THE DUPUIS PARADOX AND MATHEMATICAL SIMULATION OF UNSTEADY FILTRATION IN A HOMOGENEOUS CLOSING DIKE

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The aim of this study is to determine a flow rate and a shape of a depression curve in conditions of filtration through a rectangular closing dike using aperiodic solutions of the Boussinesq limit problem. We have established that the formation of this curve and the seepage area (the final jump of continuity or interruption of the curve at the minimum pressure point) on the border of the downstream and porous medium, in the closing dike of finite length, occurs for a finite amount of time proportional to the square of the closing dike length. Therefore, in the short closing dike, a cut-out point does not have time to fall into the downstream during the time, it takes for the depression curve to touch the water level in the upstream. The continuous curve without seepage area always reaches the steady state in the semi-infinite closing dike for a finite amount of time.

Key words: filtration of subsoil water; porous medium; depression curve; seepage area

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Introduction

Seepage problems have major importance for power engineering and construction, with a wide range of applications extending from hydraulic engineering and melioration [1 – 6] to construction technologies (rapid pumping of groundwater from the pit, drainage of construction sites) [8 – 12]. Seepage theory as a section of fluid mechanics includes two branches: hydromechanical and hydraulic. These branches overlap in solving stationary problems [4, 6, 7, 13 – 18].

Hydromechanical seepage theory encompasses methods for solving mixed limit problems of theory of analytic functions in regions with free boundaries [1]. An important result of hydromechanical theory is Devison's conclusion on the interruption of the limiting boundary streamline in the location (the cut-out point) where it falls into the tailwater (the waters downstream from a hydraulic structure). The magnitude of this interruption is called the seepage area (δh in Fig. 1). The limiting streamline separating the saturated medium from the unsaturated one is called the depression curve. In other words, the depression curve is a line characterizing the level of groundwater in the plane of water movement.

Instead of density distributions (vector bundles) of the velocity fields, hydraulic seepage theory uses trivial (scalar) bundles obtained as cross-section-averaged distribution values (fluid flow rate and its average velocity instead of a velocity layer, flow instead of a streamline bundle). This theory is based on Dupuit equations for the mean rates of seepage and flow in steady-state problems [12]. This theory does not have the concepts of a discontinuous depression curve and a seepage area, since in this case the depression curve is smooth, i.e., differentiable at each point.

Implicit techniques are used to calculate the height of the seepage area at the boundaries of this region. For this reason, it is natural to use the stabilization problem for finding the solution of the unsteady Boussinesq problem in order to determine the shape of the depression curve and the fluid flow rate through the closing dike.

Depression curve

Dupuit's theory considers the seeping motion of water with an instantaneous mean velocity v, which, according to the Dupuit formula, is expressed as

$$v = kJ,$$

where k is the hydraulic conductivity (k = const

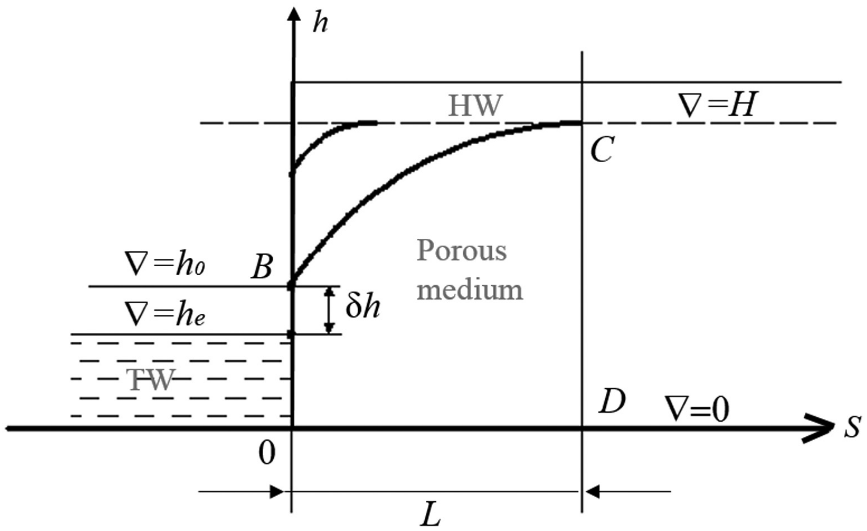

Fig. 1. Schematic of the problem statement:
TW, HW are the tailwater and the headwater; s, h are the coordinate axes (h is the seepage flow depth); ∇ is the elevation; H, h_0, h_e are the levels of the seepage flow; δh is the seepage area; L is the length of the homogeneous closing dike; BC is the depression curve; CD is the flow area; the dashed line indicates the headwater level

in the classical Dupuit theory); J is the hydraulic gradient.

The latter is determined as

$$J := -\partial h / \partial s,$$

where $h = h(t, s)$ is the depth of the seepage flow (t is the time, s is the horizontal coordinate).

The Dupuit paradox consists in the following. Let there be a homogeneous closing dike of length L carrying the seepage flow from $h = H$ on the right to $h = h_e$ on the left (see Fig. 1).

The depression curve (the free surface of the seepage flow), or the limiting streamline, must touch the horizontal straight line in the cross-section $s = L$, since the face $s = L$ (the CD segment in Fig. 1) is the constant pressure surface (flow area). However, according to the Dupuit formula, the velocity $v = 0$ and the seepage rate $q = 0$ are obtained in the cross section $s = L$. This paradox is ignored in the traditional theory, and the continuity condition is used to construct the depression curve. This condition is treated as a differential equation with respect to h:

$$\frac{dq}{ds} = \frac{d}{ds}(vh) = 0.$$

The depression curve $u = f(s)$ has the following form:

$$u = \sqrt{u_e^2 + \frac{s}{L}(1 - u_e^2)},$$
$$u := h / H \in (0 \le u_e := h_e / H, 1). \quad (1)$$

The quantity of the seepage flow is expressed as

$$q = k\,\frac{H^2 - h_e^2}{2L},$$

and the derivation of this formula does not depend on the type of the depression curve.

The reasoning is as follows.

The mean seepage rate over the length of the closing dike follows the expression

$$v = k(H - h_e)/L;$$

and therefore the flow rate value can be obtained by multiplying the mean velocity over the length of the closing dike by the mean depth h_m, calculated as the arithmetic mean of the depth limits:

$$h_m = (1/2)(H + h_e).$$

We immediately obtain the Dupuit formula for flow rate.

The Dupuit depression curve (see formula (1)) intersects the cross-section $s = 0$ at the elevation level $h = h_e$ ($u = u_e$) and the straight line $h = H$ ($u = 1$) in the cross-section $s = L$.

In other words, the following conditions are fulfilled on the Dupuit parabola (1):

a) no seepage area;

b) at the point $s = L$, $h = H$ (point C in Fig. 1), the Dupuit depression curve does not touch the straight line $h = H$ (the dashed line in Fig. 1), so this curve cannot be a streamline orthogonally intersecting the flow area $s = L$.

Both of these conditions have to be fulfilled for seepage flow to exist. This is precisely what the Dupuit paradox is.

Explaining the paradox by the singular character (singularity) of the point $s = L$, $h = H$ seems untenable.

In this study we propose an alternative scheme that explains the instantaneous configuration of the depression curve by the variable character of the instantaneous seepage rate over the length of the porous medium.

The explanation is as follows.

Let the seepage flow depth in a semi-enclosed body $s > 0$ filled with a porous medium be equal to H ($h = H$) before the initial time $t = 0$. At time $t = 0$, the fluid level in the tailwater $s < 0$ instantly drops from $h = H$ to $h = h_e$. The fluid starts to flow out of the porous medium, where $s > 0$, to the tailwater, where $s < 0$. The depression curve is deformed (Fig. 2). Its initial length is equal to zero and increases with time. At any time $t > 0$, the left end of the depression curve intersects the vertical slope $s = 0$ on the ordinate $h = h_0$ (the h_0 value lies in the region $h_e < h_0 \leq H$), where $dh_0/dt < 0$, and the right end of this curve touches the straight line $h = H$ at the cross-section $s = L > 0$, with $dL/dt > 0$. The left end of the depression curve falls down at a rate $c_0 = - dh_0/dt$, and the right end touches the straight line $h = H$ at a velocity $c_\lambda := dL/dt$.

In other words, the depression curve acts as a flexible (deformable) impermeable "piston" that squeezes the fluid out of the porous medium by turning counterclockwise around the point $s = L$, $h = H$. If the right end of the depression curve reaches the cross-section $s = L_\infty$ (L_∞ is the length of the closing dike) at the time $t = t_\lambda$, further movement of the right end stops, the depression curve stabilizes (the semi-infinite closing dike is cut off by the abscissa $s = L_\infty$). Two possible scenarios can happen at this time at the left end of the depression curve:

1. The value $h_0 > h_e$, and a final discontinuity is formed at the left end of the depression curve (seepage area);

2. $h_0 \to h_e + 0$, and the seepage area is small.

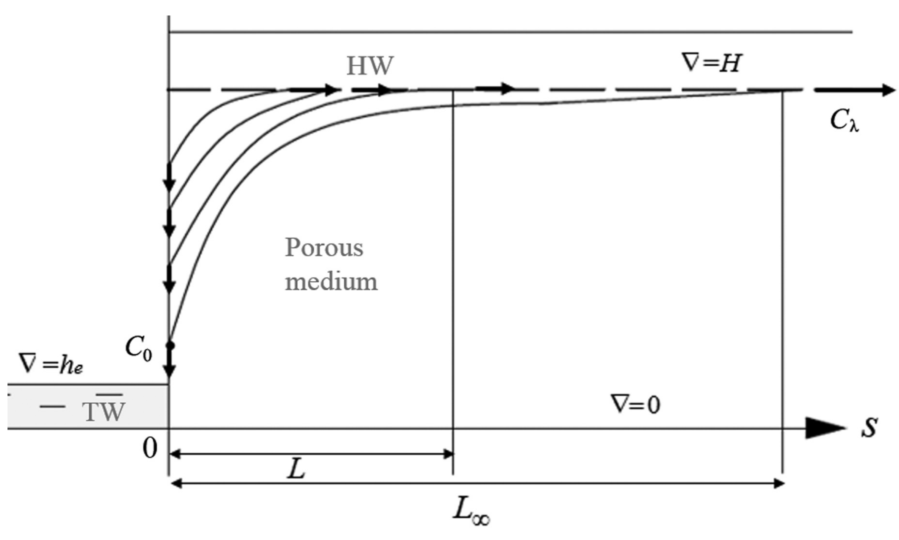

Fig. 2. The system under consideration, showing the deformation of the depression curve over time: its right end slides along the straight line $h = H$ from the tailwater (TW) to the highwater (HW), and the left slides down (L_∞ is the length of the closing dike); C_0, C_λ are the velocities of the left and right ends of the depression curve

The first case happens if the cut-out point does not have time to descend (fall) to the water level downstream. This is possible if the time T_Λ is small, the closing dike is short and the initial level difference $H - h_e$ is a finite value.

For the second case to happen, it is sufficient for the closing dike to be long and the initial difference of the levels $H - h_e$ to be small. Sufficient conditions for the existence of the seepage area correspond to the results of hydromechanical seepage theory.

Indeed, if other conditions are unchanged, the height of the seepage area is the greater, the shorter the closing dike. If the length of the closing dike is invariant, the height of the seepage area decreases together with the $H - h_e$ value, and the height of the seepage area $\Delta := h_0 - h_e$ for an infinitely thin closing dike is equal to $H - h_0 (\Delta = H - h_0)$, i.e., $h_0 = H$.

Thus, we suggest to regard seepage through the closing dike as unsteady motion in the porous medium bounded by a moving (descending and stretching) depression curve.

If the motion of the depression curve stops, steady seepage develops.

The goal of the study is to determine the flow rate and the instantaneous shape of the depression curve under unsteady seepage through a rectangular closing dike.

The solutions of the Boussinesq limit problem

The Boussinesq equation of unsteady seepage has to be integrated to calculate the integral characteristics of seepage (the flow rate, the height of the seepage area, and the shape of the depression curve). For plane one-dimensional flow, the continuity condition is fulfilled:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial s}(vh) = 0.$$

It is assumed that the Dupuit formula

$$v = -k \cdot \partial h / \partial s$$

is valid for unsteady motion, and the equality

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial s}\left(k \frac{\partial h}{\partial s}\right) \qquad (2)$$

is satisfied in this case.

Eq. (2) is considered in the region $0 < s < L \le L_\infty$, $t > 0$, and the boundary conditions have the form

$$h(0, s) - H = h(t, 0) - h_0 = \left(\frac{\partial h}{\partial s}\right)_{s=0} = 0. \qquad (3)$$

If we start using dimensionless coordinates

$$u := h / H, u_e < u_0 < u < 1,$$
$$\tau = kt / H > 0, \sigma = s / H, 0 < \sigma < \lambda \le \lambda_\infty,$$
$$\lambda := L / H, \lambda_\infty = L_\infty / H,$$

then we obtain, instead of Eq. (2), the equation

$$\frac{\partial u}{\partial \tau} = \frac{\partial}{\partial \sigma}\left(u \frac{\partial u}{\partial \sigma}\right), \qquad (2a)$$

and, instead of boundary conditions (3), boundary conditions of the form

$$u(0, \sigma) - 1 = u(\tau, 0) - u_0(\tau) = \left(\frac{\partial u}{\partial \sigma}\right)_{\sigma=\lambda} = 0. \qquad (3a)$$

Boundary problem (2a), (3a) can be complicated if we assume that the hydraulic conductivity is a function of the pressure head, for example,

$$k = k_0 \cdot f(u)/u$$

with an arbitrary function $f(u)$.

In this case, Eq. (2a) takes the form

$$\frac{\partial u}{\partial \tau} = \frac{\partial}{\partial \sigma}\left(f(u) \frac{\partial u}{\partial \sigma}\right). \qquad (2b)$$

It can be proved that boundary problem (2b), (3a) is equivalent to the following typical Crocco boundary problem:

$$2\varphi \frac{d^2\varphi}{du^2} + f(u) = 0, \varphi(u) := \int_u^1 \zeta dz, \zeta := \frac{\sigma}{2\sqrt{\tau}},$$

$$\mathfrak{D}(\varphi) = (u : u_0 < u < 1), \left(\frac{d\varphi}{du}\right)_{u=u_0} = \varphi(1) = 0.$$

The solutions of the typical Crocco boundary problem are known in terms of weak solutions (or weak approximations of solutions). For example, let $u_0 = 0$. Then we obtain that

$$\varphi^2(u) = \int_u^1 F(v)dv + \int_0^u \ln \frac{u}{v} \cdot F(v)dv -$$
$$-\int_0^1 \ln \frac{1}{v} F(v)dv,$$

where

$$F(u) := \int_0^u f(v)dv$$

is the antiderivative for $f(u)$.

In particular, if $f(u) = u$ (the classical case, the hydraulic conductivity is a constant), then it follows from the previous formula that

$$\varphi(u) = 1 / 3\sqrt{1-u^3},$$

and then the following expression is obtained for the instantaneous depression curve:

$$\zeta := -\frac{d\varphi}{du} = \frac{u^2}{2\sqrt{1-u^3}}. \quad (4)$$

Let $c = c(\tau,\sigma) := -\partial u / \partial\tau \geq 0$ be the descent rate of the depression curve and $c_0 = c(\tau,0) := -du_0 / d\tau \geq 0$ the descent (falling) rate of the left end of this curve (in fractions of the hydraulic conductivity k).

Let us set the descent rate distribution of the depression curve along its length in the form of a binomial:

$$c(\tau,\sigma;\lambda) = c_0(\tau)(1-\sigma/\lambda)^{\alpha}. \quad (4a)$$

Instead of Eq. (2a), taking into account expression (4a), we obtain the following equation:

$$\frac{d}{d\sigma}\left(u\frac{du}{d\sigma}\right) = -c_0(1-\sigma/\lambda)^{\alpha}, \quad (2c)$$

whose solution has the form

$$\theta(\tau,\sigma) := u\frac{\partial u}{\partial\sigma} = \frac{c_0\lambda}{\alpha+1}(1-\sigma/\lambda)^{\alpha+1}, \quad (5)$$

where $\theta = \theta(\tau,\sigma)$ is the dimensionless fluid flow (in fractions of kH^2/L).

The second integration leads to the expression

$$u^2/2 = u_0^2/2 + \frac{c_0\lambda^2}{(\alpha+1)(\alpha+2)} \times (1-(1-\sigma/\lambda)^{\alpha+2}). \quad (5a)$$

Consequently, the instantaneous depression curve (5a) differs from the Dupuit parabola and coincides with the Dupuit parabola (1) for $\alpha = -1$

For the flow rate θ, we obtain, due to the solution of (5), the following expression:

$$\theta(\tau,\sigma) = \frac{1-u_0^2}{2\lambda}(\alpha+2)(1-\sigma/\lambda)^{\alpha+1}. \quad (5b)$$

As a result, the flow rate varies from the maximum $\theta(\tau)$ value in the cross-section $\sigma = 0$ which is expressed as

$$\theta_0(\tau) := \theta(\tau,0) = \frac{(\alpha+2)(1-u_0^2)}{2\lambda},$$

to zero in the cross-section $\sigma = \lambda$.

The mean flow rate $\theta_m(\tau)$ over the length of the closing dike, due to expression (5b), has the form

$$\theta_m(\tau) = \frac{1-u_0^2}{2\lambda} \quad (6)$$

and coincides with the value of the Dupuit flow rate, if $u_0 = u_e$ and $\lambda = \Lambda$.

Thus, we have established that the mean flow rate does not depend on the value of the parameter α and formally coincides with the Dupuit flow rate.

The following technique is important for further calculations. Let us introduce the thickness of the seepage boundary layer as a thickness of a porous medium layer adjacent to the cross-section $\sigma = 0 (\zeta = 0)$, where a finite change in the pressure is localized, namely, let δ be the thickness of the seepage boundary layer:

$$\delta = \frac{\lambda}{2\sqrt{\tau}}.$$

Therefore, by definition, the thickness of the seepage boundary layer is determined as follows:

$$\forall\zeta > \delta, \exists\varepsilon(\zeta) > 0 \Rightarrow 1-\varepsilon < u < 1.$$

Fig. 3 shows the thickness of the seepage boundary layer for the case $u_0 > 0$.

So, according to the definition and using Crocco's equation, we obtain:

$$\delta := \int_0^{\infty}(1-u)d\zeta = (1-u_0)\lambda(\tau) =$$

$$= \int_0^1 (1-u)(-\varphi''(u))du = \frac{1}{2}\int_0^1 \frac{f(u)(1-u)}{\varphi(u)}du.$$

It can be proved that the thickness of the seepage boundary layer $\delta < \infty$ for any summa-

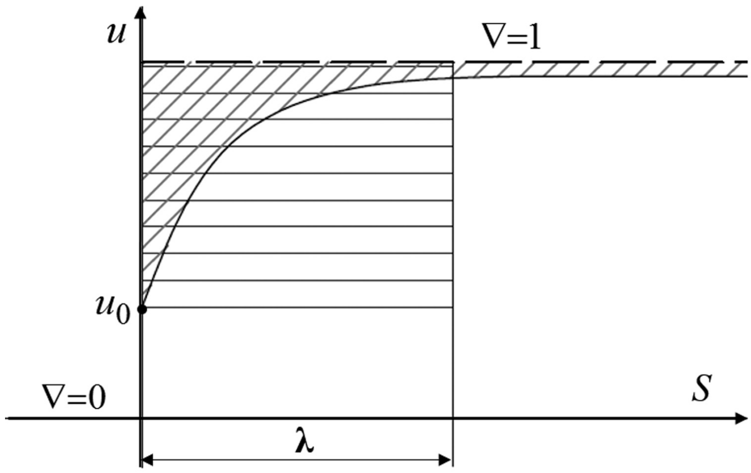

Fig. 3. Thickness of the seepage boundary layer λ (the shaded area between the dashed line and the depression curve is equal to the area of the rectangle with the sides $1 - u_0$ and λ); $u_0 = h_0/H$, $\lambda = L/H$ is the dimensionless length of the closing dike

ble and bounded function *f*. Hence, the dimensionless length of the closing dike

$$\lambda = 2\delta\sqrt{\tau} = a\sqrt{\tau}, a = O(1),$$

and then

$$\Lambda = a\sqrt{\mathrm{T}_\Lambda}.$$

The constant a is bounded, i.e., $a = O(1)$, therefore, due to expression (6), the flow rate takes the form

$$\theta_m(\tau) = \frac{1-u_0^2}{2a\sqrt{\tau}} = $$

$$= \frac{1-(1-c_0\tau)^2}{2a\sqrt{\tau}} \underset{\tau\to\mathrm{T}_\Lambda}{\to} \frac{2c_0\mathrm{T}_\Lambda - c_0^2\mathrm{T}_\Lambda^2}{2\Lambda}.$$

If $c_0 = 1/T_\Lambda$, this expression reaches maximum in the form

$$\theta_m(\mathrm{T}_\Lambda) = \frac{1}{2\Lambda}, \qquad (6a)$$

which coincides with the Dupuit flow rate.

Therefore, we can draw the following conclusions:

the mean instantaneous flow rate $\theta_m(\tau)$ over the length of the closing dike does not depend on the parameter α, i.e., on the instantaneous shape of the depression curve;

the limit expression (i.e., with $\tau \to T_\Lambda - 0$) for the mean flow rate over the length of the closing dike coincides with the Dupuit rate;

The Dupuit formula for flow rate is applicable only under conditions of steady-state seepage.

The main integral relations

The Dupuit equation (2a) implies an integral identity expressing the flow rate balance (the entire fluid forced out from the closing dike by the descending depression curve flows through the cross-section $s = 0$):

$$\frac{d}{d\tau}\int_0^\lambda (1-u)d\sigma = \theta_0(\tau) = \frac{c_0\lambda}{\alpha+1}. \qquad (7)$$

Let us approximately suppose, somewhat overestimating the $\lambda(\tau)$ value, that

$$\int_0^\infty (1-u)d\sigma \doteq \frac{1-u_0(\tau)}{2}\lambda(\tau).$$

Then we obtain a simple equation for λ^2:

$$\frac{d\lambda^2}{d\tau} = n(1+u_0), n(\alpha) = (1-\alpha)(2+\alpha) > 0, \quad -2 < \alpha < 1, \qquad (8)$$

where $\lambda(0) = 0$.

The value of the parameter $n(\alpha)$ varies from zero at $\alpha = 1$, -2 to the maximum value $n = 9/4$ at $\alpha = -1/2$; the arithmetic mean of $n(\alpha)$ is 3/2.

Then solution (8) takes the following form:

$$\lambda^2(\tau) = n(\alpha)\left(2\tau - \int_0^{\tau} c_0(\omega)(\tau-\omega)d\omega\right). \quad (9)$$

Let $\lambda = \Lambda$, then the quantity τ reaches the value $\tau = \mathrm{T}_\Lambda$. Due to expression (9), the parameters $\Lambda, \mathrm{T}_\Lambda$ and the velocity $c_0(\tau)$ are related by the following condition:

$$\Lambda^2 = n(\alpha)\left(2\mathrm{T}_\Lambda - \int_0^{\mathrm{T}_\Lambda} c_0(\tau)(T_\Lambda - \tau)d\tau\right); \quad (9a)$$

and if $u_0 = u_e$, then $\tau = \mathrm{T}_\Delta$.

Consequently,

$$1 - u_e - \int_0^{\mathrm{T}_\Delta} c_0(\tau)d\tau = 0. \quad (9b)$$

Obviously, $c_0 \le 1$; then we obtain the following from formulae (9a), (9b), respectively:

$$\Lambda^2 / n = 2\mathrm{T}_\Lambda - 1/2\mathrm{T}_\Lambda^2, \mathrm{T}_\Lambda = 2 - \sqrt{4 - \frac{2\Lambda^2}{n}},$$

$$\mathrm{T}_\Lambda = 2\left(1 - \sqrt{1 - \frac{\Lambda^2}{2n}}\right) \doteq \frac{\Lambda^2}{2n}, \quad (10)$$

$$\mathrm{T}_\Delta = 1 - u_e \le 1,$$

and then, due to expression (6a), the limit value of the mean flow rate over the length of the closing dike takes the form

$$\lim_{\tau \to \mathrm{T}_\Lambda} \theta_m = \frac{2}{\Lambda} - \sqrt{\frac{4}{\Lambda^2} - \frac{2}{n}} \le \frac{\Lambda}{2n} = \frac{1 - u_0^2(\mathrm{T}_\Lambda)}{2\Lambda}.$$

For the final expression, the ordinates of the left end of the depression curve are:

$$u_0(\mathrm{T}_\Lambda) = \sqrt{1 - \Lambda^2 / n}. \quad (6b)$$

Formulae (10) make sense if the inequality

$$\Lambda \le \sqrt{2n} \le \sqrt{9/2} = 2.121$$

holds true.

Otherwise, the closing dike is assumed to be long, i.e., $\mathrm{T}_\Lambda = \infty$, and $u_0 \to u_e + 0$.

If $\Lambda \ll 1$, then

$$\mathrm{T}_\Lambda = \frac{\Lambda^2}{2n} < \mathrm{T}_\Delta = 1 - u_e.$$

Therefore, the right end of the depression curve in a short closing dike reaches the headwater level faster than the cut-out point of the depression curve falls to the tailwater level.

Let $\tau = \mathrm{T}_\Lambda = \Lambda^2/(2n)$, and then the following expression holds true:

$$u_0(\mathrm{T}_\Lambda) = 1 - \Lambda^2/(2n),$$

$$\Delta(\mathrm{T}_\Lambda) := u_0(\mathrm{T}_\Lambda) - u_e = 1 - u_e - \Lambda^2/(2n).$$

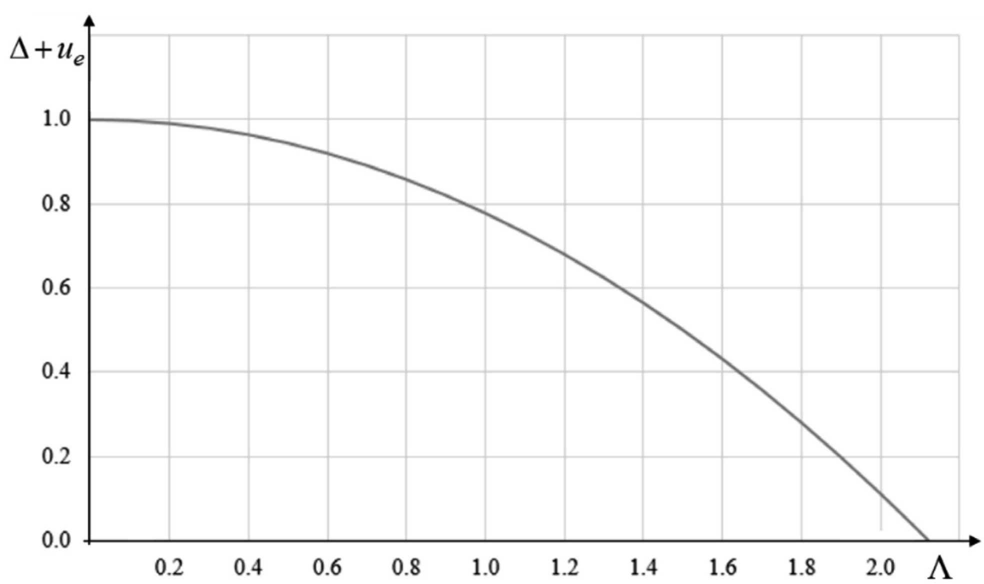

Fig. 4. Dependence of the height of the seepage area Δ on the length of the closing dike $\Lambda = L_\infty / H$ with a fixed level of u_e

Finally, we obtain the formula

$$\Delta = 1 - u_e - \Lambda^2 / (2n) = 1 - u_e - 2 / 9\Lambda^2.$$

Let $\Lambda \to +0$, then $\Delta = 1 - u_e$. The graph of function $\Delta = \Delta(\Lambda)$ is shown in Fig. 4. We assume that $\Delta = 0$ for the value $\Lambda > (9/2)^{1/2}$.

We should note that it is possible to improve the estimates. Namely, since solution (9) is valid, we have:

$$\lambda^2 = n(2\tau - \tau^2 / 2). \qquad (11)$$

Let $\tau = 2 = \mathrm{T}_\Lambda$. Then the length λ of the closing dike reaches its maximum value $\lambda = \Lambda$, while $\Lambda^2 = 2n = 9/2$. This Λ estimate coincides with the previous one. If we formally set $\mathrm{T}_\Lambda = 2$ in formula (11), then we obtain that $\Lambda^2 = 4n = 9$.

So, the closing dike is considered to be long if

$$9/2 < \Lambda^2 < 9, \text{ i.e., } 2{,}12 < \Lambda < 3{,}00,$$

and the mean value of $\Lambda = 2{,}52$. These estimates are close to the ones obtained in hydromechanical theory, where with $\Lambda \approx 2.7, ..., 2.8$, the seepage area disappears for nearly all values of u_e [12, 16].

Conclusions

The following conclusions can be drawn based on determining the flow rate and the shape of the depression curve under seepage through a rectangular closing dike:

the height of the seepage area Δ is uniquely determined by the length of the closing dike. The height of the seepage area in a short closing dike, when $\Delta = 1 - u_e$, is determined only by the upstream water level;

the seepage area stabilizes during the time $\mathrm{T}_\Lambda = O(\Lambda^2)$, so in a short closing dike this time is less than the time T_Δ that it takes for the cut-out point to fall downstream, i.e., $\mathrm{T}_\Lambda < \mathrm{T}_\Delta$;

the cut-out point in a long closing dike succeeds in falling downstream in time T_Δ shorter than the time T_Λ that it takes for the right end of the depression to touch the top-water;

the height of the seepage area monotonically decreases during stabilization from the value $1 - u_e$ at the time $\tau = 0$ to the value $\Delta(\mathrm{T}_\Lambda) \geq 0$.

The proposed alternative scheme explaining the instantaneous configuration of the depression curve by the variable nature of unsteady seepage flow along the porous medium is fully justified and allows to obtain new important results, in particular, the seepage area, the instantaneous local and mean seepage flow.

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