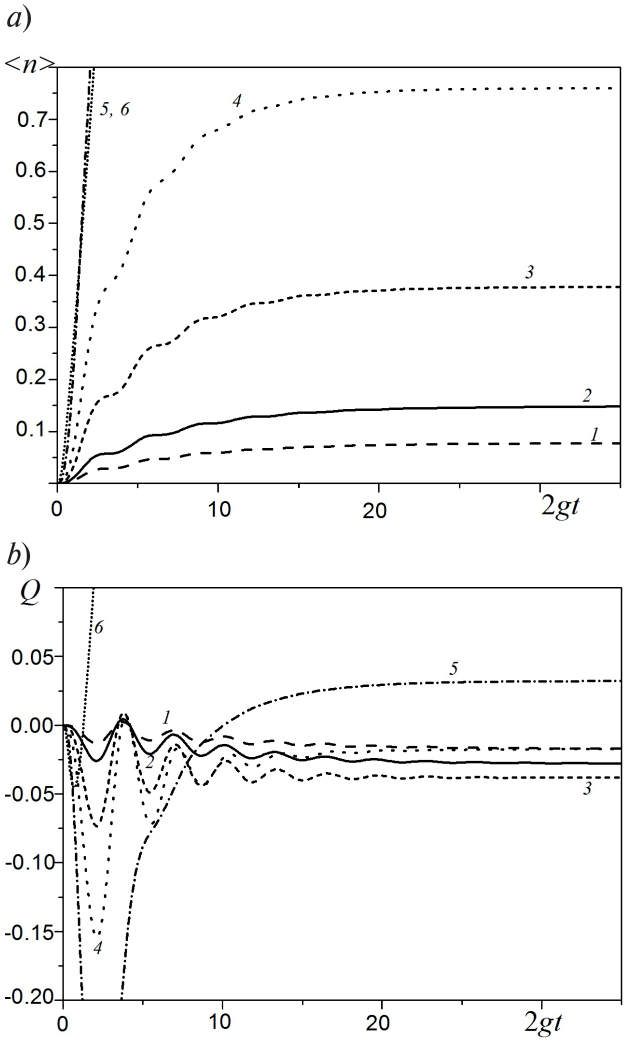

Fig. 2. Time evolution of the mean number of photons in the cavity (*a*) and the Mandel *Q* parameter (*b*) for different pumping rates *r* with fixed values $c = 400$, $I_s = 0.2$.
$r = 0.5$ (*1*); 1.01 (*2*); 3.0 (*3*); 7 (*4*); 30 (*5*); 199 (*6*)

"self-quenching". The values $r = 3$; 199 are the same as in [6], where they were used for analysis of the amplitude fluctuation spectrum.

Relaxation oscillations predicted by the linear theory [6] can be seen in Fig. 2. The oscillation amplitude increases with increasing pumping rate *r*, and the oscillations almost disappear with $r >> r_{th}$. Estimating the relaxation oscillation frequency, we can confirm that it is several times higher than the frequency found in [6], namely,

$$\Omega_{osc} / 2g = \sqrt{I_s(r-1)/2},$$

and, according to our estimate, it does not depend so much on the pumping rate. The reason for this discrepancy is most likely that linearization of the Heisenberg–Langevin equations carried out in [6] implies a high intracavity intensity, while the mean photon number is small near the threshold r_{th}.

Conclusion

We have considered the behavior of a single-atom laser with incoherent pumping, operating in the strong-coupling regime ($c >> I_s$), with a low-finesse cavity ($I_s << 1$). We have found the parameter values of the laser at which the photon statistics in the cavity becomes sub-Poisson, which is explained by the antibunching effect. The corresponding numerical calculations have yielded good qualitative agreement with the experiment in [10].

We have established that the approximate expression for the Glauber *P* function found in [6] cannot be used to describe lasing in case of a low-finesse cavity.

Additionally, we have numerically confirmed the prediction that relaxation oscillations occur in a single-atom laser operating in the strong-coupling regime [6]. However, applying the analytical formula derived in [6] for the frequency of these oscillations did not confirm the results of numerical calculations obtained in our study. It follows from numerical analysis of transient processes that the frequency of relaxation oscillations depends very little on the pumping rate r, while and the oscillation amplitude increases with increasing *r*.

Notably, interest has been growing in atomic systems placed in a cavity and interacting with its modes. Such systems include cold atomic ensembles or single atoms embedded in special matrices. Studies on this subject (see, for example, [21, 22]) found substantial modification of the interaction between the atoms and the field, particularly pronounced in near-field effects (dipole-dipole interaction) or if one of the surfaces (cavity wall) is charged.

Placing several emitters prepared in a certain entangled state in a cavity should affect the squeezing of amplitude fluctuations of the field subsystem in such a laser and possibly improve the degree of squeezing compared to that found for a single-atom laser [4].

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REFERENCES

[1] **Yi Mu, C.M. Savage,** One-atom lasers, Phys. Rev. A. 46 (9) (1992) 5944.

[2] **S.Ya. Kilin, T.B. Karlovich,** Single-atom laser: Coherent and nonclassical effects in the regime of a strong atom-field correlation, J. Exp. Theor. Phys. 95 (5) (2002) 805.

[3] **T.B. Karlovich, S.Ya. Kilin,** Quantum statistical properties of one-atom lasers, Opt. Spectra. 91 (91) (2001) 343–351.

[4] **A.V. Kozlovskiy, A.N. Orayevskiy**, Sub-Poissonian radiation of a one-atom two-level laser with incoherent pumping, JETP. 88 (4) (1999) 666–671.

[5] **N.V. Larionov, M.I. Kolobov,** Analytical results from the quantum theory of single-emitter nanolaser, Phys. Rev. A. 2011. Vol. 84 (5) (2011) 055801.

[6] **N.V. Larionov, M.I. Kolobov,** Quantum theory of a single-emitter nanolaser, Phys. Rev. A. 88 (1) (2013) 013843.

[7] **E.N. Popov, N.V. Larionov,** Glauber-Sudarshan *P* function in the model of a single-emitter laser generating in strong coupling regime, Proceeding of SPIE. 9917 (2016) 99172X.

[8] **S.Ya. Kilin, A.B. Mikhalychev,** Single-atom laser generates nonlinear coherent states, Phys. Rev. A. 85 (6) (2012) 063817.

[9] **J. McKeever, A. Boca, A.D. Boozer, et al.,** Experimental realization of a one-atom laser in the regime of strong coupling, Nature (London). 425 (2003) 268–271.

[10] **F. Dubin, C. Russo, H.G. Barros, et al.,** Quantum to classical transition in a single-ion laser, Nat. Phys. 6 (2010) 350–353.

[11] **M. Nomura, N. Kumagai, S. Iwamoto, et al.,** Photonic crystal nanocavity laser with a single quantum dot gain, Opt. Express. 17 (18) (2009) 15975–15982.

[12] **F. Riehle,** Frequency standards: Basic and applications, Wiley-VCH Verlag GmbH & Co., Weinheim (2004).

[13] **G.M. Saxena, A. Agarwal,** Laser-cooling in noisy quadrature of squeezed vacuum for cesium fountain clock, IEEE International Frequency Control Sympposium and PDA Exhibition Jointly with the 17th European Frequency and Time Forum. 2003. Proceedings of the 2003. doi:10.1109/freq.2003.1275075.

[14] **H. Vahlbruch, D. Wilken, M. Mehmet, B. Willke,** Laser power stabilization beyond the shot noise limit using squeezed light, Phys. Rev. Lett. 121 (17) (2018) 173601.

[15] **D.V. Brazhnikov, S.M. Ignatovich, V.I. Vishnyakov, et al.,** High-quality electromagnetically-induced absorption resonances in a buffer-gas-filled vapour cell, Las. Phys. Lett. 15 (2) (2018) 025701.

[16] **K.A. Barantsev, E.N. Popov, A.N. Litvinov, V.M. Petrov,** Analysis of short-term stability and light shifts of quantum frequency standarts based on coherent population trapping effect in ^{87}Rb buffer cell, Radioengineering. (12) (2016) 164–170.

[17] **S. Ritter, C. Nlleke, C. Hahn C., et al.,** An elementary quantum network of single atoms in optical cavities, Nature. 484 (7393) (2012) 195–200.

[18] **G. Bjork, A. Karlsson, Y. Yamamoto,** Definition of a laser threshold, Phys. Rev. A. 50 (2) (1994) 1675.

[19] **I.E. Protsenko, M. Travagnin,** Microscopic model of semiconductor laser without inversion, Phys. Rev. A. 65 (1) (2001) 013801.

[20] **B. Ellis, M.A. Mayer, G. Shambat, et al.,** Ultralow-threshold electrically pumped quantum-dot photonic-crystal nanocavity laser, Nat. Photonics 5 (2011) 297–300.

[21] **A.S. Kuraptsev, I.M. Sokolov,** Microscopic theory of dipole-dipole interaction in ensembles of impurity atoms in a Fabry–Perot cavity, JETP. 123 (2(8)) (2016) 237–248.

[22] **A.S. Kuraptsev, I.M. Sokolov,** The features of interatomic dipole-dipole interaction near an ideally conducting charged surface, JETP. 127 (3) (2018).

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СПИСОК ЛИТЕРАТУРЫ

1. **Mu Yi, Savage C.M.** One-atom lasers // Phys. Rev. A. 1992. Vol. 46. No. 9. P. 5944.

2. **Kilin S.Ya., Karlovich T.B.** Single-atom laser: Coherent and nonclassical effects in the regime of a strong atom-field correlation // J. Exp. Theor. Phys. 2002. Vol. 95. No. 5. P. 805.

3. **Karlovich T. B. and Kilin S. Ya.** Quantum statistical properties of one-atom lasers // Opt. Spectr. 2001. Vol. 91. No. 91. Pp. 343–351.

4. **Козловский А.В, Ораевский А.Н.** Субпуассоновское излучение одноатомного двухуровневого лазера с некогерентной накачкой // ЖЭТФ. 1999. Т. 115. Вып. 4. С. 1210–1220.

5. **Larionov N.V., Kolobov M.I.** Analytical results from the quantum theory of single-emitter nanolaser // Phys. Rev. A. 2011. Vol. 84. No. 5. P. 055801.

6. **Larionov N.V., Kolobov M.I.** Quantum theory of a single-emitter nanolaser // Phys. Rev. A. 2013. Vol. 88. No. 1. P. 013843.

7. **Popov E.N., Larionov N.V.** Glauber-Sudarshan *P* function in the model of a single-emitter laser generating in strong coupling regime // Proceeding of SPIE. 2016. Vol. 9917. P. 99172X.

8. **Kilin S.Ya., Mikhalychev A.B.** Single-atom laser generates nonlinear coherent states // Phys. Rev. A. 2012. Vol. 85. No. 6. P. 063817.

9. **McKeever J., Boca A., Boozer A.D., Buck J.R., Kimble H.J.** Experimental realization of a one-atom laser in the regime of strong coupling // Nature (London). 2003. Vol. 425. Pp. 268–271.

10. **Dubin F., Russo C., Barros H. G., Stute A., Becher C., Piet O.S., Blatt R.** Quantum to classical transition in a single-ion laser // Nat. Phys. 2010. Vol. 6. Pp. 350–353.

11. **Nomura M., Kumagai N., Iwamoto S., Ota Y., Arakawa Y.** Photonic crystal nanocavity laser with a single quantum dot gain // Opt. Express. 2009. Vol. 17. No. 18. Pp. 15975–15982.

12. **Риле Ф.** Стандарты частоты. Принципы и приложения. М.: Физматлит, 2009. 512 с.

13. **Saxena G. M., Agarwal A.** Laser-cooling in noisy quadrature of squeezed vacuum for cesium fountain clock // IEEE International Frequency Control Sympposium and PDA Exhibition Jointly with the 17th European Frequency and Time Forum. 2003. Proceedings of the 2003. doi:10.1109/freq.2003.1275075.

14. **Vahlbruch H., Wilken D., Mehmet M., Willke B.** Laser power stabilization beyond the shot noise limit using squeezed light // Phys. Rev. Lett. 2018. Vol. 121. No. 17. P. 173601.

15. **Brazhnikov D.V., Ignatovich S.M., Vishnyakov V.I., Skvortsov M.N., Andreeva Ch., Entin V.M., Ryabtsev I.I.** High-quality electromagnetically-induced absorption resonances in a buffer-gas-filled vapour cell // Las. Phys. Lett. 2018. Vol. 15. No. 2. P. 025701.

16. **Баранцев К.А., Попов Е.Н., Литвинов А.Н., Петров В.М.** Анализ кратковременной стабильности и световых сдвигов квантового стандарта частоты на основе эффекта когерентного пленения населенностей в газовой ячейке в атомами ^{87}Rb // Радиотехника. 2016. № 12. **С. 164**–170.

17. **Ritter S., Nlleke C., Hahn C., et al.** An elementary quantum network of single atoms in optical cavities // Nature. 2012. Vol. 484. No. 7393. Pp. 195–200.

18. **Bjork G., Karlsson A., Yamamoto Y.** Definition of a laser threshold // Phys. Rev. A. 1994. Vol. 50. No. 2. P. 1675.

19. **Protsenko I.E., Travagnin M.** Microscopic model of semiconductor laser without inversion // Phys. Rev. A. 2001. Vol. 65. No. 1. P. 013801.

20. **Ellis B., Mayer M.A., Shambat G., Sarmiento T., Harris J., Haller E.E., Vuckovic J.** Ultralow-threshold electrically pumped quantum-dot photonic-crystal nanocavity laser // Nat. Photonics. 2011. Vol. 5. Pp. 297–300.

21. **Курапцев А.С., Соколов И.М.** Микроскопическая теория диполь-дипольного взаимодействия в ансамблях примесных атомов в резонаторе Фабри – Перо // ЖЭТФ. 2016. Т.

150. Вып. 2. С. 275–287.

22. **Курапцев А.С., Соколов И.М.** Особенности межатомного диполь-дипольного взаимодействия вблизи идеально проводящей заряженной поверхности // ЖЭТФ. 2018. Т. 154. **Вып**. 3. С. 534–542.

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