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INTERNAL TIME IN RELATIVISTIC QUANTUM MECHANICS

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A formulation for spinless particles, that holds true the formal equivalency of spatial and time particle's coordinates achieved through the introduction of an auxiliary parameter of evolution, has been put forward in terms of relativistic quantum mechanics. The proposed modification of theory gave a space-time picture of elementary processes involved in a finite region of the Minkowski space in the form of scattering amplitudes. In order to define the evolution parameter, an additional condition was introduced. That was the presence of an extreme of the scattering amplitude phase. The probabilistic interpretation of the scattering amplitude includes a contraction of integration measure in the Minkowski space on a 3D surface given by experimental conditions. The nonrelativistic limit of the modified theory was shown to coincide with the Schrödinger theory, including the dynamical model of the stationary scattering problem in terms of wave packets movement.

Keywords: Minkowski space, spinless particle, wave packet, internal time, scattering amplitude.

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СОБСТВЕННОЕ ВРЕМЯ В РЕЛЯТИВИСТСКОЙ КВАНТОВОЙ МЕХАНИКЕ

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В рамках релятивистской квантовой механики предложена формулировка для бесспиновых частиц, которая сохраняет формальное равноправие пространственных и временной координат такой частицы благодаря введению вспомогательного параметра эволюции. Предлагаемая модификация теории дает пространственно-временную картину элементарных процессов, происходящих в конечных областях пространства Минковского в форме амплитуд рассеяния. Для определения указанного параметра введено дополнительное условие – наличие экстремума фазы амплитуды рассеяния. Вероятностная интерпретация амплитуды рассеяния включает ограничение меры интегрирования в пространстве Минковского трехмерной поверхностью, задаваемой условиями эксперимента. Показано, что нерелятивистский предел модифицированной теории совпадает с теорией Шрёдингера, в том числе для динамической модели стационарной задачи рассеяния в форме движения волновых пакетов.

Ключевые слова: пространство Минковского, бесспиновая частица, волновой пакет, собственное время, амплитуда рассеяния.

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Introduction

The most impressive result of Special Relativity is combining space and time into a single four-dimensional space-time, called the Minkowski space. In this case, separating time again as an evolution parameter that describes particle dynamics would be a step back in a way. This can be avoided in classical (non-quantum) relativistic mechanics, where motion may be described in terms of invariant proper time, measured by clocks associated with each particle.

Such a description would be difficult in quantum mechanics, if only because it is impossible to provide a clock for each elementary particle. The coordinate time of the Minkowski space acts as an evolution parameter here again. This is the method used in relativistic quantum mechanics (RQM), based on the Klein–Gordon (KG) equation for spinless particles and on the Dirac equation for spin-S particles [1].

We shall confine our consideration to spinless charged particles in this paper. Asymptotic (free, since interaction is eliminated) solutions of the Klein–Gordon equation with $t \to +\infty$ are matter waves, and the solution of the KG equation in the entire Minkowski space, in particular, within the framework of perturbation theory, allows to determine the scattering amplitudes of these waves. This scattering theory is covariant precisely due to infinite limits of the time coordinate.

However, this formulation of the scattering problem is rather incomplete. The asymptotic state in the form of a matter wave is an idealization, similar to the point mass in classical mechanics. Particles are actually produced (i.e., escape from a source) and detected in a free state in a bounded domain of space-time. Schwinger's theory on sources (sinks) [2] is the most suitable for describing these processes. The state evolving in the source is a "trimmed" matter wave, or a wave packet. We are going to refer to the dimensions of the wave packet in all four dimensions as coherence parameters. They depend on the spatial and temporal characteristics of the elementary process of particle escaping from a bound state in the source. The wave packet is not a solution of the KG equation, which means that it keeps evolving until a particle is detected after scattering at some point in Minkowski space (wave packet reduction). The whole elementary process of scattering is localized in a finite region of this space. To describe the motion of a wave packet in this space, we have to introduce proper time as an auxiliary parameter of evolution.

Proper time has been long used in relativistic quantum mechanics, starting with Dirac's works [3]. The formalism of auxiliary time was further developed by Fock [4], Stueckelberg [5], Feynman [6], Schwinger [7] and Kyprianidis [8], associating it with indefinite mass. The relationship between this auxiliary parameter and the proper time of a particle was established in [9]. Proper time was proposed in [10, 11] as a parameter of wave packet evolution in Minkowski space. Since this time is not observable, it should be excluded and expressed in terms of scattering parameters, which is achieved by imposing an additional condition on the scattering amplitude for the extremum of its phase with respect to proper time. Here we draw an analogy with the classical principle of least action in relativistic mechanics, where proper time is an independent dynamic variable [4], relying on Dirac's remark that the phase of a wave function is a quantum equivalent of the action functional [12]. Imposing this additional condition on proper time means that an additional condition has to be introduced for normalization of the scattering amplitude. An integration measure on an arbitrary $3D$ surface in Minkowski space is used for this purpose. Choice of surface, and, therefore, the probabilistic interpretation of the scattering amplitude depends on the experimental setting.

The goal of this study has been to refine the new formulation of relativistic quantum mechanics with an additional evolution parameter.

In addition to substantiating the formalism, we have obtained the nonrelativistic limit for this parameter, coinciding with the limit in Schrödinger's theory. We have also established the relationship between this formalism and the formulation of the scattering problem in terms of wave packet motion in $3D$ space (this problem was considered in [13]).

Proper time in classical and quantum mechanics

Let us start our consideration with the canonical form of action of a charged particle in an external field:

$$I = \int_0^S \left(\theta_\mu p_\mu \dot{x}_\mu - H\right) d\tau \qquad (1)$$

where the dot denotes the derivative with respect to the parameter $\tau \in [0,S]$, and

$$H \equiv \theta_\mu \left(p_\mu - eA_\mu\right)^2 - mc^2 \qquad (2)$$

is a Hamiltonian function.

The repeating indices $\mu = 0,1,2,3$ are to be summed, and the Minkowski signature

$$\theta_\mu = (+1,-1, -1, -1)$$

is introduced explicitly.

Following Fock [4], let us consider the upper integration limit for S as an independent dynamic variable, found from the principle of least action after solving the equations for charge motion in Minkowski space between the given endpoints

$$x_\mu(0) = x_{0\mu},\ x_\mu(S) = x_{1\mu}.$$

For example, in the absence of an external field, we find:

$$S = \pm \frac{\sqrt{\Delta x^2}}{2mc}, \qquad (3)$$

where

$$\Delta x^2 = \theta_\mu \left(x_{1\mu} - x_{0\mu}\right)^2 \qquad (4)$$

is the distance between the endpoints in Minkowski space. The plus or minus signs correspond to the motion of the charge back and forth in time τ.

Action (1) corresponds to a Schrödinger-type wave equation in quantum mechanics:

$$i\hbar \frac{\partial \psi}{\partial \tau} = \hat{H}\psi; \qquad (5)$$

it describes the evolution of the wave function $\psi(\tau,x)$ in Minkowski space over time $\tau \in [0, S]$.

Here

$$\hat{H} \equiv \theta_\mu \left[\frac{\hbar}{i}\nabla_\mu - eA_\mu\left(x_\mu\right)\right]^2 - m^2c^2 \qquad (6)$$

is the Hamiltonian operator.

We assume that stationary states of a particle in a static external field $A_\mu(x_k)$ are still solutions of the KG equation $\hat{H}\psi = 0$. This also includes the standard formulation of the stationary scattering problem in terms of de Broglie asymptotic plane waves [1].

Here we are going to describe elementary scattering processes localized in bounded domains of Minkowski space. Let us consider a problem similar to the boundary problem in classical mechanics for this equation. The initial state of the charge is given by a Gaussian wave packet with the center at the point $x_{0\mu}$:

$$\psi_0\left(x\right) = A\,exp\left[-\frac{\left(x_\mu - x_{0\mu}\right)^2}{2\sigma_\mu^2} + \frac{i}{\hbar}\theta_\mu p_{0\mu} x_\mu\right]. \qquad (7)$$

This state is a “trimmed” matter wave with a four-momentum $p_{0\mu}$ localized around the starting point $x_{0\mu}$. The size of the localization region is given by parameters σ_μ, referred to above as the coherence parameters of a matter wave. They are governed by the physical process that “releases” a particle from a bound state in the source. Notably, the dimensions of the package σ_μ obey the Lorentz transformations, so it is assumed that the frame of reference is fixed.

State (7) has a finite norm with a standard integration measure in Minkowski space:

$$\left\|\psi\right\|^2 \equiv \int \prod_\mu dx_\mu \left|\psi\right|^2 < \infty. \qquad (8)$$

and this norm is preserved during the evolution of the state, described by equation (5).

However, we cannot regard the quantity $|\psi(\tau, x_\mu)|^2$ as the density of the probability for detecting a particle in the neighborhood of a point x_μ in Minkowski space at a given time τ because this time itself is not observable and is yet to be determined.

For this purpose, we need the quantum equivalent to the classical principle of least action. Following Dirac [12], let us consider phase I of the wave function in its exponential representation as the quantum action:

$$\psi = R \cdot exp\left(\frac{i}{\hbar} I\right). \qquad (9)$$

Let us take the value of the wave function at the final time $\tau = S$ at some endpoint $x_{1\mu}$. We obtain the following system of equations for the corresponding modulus R and phase I of the wave function, from wave equation (5):

$$\frac{\partial R}{\partial S} = -2\theta_\mu \nabla_\mu R \cdot \left(\nabla_\mu I - eA_\mu\right) - R\theta_\mu \nabla_\mu^2 I, \quad (10)$$

$$\frac{\partial I}{\partial S} = -\theta_\mu \left(\nabla_\mu I - eA_\mu\right)^2 + m^2c^2 + \hbar^2 \frac{\theta_\mu \nabla_\mu^2 R}{R}. \quad (11)$$

The value of the wave function at endpoint $\psi(S, x_{1\mu})$ has the sense of the amplitude for scattering of a particle from initial state (7) in Minkowski space to the final state

$$\delta^4(x_\mu - x_{1\mu}),$$

localized at the endpoint $x_{1\mu}$ (preliminary, until the proper time S is fixed).

From a physical standpoint, this means that a detector (for example, a Faraday cup) “worked” at the endpoint, transferring the particle to a bound state. We believe that this process of “binding” is localized in a region of Minkowski space that is much smaller than the coherence parameters σ_μ of the initial matter wave. It is in this context that we can speak of wave-particle duality in quantum mechanics: while a matter wave (bounded by the coherence parameters) is generated in the source, it is a point particle that is detected.

We have now prepared the conditions for fixing the parameter S, so, with this in mind, let us introduce an additional condition for the extremum:

$$\frac{\partial I}{\partial S} = 0, \quad (12)$$

which we called the quantum principle of least action in our earlier study [10].

After the (preliminary) scattering amplitude $\psi(S, x_{1\mu})$ is found, Eq. (12) determines the proper time S as a function of kinematic parameters of the experiment: source coordinates $x_{0\mu}$ and detector $x_{1\mu}$, source sizes σ_μ and initial four-momentum $p_{0\mu}$. Below, we are going to demonstrate that if the source is far from the scattering center, then the initial momentum $p_{0\mu}$ satisfies the condition

$$p_{0\mu}^2 = m^2c^2. \quad (13)$$

After substituting the proper time found to the amplitude $\psi(S, x_{1\mu})$, the latter becomes the scattering amplitude of the particle in the given experimental conditions.

However, we should now refine the probabilistic interpretation of this scattering amplitude, since we cannot be sure that norm (8) of the wave function is preserved in Minkowski space, given the additional condition (12). It was proposed in [11] to determine the integration measure in Minkowski space in accordance with the given experimental setting. In general form, this condition is reduced to restricting integration to some $3D$ surface $F(x_\mu) = 0$ in Minkowski space. We can now determine the scattering amplitude norm more accurately:

$$\|\psi\|_F^2 \equiv \int_{t_0}^{\infty} dt \int \prod_k dx_k \delta(F) |\psi|^2. \quad (14)$$

This definition takes into account that the experiment takes place after the initial state evolves at time t_0. We are dealing with a particle if $p_0 > 0$, and with an antiparticle if $p_0 < 0$. In the next paragraph, this definition of the norm is established for the nonrelativistic limit.

Let us conclude this section by presenting a solution for the case of free motion of a wave packet. It is described by the amplitude

$$\psi\left(S, x_{1\mu}\right) = A\left(\prod_\mu \sqrt{1 + 2i\hbar \frac{\theta_\mu S}{\sigma_\mu^2}}\right)^{-1} \times$$
$$\times \exp\left\{ \begin{aligned} & -\frac{\left(\Delta x_\mu - 2Sp_{0\mu}\right)^2}{2\left(\sigma_\mu^2 + 4i\hbar\theta_\mu S\right)} + \\ & + \frac{i}{\hbar}\left[\theta_\mu p_{0\mu}\Delta x_\mu - \left(\theta_\mu p_{0\mu}^2 - m^2c^2\right)S\right] \end{aligned} \right\}. \quad (15)$$

From here, we obtain the quantum action:

$$I = -\frac{1}{2} arctg \frac{2\hbar\theta_\mu S}{\sigma_\mu^2} + \frac{\hbar^2 \left(\Delta x_\mu - 2Sp_{0\mu}\right)^2 S\theta_\mu}{\sigma_\mu^4 + 4\hbar^2 S^2} +$$
$$+\theta_\mu p_{0\mu}\Delta x_\mu - \left(\theta_\mu p_{0\mu}^2 - m^2c^2\right)S. \quad (16)$$

Recall that summation is intended over the index μ. Its classical limit ($\hbar \to 0$) coincides with classical action (1) in case of free motion of a particle, in which case condition (13) follows from condition (12) for the extremum. Decomposing quantum action (16) in a series in terms of powers of $\hbar$, the first nonzero correction has the second order:

$$I = I_0 + \hbar^2 I_2 + \ldots, \tag{17}$$

where

$$I_2 = -\left[\begin{array}{c} 4\left(\theta_\mu p_{0\mu}\Delta X_\mu - \theta_\mu p_{0\mu}^2 S\right)\dfrac{S^2}{\sigma_\mu^4} \\ +\left(\dfrac{\theta_\mu}{\sigma_\mu^2} - \dfrac{\theta_\mu \Delta X_\mu^2}{\sigma_\mu^4}\right)S \end{array}\right]. \tag{18}$$

Then, if condition (13) is satisfied, we obtain from extremum condition (12) for quantum action with the given accuracy:

$$S = \frac{2\sum\limits_\mu \dfrac{\theta_\mu p_{0\mu}\Delta x_\mu}{\sigma_\mu^4} \pm \sqrt{D}}{6\sum\limits_\mu \dfrac{\theta_\mu p_{0\mu}^2}{\sigma_\mu^4}}, \tag{19}$$

where

$$D \equiv 4\left(\sum_\mu \frac{\theta_\mu p_{0\mu}\Delta x_\mu}{\sigma_\mu^4}\right)^2 - -3\left(\sum_\mu \frac{\theta_\mu p_{0\mu}^2}{\sigma_\mu^4}\right)^2\left(\sum_\mu \theta_\mu\left(\frac{\Delta x_\mu^2}{\sigma_\mu^4} - \frac{1}{\sigma_\mu^2}\right)\right). \tag{20}$$

The choice of sign in expression (19) is now determined by the sign of p_{00}: it is positive if $p_{00} > 0$ (particle) and negative if $p_{00} < 0$ (antiparticle).

The resulting expression (19) should be substituted into the formula for the initial wave function (15), whose modulus

$$R = A\left[\prod_\mu \sqrt{\sigma_\mu^2 + \hbar^2 S^2}\right]^{-1} \times \times exp\left[-\frac{\left(\Delta x_\mu - 2Sp_{0\mu}\right)^2}{2\left(\sigma_\mu^2 + 4\hbar^2 S^2/\sigma_\mu^2\right)}\right] \tag{21}$$

determines the motion of the wave packet in Minkowski space. Summation over μ is to be carried out in the exponent here.

Nonrelativistic approximation

Let us first consider the nonrelativistic limit for the case of free motion of the wave packet. This limit is achieved if the initial three-momentum of the particle is small, i.e.,

$$\left|p_{0k}\right| << mc. \tag{22}$$

According to expression (21), the center of the packet also passes a short distance in $3D$ space in a finite period of time, namely,

$$\left|\Delta x_k\right| << \Delta x_0. \tag{23}$$

Then, for a sufficiently long period of time, i.e.,

$$\Delta x_0 >> \sigma_0, \tag{24}$$

we obtain with high accuracy from expression (19):

$$S \cong \frac{\Delta t}{2m}. \tag{25}$$

Thus, the proper time of the particle in this limit coincides (up to a factor of $1/2m$) with the coordinate time, as we might have expected.

In this case, the modulus of wave function (21) has the form

$$R \sim A\,exp\left[-\frac{\left(\Delta x_k - \Delta t p_{0k}/m\right)^2}{2\left(\sigma_k^2 + \hbar^2\Delta t^2/m^2\sigma_k^2\right)}\right], \tag{26}$$

where summation over k is intended in the exponent.

The solution of the non-stationary Schrödinger equation for Gaussian wave packet motion yields this exact result.

If an external electromagnetic field is present, inequalities (22) in the nonrelativistic approximation should also include the following:

$$\left|eA_\mu\right| << mc. \tag{27}$$

The same as in the case of the Klein–Gordon equation, transition to the nonrelativistic limit accomplished by separating a rapidly oscillating term in the wave function [1, 14]:

$$\psi\left(t, x_k\right) \sim exp\left(\frac{i}{\hbar}mc^2 t\right)\psi'\left(t, x_k\right). \tag{28}$$

As for the function's slowly changing component $\psi'(t, x_k)$, let us confirm that it satisfies the Schrödinger equation

$$i\hbar\frac{\partial\psi'}{\partial t} = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla_k - eA_k\right)^2\psi' + ecA_0\psi'. \tag{29}$$

A rapidly changing multiplier in the given formalism appears as the stationary value of the component of the action I corresponding to the time coordinate Δt. Indeed, by completely neglecting the motion of a particle in $3D$ space, taking into account inequalities (22), (27) and the action of the external field, we obtain free "motion" of the coordinate time t with the "flow" of the proper time τ. Let us call this approximation ultranonrelativistic.

All the equations for the case of free motion of a wave packet containing only the contribution $\mu = 0$ are valid for this approximation. In particular, formula (19) is in this case reduced to the expression

$$S \cong \frac{\Delta t}{3m} + \sqrt{\left(\frac{\Delta t}{3m}\right)^2 - \frac{\left(\Delta t^2 - \sigma_0^2\right)}{12m^2}}. \tag{30}$$

It has a nonzero limit with small values of Δt, and is transformed to expression (25) with $\Delta t >> \sigma_0$. With larger values of Δt, a transition to the limit $\sigma_0 \to 0$ can be made in the initial wave packet (7), which then takes the form

$$\psi_0\left(x_\mu\right) = \delta\left(t - t_0\right)\psi_0'\left(t, x_k\right), \tag{31}$$

where $\psi'_0(t, x_k)$ is the initial wave packet in $3D$ space.

Recall that the term "initial" here refers to proper time τ. Then the solution of Eq. (5) in the given ultranonrelativistic limit takes the form

$$\psi\left(t\right) = \sqrt{\frac{2mc^2}{i\pi\hbar t}}\, exp\left(\frac{i}{\hbar}mc^2 t\right). \tag{32}$$

The remaining task is to find nonrelativistic corrections to this solution, taking into account slow motion of the particle and the action of the external field.

For this purpose, let us consider Eqs. (10) and (11). If we assume that the extremum condition (12) is satisfied and the expression for proper time has the form (25), then we can extract the square root in equation (11):

$$\frac{\partial I}{\partial t} = \sqrt{Z} + e\phi, \tag{33}$$

where

$$Z = m^2c^4 + c^2\left(\nabla_k - eA_k\right)^2 + \hbar^2c^2\frac{\theta_\mu \nabla_\mu^2 R}{R}. \tag{34}$$

The square root in the sought-for nonrelativistic limit is approximated as follows:

$$\frac{\partial I}{\partial t} \cong mc^2 + \frac{1}{2m}\left(\nabla_k - eA_k\right)^2 + e\phi - \hbar^2\frac{\Delta R}{R}, \tag{35}$$

with the second derivative also dropped

$$\frac{1}{c^2R}\frac{\partial^2 R}{\partial t^2}. \tag{36}$$

Let us now turn to Eq. (10). According to Eq. (35), the zero component of the first term in its right-hand side ($\mu = 0$) can be approximated by the product

$$-2m\frac{\partial R}{\partial t}. \tag{37}$$

If we also neglect the zero component in the second term and divide both sides of the equation by $2m$, the equation takes the form

$$\begin{aligned}\left(\frac{1}{2m}\frac{\partial R}{\partial S} + \frac{\partial R}{\partial t}\right) &= \frac{1}{m}\left(\nabla_k I - eA_k\right)\nabla_k R \\ &+ \frac{1}{2m}R\nabla_k\left(\nabla_k I - eA_k\right).\end{aligned} \tag{38}$$

The expression in brackets on the left-hand side of Eq. (38) is equal to the total partial derivative R with respect to time. In this form, Eqs. (35) and (37) are equivalent to the Schrödinger equation (29) for a wave function $\psi'(t, x_k)$ slowly varying with time, if we assume that

$$I = mc^2t + I'$$

(I' is its phase):

$$\psi' = R \cdot exp\left(\frac{i}{\hbar}I'\right). \tag{39}$$

This establishes the nonrelativistic limit for wave equation (5) within the framework of the given theory.

Now we have to refine the probabilistic interpretation of the amplitude $\psi(x_\mu)$ in the non-relativistic limit. There are two possibilities here. The first is the typical case when time t is regarded as a classical evolution parameter with large values of Δt:

$$\Delta t \equiv T >> \sigma_0. \tag{40}$$

To consider this possibility, let us imagine the following experiment: detectors are turned on in the whole space after a time interval T after a particle is produced. Before this, since the time t_0 (σ_0 can be taken equal to zero in this case) when the particle was produced, it moved in a given potential field in the absence of an observer. This experiment corresponds to a certain choice of integration surface in expression (14):

$$F \equiv \Delta t - T = 0. \tag{41}$$

In this case, integration over time is eliminated in (14) and the wave function $\psi'(t, x_k)$ that is the solution to Schrödinger equation (29) is interpreted in the standard manner as the probability distribution density in $3D$ space.

Another interpretation is more consistent with the standard experiment on scattering of particles by a static target. The target is usually placed in the center of a vacuum chamber, and the detectors (Faraday cups) are located on the surface of a spherical screen. The diffraction pattern is obtained through recording the results of elementary scattering events over a long observation period. If ρ is the radius of the screen with the center in the target, then the integration surface in (14) is a $3D$ sphere:

$$F \equiv \sqrt{x_k^2} - \rho = 0. \tag{42}$$

Thus, integration over the radial variable is eliminated in $3D$ space is removed in (14) but integration over time is preserved. We might call this a good scenario: the elementary scattering of a particle is completed only when the entire packet passes through the detector. Such case corresponds to dynamic interpretation of the stationary scattering problem in terms of the motion of wave packets, considered in [13]. In practice, elementary scattering events are also averaged over the exposure time.

Conclusion

The formulation of relativistic quantum mechanics discussed in detail in this study describes the space-time characteristics of elementary processes localized in finite regions of Minkowski space. This description also includes the characteristics of the initial state, such as the coherence parameters σ_μ, which are observable quantities. Introducing an additional evolution parameter to preserve equivalent spatial and temporal coordinates of Minkowski space means that another additional condition has to be imposed, in order to exclude this parameter as an unobservable quantity, and to additionally determine the probability measure.

Choice of this measure depends on the experimental setting. Given these additional conditions, the solutions of Eq. (5) acquire the physical meaning of scattering amplitudes. The formalism proposed should serve as an addition to stationary theory of scattering of asymptotic matter waves based on the Klein – Gordon equation, which takes into account finite spatial and temporal characteristics of elementary scattering processes. One of the rationales behind the proposed theory is that scattering amplitudes, together with their probabilistic interpretation, have the correct nonrelativistic limit.

The formulation of relativistic quantum mechanics for spinless particles considered in this study can be generalized to the case of spin particles.

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REFERENCES

[1] **J.D. Bjorken, S.D. Drell,** Relativistic quantum theory, Vol. I. Relativistic quantum mechanics, McGraw-Hill, New York, 1964.

[2] **J. Schwinger,** Particles, sources, and fields, Vol. I. Addison-Wesley PC, London, Amsterdam, 1973.

[3] **P.A.M. Dirac,** On the theory of quantum mechanics, Proc. R. Soc. Lond., Ser. A. 112 (762) (1926) 661–677.

[4] **V. Fock,** Die Eigenzeit in der Klassischen und in der Quantenmechanik, Sow. Phys. 12 (1937) 404–425.

[5] **E.C.G. Stueckelberg,** Remark of the creation of pairs of particles in the theory of relativity, Helv. Phys. Acta. 14 (7) (1941) 588–594.

[6] **R.P. Feynman,** Mathematical formulation of the quantum theory of electromagnetic interaction, Phys. Rev. 80 (3) (1950) 440–456.

[7] **J. Schwinger,** On gauge invariance and vacuum polarization, Phys. Rev. 82 (5) (1951) 664–679.

[8] **A. Kyprianidis,** Scalar time parametrization of relativistic quantum mechanics: The covariant Schrödinger formalism, Phys. Reports. 155 (1) (1987) 1–27.

[9] **J.P. Aparicio, F.H. Gaioli, G.E.T. Alvarez,** Interpretation of the evolution parameter of Feynman parametrization of the Dirac equation, arXiv: 9502015v1 quant-ph. 16 Fev1995.

[10] **N.N. Gorobey, A.S. Luk'yanenko,** The least action principle in relativistic quantum mechanics, St. Petersburg Polytechnical University Journal. (6(70)) (2008) 7–10.

[11] **N. Gorobey, A. Lukyanenko, I. Lukyanenko,** Quantum action principle in relativistic mechanics (II), arXiv:1010.3824v1quant-ph., 2010.

[12] **P. Dirac,** The principles of quantum mechanics, 4th Ed, Oxford at the Clarendon Press, 1958.

[13] 13. **L.D. Faddeev, O.A. Yakubovskii,** Lectures on quantum mechanics for mathematics students, Transl. by H. McFaden, Student Mathematical Library 47. Providence, RI: American Mathematical Society (AMS). xii, 2009.

[14] **V.G. Levich, Yu.A. Vdovin, V.A. Myamlin,** Kurs teoreticheskoy fiziki. T. 2. [A course of lectures on theoretical physics] Vol. 2., Nauka, Moscow, 1971.

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СПИСОК ЛИТЕРАТУРЫ

1. **Бьёркен Дж.Д., Дрелл С.Д.** Релятивистская квантовая теория. В 2-х тт. Пер. с англ. Т. 1. Релятивистская квантовая механика. М.: Наука, 1978. 295 с.

2. **Швингер Ю.** Частицы, источники, поля. В 2-х тт. Пер. с англ. Т. 1. Общий курс теории элементарных частиц и квантовой теории поля. М.: Мир, 1973. 502 с.

3. **Dirac P.A.M.** On the theory of quantum mechanics // Proc. R. Soc. Lond. Ser. A . 1926. Vol. 112. No. 762. Pp. 661–677.

4. **Фок В.А.** Собственное время в классической и квантовой механике // Изв. АН СССР. 1937. № 4–5. С. 551–568.

5. **Stueckelberg E.C.G.** Remark of the creation of pairs of particles in the theory of relativity // Helv. Phys. Acta. 1941. Vol. 14. No. 7. Pp. 588–594.

6. **Feynman R.P.** Mathematical formulation of the quantum theory of electromagnetic interaction // Phys. Rev. 1950. Vol. 80. No. 3. Pp. 440–456.

7. **Schwinger J.** On gauge invariance and vacuum polarization // Phys. Rev. 1951. Vol. 82. No. 5. Pp. 664–679.

8. **Kyprianidis A.** Scalar time parametrization of relativistic quantum mechanics: The covariant Schrödinger formalism // Phys. Reports. 1987. Vol. 155. No. 1. Pp. 1–27.

9. **Aparicio J.P., Gaioli F.H., Alvarez G.E.T.** Interpretation of the evolution parameter of Feynman parametrization of the Dirac equation // arXiv : 9502015v1 [quant-ph] 16 Fev1995.

10. **Горобей Н.Н., Лукьяненко А.С.** Квантовый принцип наименьшего действия в релятивистской квантовой механике // Научно-технические ведомости СПбГПУ. 2008. № 6 (70). С. 7–10.

11. **Gorobey N., Lukyanenko A., Lukyanenko I.** Quantum action principle in relativistic mechanics (II) //arXiv:1010.3824v1[quant-ph], 2010.

12. **Дирак П.** Принципы квантовой механики. М.: Наука, 1979. 479 с.

13. **Фаддеев Л.Д., Якубовский О.А.** Лекции по квантовой механике для студентов-

математиков. Л.: Изд-во ЛГУ, 1980, 199 с.

14. **Левич В.Г., Вдовин Ю.А., Мямлин В.А.** Курс теоретической физики. В 2-х тт. Т. 2. Квантовая механика. Квантовая статистика и физическая кинетика. Изд. 2-е. М.: Наука, 1971. 936 с.

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