

THREE-FLUID FORMULATION AND A NUMERICAL METHOD FOR SOLVING THE STATIONARY PROBLEM OF THERMAL HYDRAULICS OF A TWO-PHASE ANNULAR DISPERSED FLOW

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The article presents a one-dimensional three-fluid model for solving the stationary flow problem on a two-phase steam-water annular dispersed stream in a vertical heated channel, based on nine balance equations with an equal phase pressure assumed. The validation of the marching algorithm of the described stationary problem has been carried out by comparison with the published experimental data relating to a two-phase flow in a circular pipe under adiabatic conditions for pressures of 3 – 9 MPa, total flow rates of 500 – 3000 kg / (m²·s) and internal diameters of 10 and 20 mm. The total pressure differences in the channel were calculated. Good qualitative agreement with experimental data was obtained. Small quantitative disagreements were found. They were shown to be reduced by the refinement of the closing relations.

Key words: two-phase steam-water flow, three-fluid model, numerical solution

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Introduction

The most common approach for describing multiphase flows in a thermal hydraulic approximation is based on the model of interpenetrating continua. The existing simulation codes for thermal hydraulics (KORSAR, TRAC, RELAP) are based on the so-called two-fluid approximation describing the case for multiphase flow represented by two “fluids”: vapor and fluid. The corresponding system consists of six differential equations of mass, momentum and energy balance (for each of the fluids). The system is closed by the phase balance equation, the thermodynamic equation of state (for each of the fluids) and empirical or semi-empirical relations (for each of the fluids). These relations, called closing, describe mass, heat and momentum transfer between the individual phases and between the phases and the channel wall.

The structure of the two-phase medium and, accordingly, the closing relations in such a model are determined by the chosen flow regime map.

The two-fluid approximation was the first one used in simulation codes developed for thermal hydraulics. It is still found in many

codes and is often discussed in literature. The two-fluid approach is fairly up-to-date, providing adequate descriptions for all flow regimes where it is more correct to divide two-phase flow into any two components. This includes, for example, bubble, slug, dispersed and stratified flows. The two-fluid approach yields acceptable results for all of these flow regimes; the descriptions differ only in the definitions of “fluids” in each case and, accordingly, in the closing relations.

On the other hand, the medium in the annular dispersed flow regime is divided into three fluids: vapor, droplets, and liquid film; accordingly, each of the fluids should have its own velocity and temperature, which is impossible in the two-fluid approximation. There are approximate methods for describing the specifics of the annular dispersed-regime for these cases, staying within the framework of the two-fluid model. For example, in the KORSAR simulation code, the emerging droplets are taken into account by an additional term in the balance equations but still do not have their own velocity and temperature.

The issue of inaccurate description of the annular dispersed regime is solved by replacing the two-fluid approach with the three-fluid one

where each of the fluids (i.e., vapor, droplets, and liquid film) is described by its own equations for the balance of mass, momentum, and energy. The three-fluid model is well-known: it has been considered in some Russian and foreign studies [1 – 7]. While using the approach with three fluids instead of two is nothing new from an algorithmic standpoint, it allows to construct a more complete physical model that is non-equilibrium with respect to the velocities and temperatures of the given fluids.

The three-fluid model is more accurate in describing the annular dispersed flow regime, which means that more accurate values can be obtained for the characteristics of heat transfer, friction loss, volume fractions of fluids, and, ultimately, the position of the dryout point can be determined more reliably. Using this model means that the problem of closing relations has to be considered again. Most of the correlations tested were formulated for the two-fluid approximation. However, formulation of the three-fluid model makes it possible to partially use the experience in accumulated solving two-phase problems. For example, closing relations describing the exchange processes between vapor and droplets in dispersed flow or exchange with the channel wall in single-phase flow can be used for describing exchange processes at the corresponding interfaces and in the three-fluid approximation. In this case, the exchange processes directly between the droplets and the film are characteristic for the three-fluid model distinguishing it from the two-fluid one, allowing to abandon the equilibrium model of generation and deposition move on to non-equilibrium models by processing the experimental data.

Our study presents a marching algorithm for the numerical solution of the stationary problem of annular dispersed flow in a one-dimensional approximation using the three-fluid approach. Systems considered in a two- and three-fluid approximation in a non-stationary formulation for the case of equilibrium pressure (mathematical models with total pressure in fluids) are known to lose their evolutionary properties with certain values of operating parameters [8]. The correctness of the Cauchy problem can be restored by different measures but all of them “perturb” the original system

of equations. Therefore, if a stationary flow regime is possible for a given problem, then the solution obtained by the marching method of integration is a reference for evolutionary problems, allowing to quantitatively estimate the distortions introduced into the solutions by the techniques applied for regularizing the problem.

Mathematical model

The developed stationary one-dimensional three-fluid model describes annular dispersed flow with masses of vapor, droplets and liquid film moving in two-phase vapor-liquid flow. The model takes into account phase transition, exchange processes with the channel wall, entrainment and deposition of droplets on the film surface. Accordingly, we are going to solve a system of equations consisting of nine differential equations of mass, momentum and energy balance for each of the three liquids.

The differential equations solved are written in the following form.

1) Mass balance equations.

For vapor:

$$\frac{\partial}{\partial x}(A\alpha_v\rho_v u_v) = m_{dv}\Pi_{id} + m_{fv}\Pi_{if}, \quad (1)$$

where x , m , is the axial coordinate; A , m^2 , is the cross-sectional area of the channel; α_v is the volume fraction of vapor; ρ_v , kg/m^3 , is its density; u_v , m/s , is its velocity; m_{dv} , m_{fv} , $kg/(m^2s)$, are the mass sources of vapor generation from the droplets and the film, respectively (positive for condensation and negative for evaporation); Π_{id} , Π_{if} , m , are the perimeters of the surface of heat transfer with vapor for the droplets and the film respectively; the subscripts v , d , f refer to vapor, droplets, and liquid film, respectively. Below we are going to use another subscript w , referring to the channel wall.

For droplets:

$$\frac{\partial}{\partial x}(A\alpha_d\rho_d u_d) = -m_{dv}\Pi_{id} - \Pi_{if}(S_d - S_e), \quad (2)$$

where α_d , ρ_d are the volume fraction and density of the droplets, respectively; u_d , m/s , is their velocity; S_e , S_d , $kg/(m^2s)$, are, respectively, the entrainment and deposition rates for droplets on the surface of the liquid film.

For liquid film:

$$\frac{\partial}{\partial x}(A\alpha_f\rho_f u_f) = -m_{fv}\Pi_{id} + \Pi_{if}(S_d - S_e), \quad (3)$$

where α_f and ρ_f are, respectively, the volume fraction and the density of the liquid film; u_f , m/s, is its flow velocity.

2) *Momentum balance equations.*

For vapor:

$$\begin{aligned} \frac{\partial}{\partial x}(A\alpha_v\rho_v u_v^2) + \bar{\alpha}_v \bar{A} \frac{\partial P}{\partial x} &= m_{dv}\Pi_{id}(u_{di} - u_v) + \\ + m_{fv}\Pi_{if}(u_{fi} - u_v) - \Pi_{if}\tau_{vf} - \Pi_{id}\tau_{vd} + A\alpha_v\rho_v g_x, \end{aligned} \quad (4)$$

where P , Pa, is the pressure; u_{di} , u_{fi} , m/s, are the interfacial velocities (for the vapor – droplets and vapor – film interfaces, respectively). We used the interfacial velocity u_{di} between the droplets and the vapor, assumed to be equal to the droplet velocity u_d . A dependence from the CARHARE code [9] was used for the velocity of the interface between the liquid film and the vapor:

$$u_{fi} = \frac{\alpha_v}{\alpha_v + \alpha_f} u_f + \frac{\alpha_f}{\alpha_v + \alpha_f} u_v;$$

τ_{vf} , τ_{vd} , kg/(s²·m) are the shear stresses (vapor – liquid film and vapor – droplets, respectively); g_x , m/s², is the projection of the gravity vector on the x axis; the overbar indicates the averaged value of a quantity.

For droplets:

$$\begin{aligned} \frac{\partial}{\partial x}(A\alpha_d\rho_d u_d^2) + \bar{\alpha}_d \bar{A} \frac{\partial P}{\partial x} &= -m_{dv}\Pi_{id}(u_{di} - u_v) + \\ + \Pi_{id}\tau_{vd} + A\alpha_d\rho_d g_x - \Pi_{if}(S_d u_d - S_e u_f). \end{aligned} \quad (5)$$

For liquid film:

$$\begin{aligned} \frac{\partial}{\partial x}(A\alpha_f\rho_f u_f^2) + \bar{\alpha}_f \bar{A} \frac{\partial P}{\partial x} &= \\ = -m_{fv}\Pi_{if}(u_{fi} - u_v) - \Pi_{wf}\tau_{wf} + \Pi_{if}\tau_{vf} + \\ + A\alpha_f\rho_f g_x + \Pi_{if}(S_d u_d - S_e u_f), \end{aligned} \quad (6)$$

where Π_{wf} , m, is the perimeter of the surface of heat transfer between the channel wall and the film; τ_{wf} , kg/(s²·m), is the shear stress between the liquid film and the channel wall.

3) *Energy balance equations.*

For vapor:

$$\frac{\partial}{\partial x}(A\alpha_v\rho_v u_v H_v) = (\text{HTC})_{vd}\Pi_{id}(T_{sat} - T_v) + \quad (7)$$

$$\begin{aligned} + \left(h_v + \frac{u_{di}^2}{2} \right) m_{dv}\Pi_{id} + (\text{HTC})_{vf}\Pi_{if}(T_{sat} - T_v) + \\ + \left(h_v + \frac{u_{fi}^2}{2} \right) m_{fv}\Pi_{if}, \end{aligned} \quad (7)$$

where H_v , J/kg, is the total specific enthalpy of vapor, $H_v = h_v + 0.5u^2$ (h_v is the specific enthalpy); $(\text{HTC})_{vd}$, $(\text{HTC})_{vf}$, W/(m²·K), are, respectively, the coefficients of heat transfer from the vapor to the interface with the droplets and from the vapor to the interface with the liquid film; T_{sat} , T_v , K, are the saturation and vapor temperatures, respectively.

For droplets:

$$\begin{aligned} \frac{\partial}{\partial x}(A\alpha_d\rho_d u_d H_d) &= (\text{HTC})_{dv}\Pi_{id}(T_{sat} - T_d) - \\ - \left(h_d + \frac{u_{di}^2}{2} \right) m_{dv}\Pi_{id} - \Pi_{if}(S_d H_d - S_e H_f), \end{aligned} \quad (8)$$

where H_d , H_f , J/kg, are, respectively, the total specific enthalpies of the droplets and the liquid film, $H_{df} = h_{df} + 0.5u^2$ (h_{df} is the specific enthalpy); $(\text{HTC})_{dv}$, W/(m²·K), is the coefficient of heat transfer from the droplets to the interface with the vapor; T_d , K, is the temperature of the droplets.

For liquid film:

$$\begin{aligned} \frac{\partial}{\partial x}(A\alpha_f\rho_f u_f H_f) &= (\text{HTC})_{fv}\Pi_{if}(T_{sat} - T_f) - \\ - \left(h_f + \frac{u_{fi}^2}{2} \right) m_{fv}\Pi_{if} + \Pi_{if}(S_d H_d - S_e H_f) + \\ + q_{wf}\Pi_{fw} - q_{wfi}\Pi_{fw}, \end{aligned} \quad (9)$$

where $(\text{HTC})_{fv}$, W/(m²·K), is the coefficient of heat transfer from the liquid film to the interface with the vapor; T_f , K, is the temperature of the liquid film; q_{wf} , W/m², is the heat flux from the channel wall to the liquid film; q_{wfi} , W/m², is the component of the heat flux from the channel wall that actually contributes to generating vapor.

The system is also complemented by a *phase balance equation*

$$\sum \alpha_k = 1 \quad (10)$$

and by *thermodynamic equations of state* taking the form

$$\rho_k = \rho_k(P, T), \quad (11)$$

$$e_k = e_k(P, T), \quad (12)$$

where e_k , J/kg, is the specific internal energy of the k th phase.

Closing relations

The system of equations (1) – (10) is closed by a set of relations describing the processes of mass, momentum and energy transfer both between the individual phases and between the phases and the channel wall.

Mass sources describing the phase transition (m_{dv} and m_{fv}) are derived by considering the heat balance at the interface. Since the interface cannot accumulate heat, we obtain:

$$m_{dv} = -[(HTC)_{dv}(T_{sat} - T_d) + (HTC)_{vd}(T_{sat} - T_v)] / (h_v - h_d). \quad (13)$$

Direct heat transfer with the channel wall only occurs for the liquid film in annular dispersed flow. Accordingly, a term $q_{wf}\Pi_{fw}$, describing the heat flux transferred to the liquid film from the channel wall is added to the right-hand side of the film's energy balance equation. A term describing the component of the heat flux from the channel wall that actually contributes to generating vapor: $q_{wfi}\Pi_{fw}$, where $q_{wfi} = \psi q_{wf}$; $\psi = 0 - 1$, is also added to the energy balance equation.

Given this term, a similar mass source for the phase transition between the liquid film and the vapor has the form

$$m_{fv} = -[(HTC)_{fv}(T_{sat} - T_f) + (HTC)_{vf}(T_{sat} - T_v) + q_{wfi}] / (h_v - h_f). \quad (14)$$

The terms of the form $(HTC)_{kv}(T_{sat} - T_k)$ are the heat flux from the liquid phase to the interface with the vapor, and $(HTC)_{vk}(T_{sat} - T_v)$ the heat flux from the vapor to the interface with the liquid phase for both mass sources (13), (14).

The phase transition model used allows for heat transfer coefficients of different magnitudes on both sides of the interface, but the following assumption is used for the sake of simplicity. Heat transfer coefficients on both sides of the interface are assumed to be the same and equal to a limiting factor that is the

coefficient of heat transfer from the vapor. It follows then that the coefficients of heat transfer from the vapor to the interface with the droplets and from the droplets to the interface with the vapor are equal and take the form of a correlation describing gas flow around spherical particles [10]:

$$(HTC)_{vd} = (HTC)_{dv} = \frac{\lambda_v}{D_d} (2 + 0,6 Re_d^{0,5} Pr_v^{0,33}), \quad (15)$$

where the Reynolds number for droplets follows the expression

$$Re_d = \frac{\rho_v |u_v - u_d| D_d}{\mu_v}$$

(D_d , m, is the mean droplet diameter; μ_v , Pa·s, is the coefficient of dynamic viscosity of vapor); λ_v , W/(m·K), is the thermal conductivity of vapor.

The coefficients of heat transfer from the vapor to the interface with the liquid film and from the liquid film to the interface with the vapor are calculated from the correlation for single-phase convection [11] applied to the droplet-laden vapor core:

$$(HTC)_{vf} = (HTC)_{fv} = \frac{\lambda_v}{(D - 2\delta)} (0,023 Re_v^{0,8} Pr_v^{0,4}), \quad (16)$$

where D , m, is the internal diameter of the channel; δ , m, is the mean film thickness; Pr_v is the Prandtl number for vapor; Re_v is the Reynolds number for vapor,

$$Re_v = \frac{\rho_v |u_v - u_f| (D - 2\delta)}{\mu_v}.$$

The expressions for the mass sources describing hydrodynamic entrainment and deposition on the surface of the liquid film (S_e and S_d) are borrowed from [2], where turbulent diffusion is assumed to be the dominant mechanism for deposition of droplets on the film surface:

$$S_d = 9 \cdot 10^{-3} u_v \left(\frac{C}{\rho_v} \right)^{-0,5} Re_v^{-0,2} Sc_v^{-2/3} C, \quad (17)$$

where C , kg/m³, is the droplet concentration,

$$C = \frac{G_d}{\frac{G_v u_d}{\rho_v u_v} + \frac{G_d}{\rho_d}} = \frac{\alpha_d \rho_d}{\alpha_v + \alpha_d};$$

Sc_v is the Schmidt number for vapor (taken to equal unity for simplicity);

$$\text{here } Re_v = \frac{\rho_v u_v \alpha_v D}{\mu_v}.$$

It is assumed that shearing off of roll wave crests is the main mechanism of hydrodynamic entrainment of droplets into the droplet-laden vapor core for a liquid with a low viscosity (like water):

$$S_e = 1,07 \frac{u_v \mu_f \tau_{vf}}{\sigma^2} \left(\frac{\rho_f}{\rho_v} \right)^{0,4} \times \begin{cases} k_s, Re_v > 10^5; \\ k_s [2,136 \lg(Re_v) - 9,68], Re_v < 10^5, \end{cases} \quad (18)$$

where $k_s = 0,57\delta + 21,73 \cdot 10^3 \delta^2 - 38,3 \cdot 10^6 \delta^3 + 55,68 \cdot 10^9 \delta^4$.

The shear stresses in the momentum balance equations (τ_{wf} between the channel wall and the liquid film; τ_{vf} between the vapor and the liquid film; τ_{vd} between the vapor and the droplets) are written in the following form:

$$\tau_{wf} = Cf_{wf} \frac{\rho_f}{2} u_f |u_f|; \quad (19)$$

$$\tau_{vf} = Cf_{vf} \frac{\rho_v}{2} (u_v - u_f) |u_v - u_f|; \quad (20)$$

$$\tau_{vd} = Cf_{vd} \frac{\rho_v}{2} (u_v - u_d) |u_v - u_d|, \quad (21)$$

where Cf_{kk} are the respective coefficients of friction (drag force).

The coefficient of friction between the vapor and the liquid film is calculated from the modified Wallis correlation [12]:

$$Cf_{vf} = \frac{0,079}{Re_v^{0,25}} \left(1 + 300 \frac{\delta}{D} \right), \quad (22)$$

where

$$Re_v = \frac{\rho_v |u_v| (D - 2\delta)}{\mu_v}.$$

The aerodynamic drag of the droplets is calculated from the dependence [13]:

$$Cf_{vd} = \left(0,4 + \frac{24}{Re_d} + \frac{4}{\sqrt{Re_d}} \right) \frac{1}{4}, \quad (23)$$

where

$$Re_d = \max \left(0,1; \frac{\rho_v |u_v - u_d| D_d}{\mu_v} \right).$$

The coefficient of friction between the liquid film and the channel wall is expressed as follows [14]:

$$Cf_{fw} = \max \left(\frac{16}{Re_{Dhf}}; \frac{0,079}{Re_{Dhf}^{0,25}} \right), \quad (24)$$

where

$$Re_{Dhf} = \frac{u_f \rho_f \alpha_f D}{\mu_f}.$$

The geometric characteristics used should also be formulated for writing the closing relationships; these include:

- perimeters of interface interactions;
- perimeter of the interaction between the liquid film and the channel wall;
- mean thickness of the liquid film;
- mean droplet diameter.

The perimeter of the interface between the vapor and the droplets follows the expression

$$\Pi_{id} = \frac{6\alpha_d}{D_d}, \quad (25)$$

The perimeter of the interface between the vapor and the liquid film:

$$\Pi_{if} = \pi(D - 2\delta), \quad (26)$$

where $\delta = 0,5D(1 - \sqrt{1 - \alpha_f})$ (mean thickness of the liquid film).

The perimeter of the interface between the liquid film and the channel wall:

$$\Pi_{wf} = \pi D. \quad (27)$$

The droplet diameter is calculated by the technique described in [14]:

$$D_d = \max(8, 4 \cdot 10^{-5}; \min[D_{d1}; D_{d2}]), \quad (28)$$

$$D_{d1} = 7,96 \cdot 10^{-3} \frac{\sigma}{\rho_v j_v} Re_v \left(\frac{\rho_d}{\rho_v} \right)^{1/3} \left(\frac{\mu_v}{\mu_d} \right)^{2/3}, \quad (29)$$

where j_v is the normalized velocity,

$$j_v = \frac{u_v \rho_v A \alpha_v}{\rho_v A} = u_v \alpha_v;$$

the Reynolds number

$$\text{Re}_v = \frac{\rho_v |j_v| D}{\mu_v}$$

$$D_{d2} = 0,254L \left[-0,13We_v + \sqrt{16 + (0,13We_v)^2} \right], \quad (30)$$

where L is the characteristic size (taken as the internal diameter D of the channel);

$$We_v = \frac{\rho_v j_v^2 L}{\sigma}$$

Numerical method

Moving on to the finite-difference formulation of the system, let us introduce the pseudovector notation of the following form:

$$\vec{W} = \begin{pmatrix} W_{k,m} \\ W_{k,i} \\ W_{k,e} \end{pmatrix} = \begin{pmatrix} A\alpha_k \rho_k u_k \\ A\alpha_k \rho_k u_k^2 \\ A\alpha_k \rho_k u_k H_k \end{pmatrix}; \quad (31)$$

$$Q = \begin{pmatrix} S_{k,m} \\ S_{k,i} \\ S_{k,e} \end{pmatrix}, \quad (32)$$

where \mathbf{W} is the flux vector; \mathbf{Q} is the right-hand side vector; the subscript k indicates the corresponding fluid (vapor, droplet, liquid film).

Using two-point approximation, let us write the finite-difference formulation of the system of equations:

$$\frac{W_{k,p}^{n+1} - W_{k,p}^n}{\Delta x} = S_{k,p}^{n+1}; p = m, e; \quad (33)$$

$$\frac{W_{k,i}^{n+1} - W_{k,i}^n}{\Delta x} + \bar{\alpha} \bar{A} \left(\frac{P^{n+1} - P^n}{\Delta x} \right) = S_{k,i}^{n+1}; \quad (34)$$

$$\sum_k (\alpha_k^{n+1}) = 1, \quad (35)$$

where the subscript p indicates the corresponding balance equation (m for mass, i for momentum, e for energy).

At the same time, the balance equations for momentum and energy are written identically in pseudovector notation, so we combined them. Since the flux vector is not linear, in order to achieve convergence of the process, at the next step we divide the sought-for quantities by the

sum of the known quantity $W_p^{n+1,s}$ from the previous iteration and their increments $\Delta W_p^{n+1,s+1}$ at the sought-for iteration, which transforms the given system to the form:

$$\frac{W_{k,p}^{n+1,s} + \Delta W_{k,p}^{n+1,s+1} - W_{k,p}^n}{\Delta x} = S_{k,p}^{n+1,s}; p = m, e; \quad (36)$$

$$\frac{W_{k,i}^{n+1,s} + \Delta W_{k,i}^{n+1,s+1} - W_{k,i}^n}{\Delta x} + \bar{\alpha} \bar{A} \left(\frac{P^{n+1,s} + \Delta P^{n+1,s+1} - P^n}{\Delta x} \right) = S_{k,i}^{n+1,s}; \quad (37)$$

$$\sum_k (\alpha_k^{n+1,s} + \Delta \alpha_k^{n+1,s+1}) = 1. \quad (38)$$

Leaving the increments on the left-hand side of the equation, we move the remaining terms to the right-hand side, thus reducing the solution of the system to finding the increments of the sought-for functions. In the same way, we represent the sought-for flux vector in terms of the vector of primitive variables:

$$f = (\alpha_k, T_k, u_k, P)^T.$$

As a result, the system is reduced to a matrix notation of the form

$$M^{n+1,s} \Delta f^{n+1,s+1} = \mathbf{B}^{n+1,s}, \quad (39)$$

which is solved by the Gauss method. The vector $\mathbf{B}^{n+1,s}$ is the right-hand side vector; $M^{n+1,s}$ is the matrix of transformation of the flux vector to the vector of primitive variables.

The coefficients of the matrix $M^{n+1,s}$ can be obtained by vector differentiation of the flux vector with respect to the vector of primitive variables:

$$\Delta W^{n+1,s+1} = \left(\frac{\partial \mathbf{W}}{\partial \mathbf{f}} \right)^{n+1,s} \Delta f^{n+1,s+1} = M^{n+1,s} \Delta f^{n+1,s+1}.$$

The matrix $M^{n+1,s}$ has a repeating block structure with a block size of 3×3 (see Table).

The given algorithm is applicable for solving problems with codirectional liquid velocities.

Model testing

The three-fluid model described in our study was used to calculate two-phase flow of water in an adiabatic circular tube; we took as a basis the experimental data obtained by

Table

Coefficients of the transformation matrix $M^{n+1,s}$

Balance equation	$\Delta\alpha_k$	ΔT_k	Δu_k	...	ΔP
mass	$A\rho u$	$A\alpha u \frac{\partial \rho}{\partial T}$	$A\alpha\rho$...	$A\alpha u \frac{\partial \rho}{\partial P}$
momentum	$A\rho u^2$	$A\alpha u^2 \frac{\partial \rho}{\partial T}$	$A\alpha\rho 2u$...	$A\alpha u^2 \frac{\partial \rho}{\partial P} + \bar{A}\bar{\alpha}$
energy	$A\rho u H$	$A\alpha u E \frac{\partial \rho}{\partial T} + A\alpha\rho u \frac{\partial e}{\partial T}$	$A\alpha\rho(H + u^2)$...	$A\alpha u \left(E \frac{\partial \rho}{\partial P} + \rho \frac{\partial e}{\partial P} + 1 \right)$
	\vdots	\vdots	\vdots	\vdots	\vdots
phases	1	0	0	...	0

Würz in [15], considering two-phase water-vapor flow in adiabatic round tubes with internal diameters of 10 and 20 mm, with pressures ranging from 3 to 9 MPa and total flow rates of 500 – 3000 kg/(m²·s).

We used the given model to calculate a total of 90 experimental points. Fig. 1 shows a comparison of the calculated dependences of the total pressure drop dp/dx on the relative flow rate G_{vrel} of vapor with the measured values. G_{vrel} is the ratio of the absolute flow rate

of vapor the sum of flow rates of all three “liquids”.

It can be seen from the given results that the calculated dependences practically coincide with the experimental ones from a qualitative standpoint: they have the same slopes, even for dependences with an alternating-sign derivative. Quantitative analysis of the data reveals slight discrepancies, which can be clearly observed by plotting the calculated total pressure drops versus the experimentally measured ones (Fig. 2).

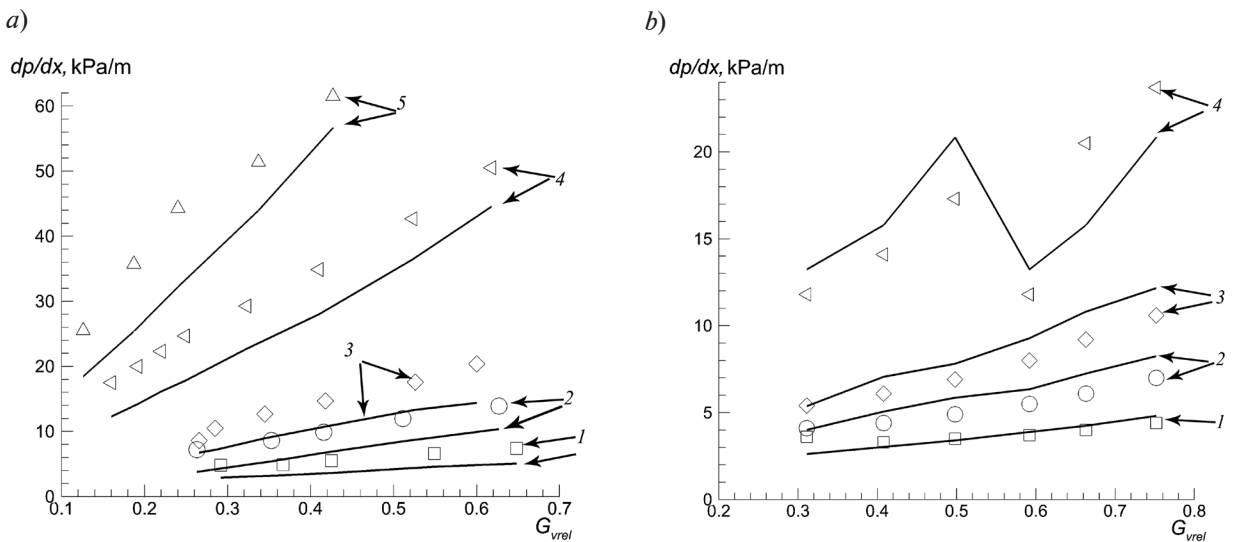


Fig. 1. Experimental (symbols) and calculated (lines) dependences of the total pressure drop in the channel versus the relative vapor flow rate for internal channel diameters $D = 10$ mm (a) and 20 mm (b), with different total flow rates, kg/(m²·s): 500 (1), 750 (2), 1000 (3), 2000 (4), 3000 (5). Pressure $P = 7$ MPa

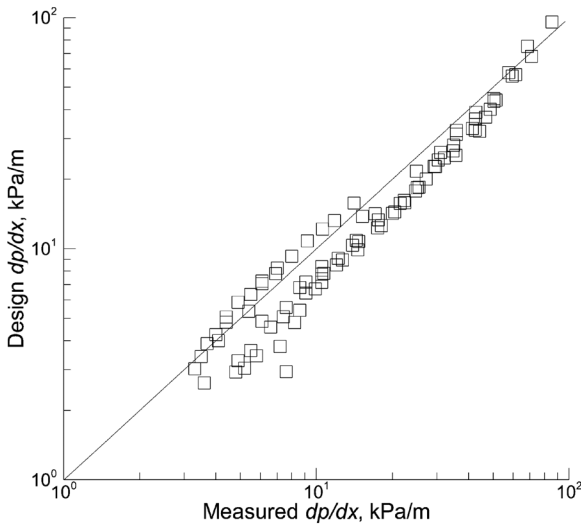


Fig. 2. Relationship between the calculated total pressure drop in the channel and the measured values for all experimental data used from [15]

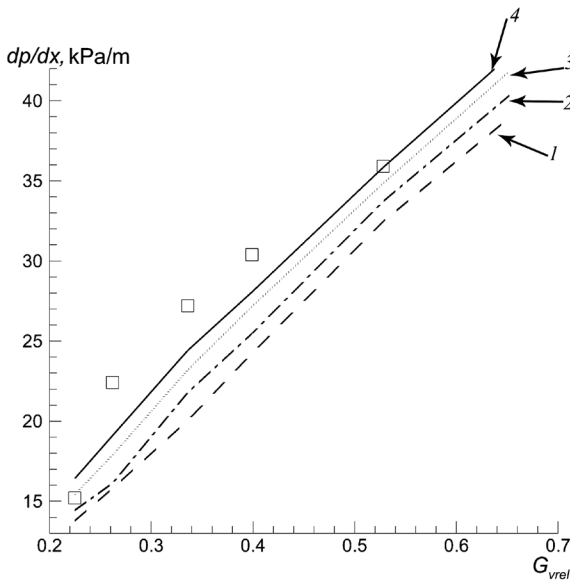


Fig. 3. Dependences similar to those shown in Fig. 1, which were obtained for different roughness coefficients: 1.00 (1), 1.20 (2), 1.50 (3), 1.85 (4); $P = 3$ MPa, $D = 10$ mm

A possible explanation for these discrepancies is that the chosen formulation of the closing relations was not optimal. It is these relations that describe the physics of the processes and, therefore, determine the calculated result in the

three-fluid model. Optimizing the closing relations was not among the goals of this study. However, we can confirm that taking into account additional factors, such as tube roughness (which complicates the expression for the coefficient of friction between the film and the channel wall), makes it possible to bring the calculated results to closer agreement with the experimentally measured values. Fig. 3. shows the results obtained by adjusting the calculated results in this manner. For simplicity, the tube roughness is taken into account in the coefficient of friction as a variation of the multiplier, and the calculations were carried out only for one series of experiments (pressure of 3 MPa, total flow rate of $1000 \text{ kg}/(\text{m}^2\cdot\text{s})$, internal channel diameter of 10 mm). The data in Fig. 3 clearly demonstrate that the calculated total pressure drops can be better fitted to the experimentally measured values by increasing the roughness coefficient.

Conclusion

We have developed a numerical procedure for solving the stationary problem of two-phase water-vapor flow in a one-dimensional approximation with an annular dispersed-flow regime using a three-fluid formulation. The integrated system of equations includes the heat transfer between the phases and the channel wall, the friction between the phases and the channel wall, entrainment and deposition on the surface of the liquid film, as well as interaction of the gravity field for channels with constant and variable cross-sections.

We have carried out initial testing of the developed computational model by comparing the simulation results with the experimental data reported by Würz [15]. The comparison confirmed that the three-fluid model considered in our study provides an adequate description of the given series of experiments with vapor-fluid flow and qualitatively speaking, the obtained dependences fully agree with the experimental ones. There are quantitative discrepancies but they do not exceed 20%. However, we have established that these discrepancies can be reduced by adjusting the closing relations.

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